MATH 1314

Section 2.5

Other Techniques for Solving Equations

Solving by Factoring:

Factoring can be used to solve many types of equations. Always begin by Factoring Completely. Then, set each factor equal to zero.

Find all solutions to

$x^3 + 3x^2 + 2x + 6 = 0$

$x^2(x+3) + 2(x+3) = 0$

$(x+3)(x^2+2) = 0$

$x+3 = 0$

$-3 \quad -3$

$x = -3$

$x^2 + 2 = 0$

$-2 \quad -2$

$\sqrt{x^2} = \sqrt{-2}$

$x = \pm\sqrt{2}\,i$

$\{-3, -\sqrt{2}\,i, +\sqrt{2}\,i\}$

Find all the solutions of $x^3 = x$

$x^3 = x$

$-x \quad -x$

$x^3 - x = 0$

$x(x^2 - 1) = 0$

$x(x+1)(x-1) = 0$

$x = 0$ $\quad x+1=0$ $\quad x+1=0$

$-1 \quad -1 \quad\quad +1 \quad +1$

$x = -1 \quad\quad x = 1$

$\{-1, 0, 1\}$

Common mistake:

$\frac{x^3}{x} = \frac{x}{x}$

$\sqrt{x^2} = \sqrt{1}$

$x = \pm 1$

We lost the $x=0$ answer!

Reason: You cannot divide (or cancel) anything that may equal zero.

Equations Involving Fractions:

Option 1: Rewrite all fractions so that they have the same denominator, then drop all denominators from the equation.

or

Option 2: Multiply the entire equation by the LCD to clear the fractions.

Then: Solve normally.

Be advised: If your answer makes any of the original fractions undefined, it must be rejected!!

$$\frac{4}{x-1}+\frac{3}{x}=\frac{3}{1}$$

LCD: $x(x-1)$

cross cancel: cancel any numerator with any denominator when multiplying.

$$\frac{4}{x-1}\cdot\frac{x(x-1)}{1}+\frac{3}{x}\cdot\frac{x(x-1)}{1}=\frac{3}{1}\cdot\frac{x(x-1)}{1}$$

$$4x+3x-3 = 3x^2-3x$$

$$7x-3 = 3x^2-3x$$

$$-7x+3 \qquad -7x+3$$

$$0 = 3x^2-10x+3$$

$$3x^2-10x+3=0$$

$3x^2 - 10x + 3 = 0$

$(3)(3) = 9$

$-9, -1$

$(3x^2 - 9x)(-x + 3) = 0$

$3x(x-3) - 1(x-3) = 0$

$(x-3)(3x-1) = 0$

$x - 3 = 0$

$+3 \quad +3$

$x = 3$

$3x + 1 = 0$

$-1 \quad -1$

$\frac{3x}{3} = \frac{-1}{3}$

$x = -\frac{1}{3}$

$\{-\frac{1}{3}, 3\}$

Equations involving radicals:

If an equation involves a square root (also called a radical), you must isolate the radical, square both sides, and solve the remaining equation. Be certain to check your answers!

Find all solutions to $\sqrt{x+8}-2=x$

$+2 \quad +2$

$(\sqrt{x+8})^2=(x+2)^2$

$x+8=(x+2)(x+2)$

$x+8=x^2+2x+2x+4$

$x+8=x^2+4x+4$

$-x-8 \quad -x-8$

$0=x^2+3x-4$

$0=(x+4)(x-1)$

$x+4=0$ reject $x=-4$ $\quad x-1=0$ $\boxed{x=1}$

Check $x=-4$

$\sqrt{-4+8}-2=-4$

$\sqrt{4}-2=-4$

$2-2=-4$

$0 \neq -4$

Check $x=1$

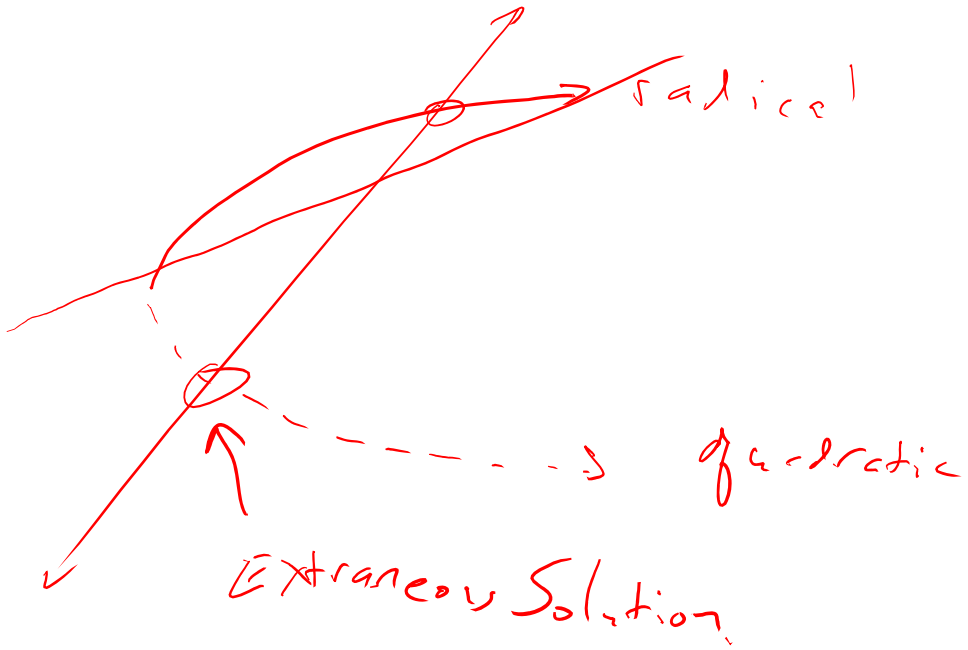
$\sqrt{1+8}-2=1$

$\sqrt{9}-2=1$

$3-2=1$

$1=1$ ✓

Extraneous Solutions: In a radical solution, you may "create" additional answers that are not correct. These must be rejected!



Find all solutions to $\sqrt{3x+1}-1=x$

$+1 \quad +1$

$(\sqrt{3x+1})^2=(x+1)^2$

$3x+1=(x+1)(x+1)$

$3x+1=x^2+x+x+1$

$3x+1=x^2+2x+1$

$-3x-1 \quad -3x-1$

$0=x^2-x$

$0=x(x-1)$

$x=0 \qquad x-1=0$

$x=1$

Check $x=0$

$\sqrt{3(0)+1}-1=0$

$\sqrt{1}-1=0$

$1-1=0$

$0=0$ ✓

Check $x=1$

$\sqrt{3(1)+1}-1=1$

$\sqrt{4}-1=1$

$2-1=1$

$1=1$ ✓

$\{0,1\}$

Popper # 5

1. Solve the following: $x^3 = 9x$

   a. 0, 3 b. 0, 9, -9 c. 0,-3, 3 d. -3, 3

$x^3 - 9x = 0$

$x(x^2 - 9) = 0$

$x = 0$ or $x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

2. Solve the following: $x + \sqrt{x+1} = 5$

   a. 8, 3 b. 3 c. 8 d. No Answer

$(\sqrt{x+1})^2 = (-x+5)^2$

$x + 1 = (-x+5)(-x+5)$

$x + 1 = x^2 - 5x - 5x + 25$

$x + 1 = x^2 - 10x + 25$

$-x \quad -1$

$0 = x^2 - 11x + 24$

$0 = (x-3)(x-8)$

$x - 3 = 0$ $\quad x - 8 = 0$

$x = 3$ $\quad x = 8$ reject

3. Solve the following: $\frac{8}{x+1} + \frac{3}{x} = 3$

   a. 3 b. $-1/3$ c. -1 d. 3, $-1/3$

Check $x = 3$

$3 + \sqrt{3+1} = 5$

$3 + \sqrt{4} = 5$

$3 + 2 = 5$ ✓

Check $x = 8$

$8 + \sqrt{8+1} = 5$

$8 + \sqrt{9} = 5 \rightarrow 8 + 3 \neq 5$

$$\frac{8}{x+1}+\frac{3}{x}=\frac{3}{1} \qquad LCD: x(x+1)$$

$$\frac{8}{x+1}\cdot\frac{x(x+1)}{1}+\frac{3}{x}\cdot\frac{x(x+1)}{1}=\frac{3}{1}\cdot\frac{x(x+1)}{1}$$

$$8x+3x+3=3x^2+3x$$

$$11x+3=3x^2+3x$$

$$-11x \quad -3$$

$$0=3x^2-8x-3$$

$$0=(3x^2-9x)+(x-3)$$

$$0=3x(x-3)+1(x-3)$$

$$0=(x-3)(3x+1)$$

$$x-3=0$$

$$x=3$$

$$3x+1=0$$

$$-x \quad -1$$

$$\frac{3x}{3}=\frac{-1}{3}$$

$$x=-\tfrac{1}{3}$$

$$\{-\tfrac{1}{3}, 3\}$$