MATH 1314

Section 2.6

In this lesson, you will learn to solve linear inequalities, graph the solution on a real number line and state the solution using interval notation.

You'll solve 4 different types of linear inequalities, involving these four symbols:

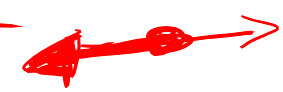
$<$ less than (the quantity to the left is less than the quantity to the right)

$\leq$ less than or equal to (the quantity to the left is less than or equal to the quantity to the right)

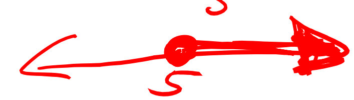
$>$ greater than (the quantity to the left is greater than the quantity to the right)

$\geq$ greater than or equal to (the quantity to the left is greater than or equal to the quantity to the right)

$x<5$ Every value less than, but not including 5 

$x\leq 5$ Every value less than, and including 5 

$x>5$ Every value greater than, but not inc. 5 

$x\geq 5$ Every value 5 or greater 

To solve an inequality containing a variable, find all values of the variable that make the inequality true. In solving linear inequalities, isolate the variable on one side of the inequality symbol by using the following rules.

1. If A < B then A + C < B + C.
2. If A < B then A – C < B – C.
3. Let C > 0. If A < B then AC < BC.
4. Let C < 0. If A < B then AC > BC.

$$\begin{aligned} x + \cancel{2} &< 3 \\ -\cancel{2} &\quad -2 \\ \hline x &< 1 \end{aligned}$$

$$\begin{aligned} x + \cancel{1} &\geq 3 \\ -\cancel{1} &\quad -1 \\ \hline x &\geq 2 \end{aligned}$$

$$\frac{\cancel{2}x}{\cancel{2}} > \frac{10}{2}$$
$$x > 5$$

$$\frac{-\cancel{2}x}{-\cancel{2}} > \frac{10}{-2}$$
$$x < -5$$

* Multiplying or Dividing both sides by a negative → switch the inequality

Reason $(-3)(2) < (5)(-3)$

$-6 > -15$

Example 1: Solve each of the following inequalities.

a. $2(7 - 4x) \geq -13 + 8x$

$$14 - 8x \geq -13 + 8x$$
$$+8x \qquad +8x$$
$$14 \geq -13 + 16x$$
$$+13 \qquad +13$$
$$27 \geq 16x$$

$$\frac{27}{16} \geq \frac{16x}{16}$$
$$27/16 \geq x \rightarrow \boxed{x \leq 27/16}$$

b. $-3 \leq 2x + 1$

$$-1 \qquad -1$$
$$\frac{-4}{2} \leq \frac{2x}{2}$$
$$-2 \leq x \quad (-2 \text{ less than } x)$$
$$x \geq -2 \quad (x \text{ greater than } -2)$$

c. $2x + 1 < 7$

$$-1 \quad -1$$
$$\frac{2x}{2} < \frac{6}{2}$$
$$x < 3$$

Next, you'll need to be able to work with interval notation. An interval is a set of real numbers. It can be a line segment, a ray or the entire number line. If it is a line segment, it can include one or both endpoints. If it is a ray it may or may not include the endpoint. We note intervals using brackets, parentheses or a combination.

The interval [*a*, *b*] is the line segment from point *a* to point *b*, including both endpoints. This corresponds to the inequality $a \leq x \leq b$.

$-2 \leq x \leq 5 \rightarrow [-2, 5]$

The interval [*a*, *b*) is the line segment from point *a* to point *b*, including point *a* but not point *b*. This corresponds to the inequality $a \leq x < b$.

$-2 \leq x < 5 \rightarrow [-2, 5)$

The interval [*a*, ∞)corresponds to $x \geq a$ and is a ray beginning at (and including) point *a* and including all real numbers to the right of the point.

$x \geq 3 \rightarrow [3, \infty)$ ± infinity always gets round parenthesis

The interval (- ∞, *a*) corresponds to $x < a$ and is a ray beginning at (and not including) point *a* and including all real numbers to the left of the point.

$x < 5$ $(-\infty, 5)$

(smaller, larger]

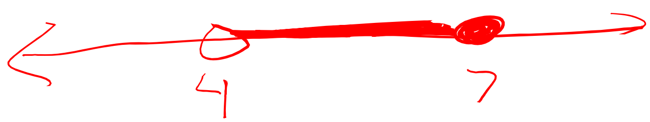
(-∞ , Number] [Number, ∞)

Example 2: Write each of these inequalities using interval notation.

a. $x \geq -2$ $[-2, \infty)$

b. $x < 3$ $(-\infty, 3)$

c. $4 < x \leq 7$



$(4, 7]$

Common mistakes

$(2, -5)$

larger, smaller

$(\infty, 5)$

larger, smaller

$(2, \infty]$ ← Round on infinity only

Popper #6

Example 3: Solve each inequality. Graph each solution on the real number line. Write your solutions using interval notation.

1. $\cancel{7}x > 35$ $\rightarrow$ $x > 5$

$\frac{\cancel{7}x}{7} > \frac{35}{7}$

a. $(-\infty, 5)$ b. $(5, \infty)$ c. $(-\infty, 5]$ d. $[5, \infty)$

2. $-\cancel{4}x \leq 48$ $\rightarrow$ $x \geq -12$

$\frac{-\cancel{4}x}{-4} \leq \frac{48}{-4}$

a. $(-\infty, -12)$ b. $(-12, \infty)$ c. $(-\infty, -12]$ d. $[-12, \infty)$

3. $5x - 4 > 2x + 7$ $\rightarrow$ $3x - \cancel{4} > 7$ $\rightarrow$ $\frac{\cancel{3}x}{\cancel{3}} > \frac{11}{3}$ $\rightarrow$ $x > 11/3$

$-2x \quad -2x \qquad\qquad +\cancel{4} \quad +4$

a. (-∞, -11/3) b. (3/11, ∞)

c. (-∞, 11/3) d. (11/3, ∞)

4. $-2(x - 5) < 3(4x - 7) + 12$

$-2x + 10 < 12x - 21 + 12$ $\rightarrow$ $\cancel{-2x} + 10 < 12x - 9$

$\cancel{+2x} \qquad\qquad +2x$

$10 < 14x \cancel{- 9}$

$+9 \qquad\qquad \cancel{+9}$

$\frac{19}{14} < \frac{\cancel{14}x}{\cancel{14}}$ $\rightarrow$ $\frac{19}{14} < x$

a. (-∞, -14/19) b. (-19/14, ∞)

c. (-∞, 19/14) d. (19/14, ∞)

$x > 19/14$

You can also solve some compound inequalities. All of the same rules apply to these problems

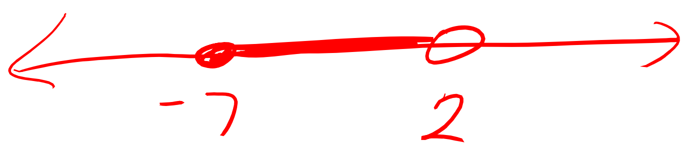
Solve each inequality. Write your solutions using interval notation. Graph each solution on the real number line.

Example 4: $-2 \leq x + 5 < 7$

$$-5 \quad -5 \quad -5$$

$$-7 \leq x < 2$$

$$[-7, 2)$$

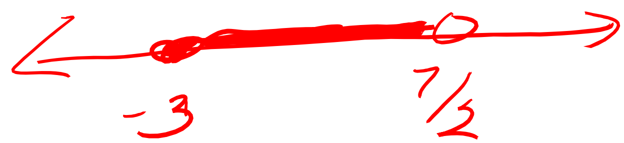


Example 5: $-4 < 3 - 2x \le 9$

$$\begin{array}{ccccc} -4 & < & 3 - 2x & \le & 9 \\ -3 & & -3 & & -3 \\ \hline -7 & < & -2x & \le & 6 \\ \hline -2 & & -2 & & -2 \\ \hline \frac{7}{2} & > & x & \ge & -3 \end{array}$$

$$-3 \le x < 7/2$$

$$[-3, 7/2)$$



Example 6: $-\frac{7}{6} < \frac{-3(-x-1)}{8} < \frac{7}{3}$

$24 \quad 4 \qquad 24 \quad 3 \qquad 24 \quad 8 \qquad LCD: 24$

$$-28 < -9(-x-1) < 56$$

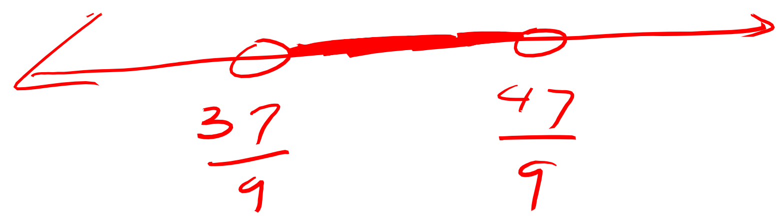
$$-28 < 9x + \cancel{9} < 56$$

$$-9 \qquad\qquad -9 \qquad -9$$

$$\frac{-37}{9} < \frac{\cancel{9}x}{\cancel{9}} < \frac{47}{9}$$

$$\frac{-37}{9} < x < \frac{47}{9}$$

$$\left(\frac{-37}{9}, \frac{47}{9}\right)$$



Example 7: $2\left(35\right) < 2\left(5x\right) - 2\left(\frac{5(x-7)}{\cancel{2}}\right) \le 2\left(70\right)$

$70 < 10x - 5(x-7) \le 140$

$70 < 10x - 5x + 35 \le 140$

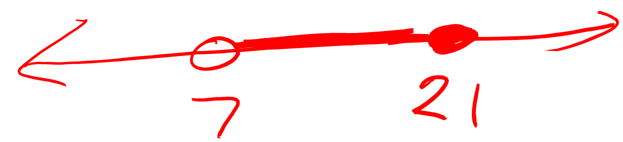
$70 < 5x \cancel{+35} \le 140$

$\underline{-35 \qquad\qquad \cancel{-35} \qquad\qquad -35}$

$\dfrac{35}{5} < \dfrac{\cancel{5}x}{\cancel{5}} \le \dfrac{105}{5}$

$7 < x \le 21$

$(7, 21]$



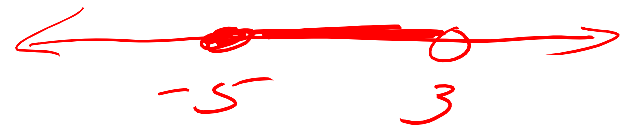
$$3\left(10\right) \le \left(\frac{2x+40}{3}\right)3 < \left(\frac{46}{3}\right)3$$

$$30 \le 2x+40 < 46$$

$$-40 \quad -40 \quad -40$$

$$\frac{-10}{2} \le \frac{2x}{2} < \frac{6}{2} \longrightarrow -5 \le x < 3$$

$[-5, 3)$



$$-21 \le -3x + 6 < 6$$

$$-6 \quad -6 \quad -6$$

$$\frac{-27}{-3} \le \frac{-3x}{-3} < \frac{0}{-3}$$

$$9 \ge x > 0 \longrightarrow 0 < x \le 9$$

$(0, 9]$

$$\frac{1}{2} < \frac{3(x+3)}{2} - 1 < 5$$

$+\frac{2}{2} \qquad +1 \qquad +1$

$$2 \times \left(\frac{3}{2}\right) < \left(\frac{3(x+3)}{2}\right) \times 2 < (6) \times 2$$

$$3 < 3(x+3) < 12$$

$$3 < 3x + 9 < 12$$

$-9 \qquad -9 \qquad -9$

$$\frac{-6}{3} < \frac{3x}{3} < \frac{3}{3}$$

$$-2 < x < 1 \rightarrow (-2, 1)$$

$$6 \le \frac{x}{3} + 5 < 9$$

$-5 \qquad -5 \qquad -5$

$$3 \cdot (1) \le \left(\frac{x}{3}\right) \times 3 < (4) \times 3$$

$$3 \le x < 12 \rightarrow [3, 12)$$