MATH 1314

Section 2.6

In this lesson, you will learn to solve linear inequalities, graph the solution on a real number line and state the solution using interval notation.

You'll solve 4 different types of linear inequalities, involving these four symbols:

<   less than (the quantity to the left is less than the quantity to the right)

≤   less than or equal to (the quantity to the left is less than or equal to the quantity to the right)

>   greater than (the quantity to the left is greater than the quantity to the right)

≥   greater than or equal to (the quantity to the left is greater than or equal to the quantity to the right)

To solve an inequality containing a variable, find all values of the variable that make the inequality true. In solving linear inequalities, isolate the variable on one side of the inequality symbol by using the following rules.

1. If $A < B$ then $A + C < B + C$.
2. If $A < B$ then $A - C < B - C$.
3. Let $C > 0$. If $A < B$ then $AC < BC$.
4. Let $C < 0$. If $A < B$ then $AC > BC$.

Example 1: Solve each of the following inequalities.

a. $2(7 - 4x) \geq -13 + 8x$

c. $2x + 1 < 7$

b. $-3 \leq 2x + 1$

Next, you'll need to be able to work with interval notation. An interval is a set of real numbers. It can be a line segment, a ray or the entire number line. If it is a line segment, it can include one or both endpoints. If it is a ray it may or may not include the endpoint. We note intervals using brackets, parentheses or a combination.

The interval $[a, b]$ is the line segment from point a to point b, including both endpoints. This corresponds to the inequality $a \leq x \leq b$.

The interval $[a, b)$ is the line segment from point a to point b, including point a but not point b. This corresponds to the inequality $a \leq x < b$.

The interval $[a, \infty)$ corresponds to $x \geq a$ and is a ray beginning at (and including) point a and including all real numbers to the right of the point.

The interval $(-\infty, a)$ corresponds to $x < a$ and is a ray beginning at (and not including) point a and including all real numbers to the left of the point.

Example 2: Write each of these inequalities using interval notation.

a. $x \geq -2$

b. $x < 3$

c. $4 < x \leq 7$

Example 3: Solve each inequality. Graph each solution on the real number line. Write your solutions using interval notation.

a. $7x > 35$

b. $-4x \leq 48$

c. $5x - 4 > 2x + 7$

d. $-2(x - 5) < 3(4x - 7) + 12$

You can also solve some compound inequalities. All of the same rules apply to these problems

Solve each inequality. Write your solutions using interval notation. Graph each solution on the real number line.

Example 4: $-2 \leq x + 5 < 7$

Example 5: $-4 < 3 - 2x \leq 9$

Example 6: $-\dfrac{7}{6} < \dfrac{-3(-x-1)}{8} < \dfrac{7}{3}$

Example 7:

$$35 < 5x - \frac{5(x-7)}{2} \leq 70$$

$$10 \leq \frac{2x+40}{3} < \frac{46}{3}$$

-21 ≤ -3x +6 < 6

$$\frac{1}{2} < \frac{3(x+3)}{2} - 1 < 5$$

$$6 \leq \frac{x}{3} + 5 < 9$$