MATH 1314

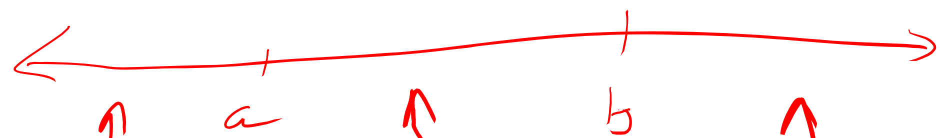
Section 2.7

Non-Linear Inequalities

In this section, we will examine how to solve inequalities involving (1) quadratic functions, and (2) rational functions.

In these examples, we will use a method known as the Number Line Test.

solution to equation is $x = a$, $x = b$



Less than a

between a & b

Greater than b.

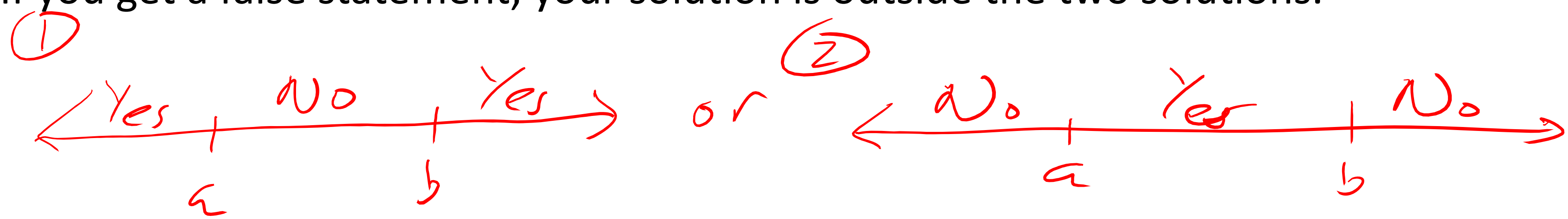
Test 1 x-value in each region.

Only values you cannot use: a, b

Solving a Quadratic Inequality

Quadratic Only:
Test 1 region only,
then alternate

- Rewrite the inequality as an equation (with an equal sign).
- Solve as done before.
- Test an x-value between the two solutions by plugging into the original inequality.
  - If you get a true statement, your solution is between the two solutions.
  - If you get a false statement, your solution is outside the two solutions.



Two ways a quadratic inequality will resolve

Try this: $x^2 - x > 6$ $\rightarrow$ $x^2 - x = 6$

Less Than without endpoints.

Answer will Not include endpoints.

$$x^2 - x = 6$$
$$\underline{\;-6 \qquad -6\;}$$
$$x^2 - x - 6 = 0$$

$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$

$$x - 3 = 0 \qquad x + 2 = 0$$
$$\underline{\;+3 \quad +3\;} \qquad \underline{\;-2 \quad -2\;}$$
$$x = 3 \qquad x = -2$$

Test Point: $x = 0$

$$0^2 - 0 > 6$$
$$0 > 6$$
False

$$(-\infty, -2) \cup (3, \infty)$$

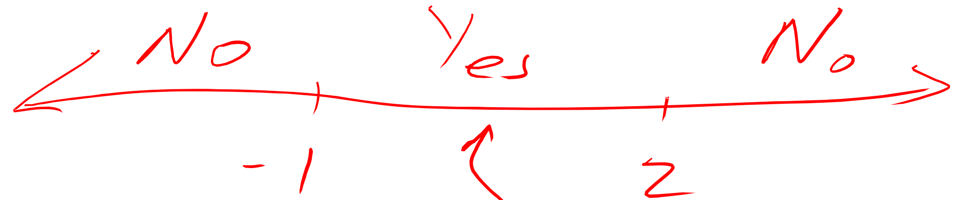
union
(means "or")

Now, try this: $x^2 - x - 2 \le 0$ $\longrightarrow$ $x^2 - x - 2 = 0$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$\begin{array}{r} x - 2 = 0 \\ +2 \quad +2 \\ \hline x = 2 \end{array} \qquad \begin{array}{r} x + 1 = 0 \\ -1 \quad -1 \\ \hline x = -1 \end{array}$$



Test Point: $x = 0$

$$0^2 - 0 - 2 \le 0$$

$$-2 \le 0$$

True

$$[-1, 2]$$

Solving a Rational Inequality (Fractional Inequalities)

- Set the denominator equal to zero and solve.
- Set the numerator equal to zero and solve.
- Plot these points on a number line (denominator is always open dot).
- Use test points between these values to determine the solution set.

↳ Must test every interval.

Note: If $\leq$ or $\geq$ is in the original inequality, any values from denominator must have (,) Try this: $\frac{x+3}{x-1} > 0$ → Greater than 0 (or Pos) is a "Yes"

Num:

$x+3=0$

$-3 \quad -3$

$x=-3$

Den:

$x-1=0$

$+1 \quad +1$

$x=1$

① Test $x=-5$: $\frac{-5+3}{-5-1} = \frac{(\text{Neg})}{(\text{Neg})} = \text{Pos} > 0$ Yes

② Test $x=0$: $\frac{0+3}{0-1} = \frac{(\text{Pos})}{(\text{Neg})} = \text{Neg} < 0$ No

③ Test $x=2$: $\frac{2+3}{2-1} = \frac{(\text{Pos})}{(\text{Pos})} = \text{Pos} > 0$ Yes

$$\boxed{(-\infty,-3)\cup(1,\infty)}$$

Now try this: $\dfrac{x-5}{x+8} \geq 0$ → Positive → Yes

Num: → square bracket

$x - 5 = 0$

$+5 \quad +5$

$x = 5$

Den: → round parenthesis

$x + 8 = 0$

$-8 \quad -8$

$x = -8$

Test $x = -10$: $\dfrac{-10-5}{-10+8} = \dfrac{Neg}{Neg} = Pos > 0$ Yes

Test $x = 0$: $\dfrac{0-5}{0+8} = \dfrac{Neg}{Pos} = Neg < 0$ No

Test $x = 10$: $\dfrac{10-5}{10+8} = \dfrac{Pos}{Pos} = Pos > 0$ Yes

$\boxed{(-\infty, -8) \cup [5, \infty)}$

since $x = -8$ makes the fraction undefined we round parenthesis

original question has endpoint included.

Now, a tough one: $\dfrac{x}{x}\cdot\dfrac{5}{x+2} + \dfrac{1}{x}\cdot\dfrac{(x+2)}{(x+2)} \geq 0 \rightarrow$ positive = Yes

CD: $x(x+2)$

$$\frac{5x}{x(x+2)} + \frac{x+2}{x(x+2)} = 0$$

$$\frac{6x+2}{x(x+2)} = 0$$

No $\underset{-2}{\circ}$ Yes $\underset{-1/3}{|}$ No $\underset{0}{\circ}$ Yes

Num:

$6x + \cancel{2} = 0$

$\quad -\cancel{2} \quad -2$

$\cancel{6}x = -2$

$\cancel{6} \quad\; 6$

$x = -1/3^*$

Den:

$x = 0^*$

$x + \cancel{2} = 0$

$\quad -\cancel{2} \quad -2$

$x = -2^*$

$\boxed{(-2, -1/3] \cup (0, \infty)}$

Test $x = -5$: $\dfrac{6(-5)+2}{-5(-5+2)} = \dfrac{N}{N\cdot N} = \dfrac{N}{P} = N < 0$ No

Test $x = -1$: $\dfrac{6(-1)+2}{-1(-1+2)} = \dfrac{N}{N\cdot P} = \dfrac{N}{N} = P > 0$ Yes

Test $x = -1/6$: $\dfrac{6(-1/6)+2}{-1/6(-1/6+2)} = \dfrac{P}{N\cdot P} = \dfrac{P}{N} = N < 0$ No

Test $x = 1$: $\dfrac{6(1)+2}{1\cdot(1+2)} = \dfrac{P}{P\cdot P} = \dfrac{P}{P} = P > 0$ Yes

Popper #7:

1. $x^2 + x – 12 < 0$ $\rightarrow x^2+x-12=0 \rightarrow (x+4)(x-3)=0$

   a. $(-\infty, -4)U(3, \infty)$ b. $(-\infty, -12)U(1, \infty)$

   c. $(-4, 3)$ d. $(-3, 4)$

$x+4=0$ $x-3=0$

$x=-4$ $x=3$

No Yes No

-4 3

Test $x=0$:

$0^2+0-12<0$

$-12<0$ True

2. $\dfrac{x+8}{x-2} > 0$

Num: $x+8=0$

$x=-8$

Den: $x-2=0$

$x=2$

   a. $(-\infty, -8)U(2, \infty)$ b. $(-\infty, -2)U(8, \infty)$

   c. $(-8, 2)$ d. $(-2, 8)$

Yes No Yes

-8 2

Test $x=-10$: $\frac{-10+8}{-10-2} = \frac{N}{N} = P > 0$ Yes

Test $x=0$: $\frac{0+8}{0-2} = \frac{P}{N} = N < 0$ No

Test $x=5$: $\frac{5+8}{5-2} = \frac{P}{P} = P > 0$ Yes

Popper #7, continued

Neg → Yes

3. $\frac{x+2}{x-1} + \underset{1(x-1)}{\overset{(x-1)}{5}} \le 0$ → $\frac{x+2}{x-1} + \frac{5x-5}{x-1} = 0 \rightarrow \frac{6x-3}{x-1} = 0$

a. (-∞, ½]

b. [½ , 1) U (1, ∞)

c. [½ , 1)

d. [½ , 1]

Num: $6x-3=0 \rightarrow 6x=3 \rightarrow x=1/2$

Den: $x-1=0 \rightarrow x=1$

No   Yes   No

$\frac{1}{2}$ 1

Test $x=0$: $\frac{6(0)-3}{0-1} = \frac{N}{N} = P > 0$ No

Test $x=\frac{3}{4}$: $\frac{6(3/4)-3}{3/4-1} = \frac{P}{N} = N < 0$ Yes

Test $x=2$: $\frac{6(2)-3}{2-1} = \frac{P}{P} = P > 0$ No

$\frac{\cancel{6}^{3}}{1} \cdot \frac{3}{\cancel{4}_{2}} = \frac{9}{2} - \frac{3}{1}$

$= \frac{9}{2} - \frac{6}{2} = \frac{3}{2}$