MATH 1314

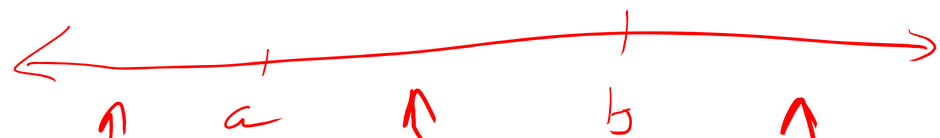
Section 2.7

Non-Linear Inequalities

In this section, we will examine how to solve inequalities involving (1) quadratic functions, and (2) rational functions.

In these examples, we will use a method known as the Number Line Test.

solution to equation is $x = a$, $x = b$



Less than a

between a & b

Greater than b.

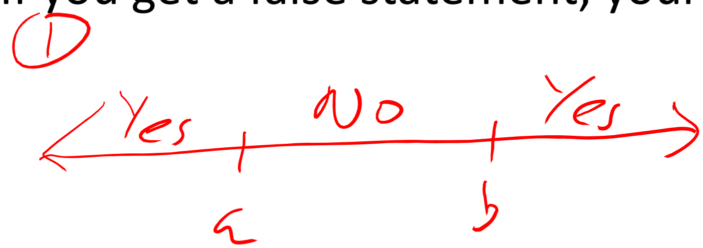
Test 1 x-value in each region.

Only values you cannot use: a, b

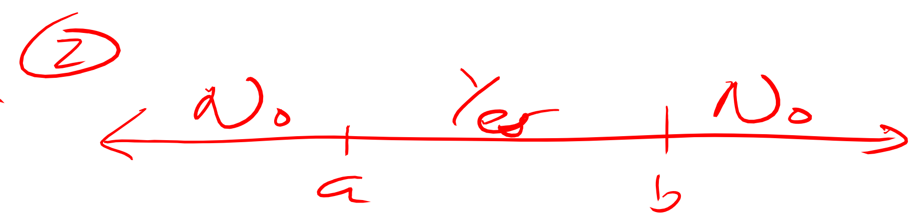
Quadratic Only:
Test 1 region only, then alternate

Solving a Quadratic Inequality

- Rewrite the inequality as an equation (with an equal sign).
- Solve as done before.
- Test an x-value between the two solutions by plugging into the original inequality.
  - If you get a true statement, your solution is between the two solutions.
  - If you get a false statement, your solution is outside the two solutions.



or



Two ways a quadratic inequality will resolve

Try this: $x^2 - x > 6$

Less Than without endpoints.

$\rightarrow x^2 - x = 6$

$\quad -6 \quad -6$

$x^2 - x - 6 = 0$

Answer will Not include endpoints.

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x - 3 = 0 \qquad x + 2 = 0$

$\quad +3 \quad +3 \qquad -2 \quad -2$

$x = 3 \qquad x = -2$

Test Point: $x = 0$

$0^2 - 0 > 6$

$0 > 6$

False

$(-\infty, -2) \cup (3, \infty)$

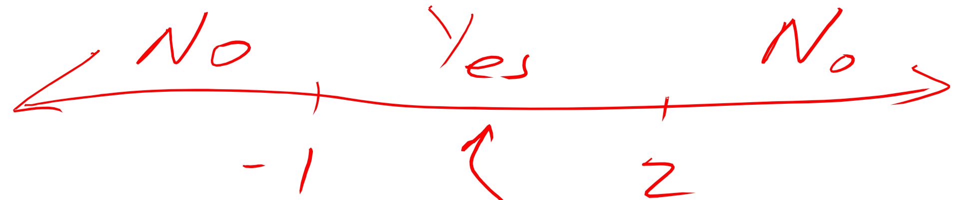
union (means "or")

Now, try this: $x^2 - x - 2 \le 0$ $\longrightarrow$ $x^2 - x - 2 = 0$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$\begin{aligned} x - \cancel{2} &= 0 \\ + \cancel{2} & \quad +2 \\ \hline x &= 2 \end{aligned} \qquad \begin{aligned} x + \cancel{1} &= 0 \\ - \cancel{1} & \quad -1 \\ \hline x &= -1 \end{aligned}$$



Test Point: $x = 0$

$$0^2 - 0 - 2 \le 0$$

$$-2 \le 0$$

True

$$[-1, 2]$$

Solving a Rational Inequality (Fractional Inequalities)

- Set the denominator equal to zero and solve.
- Set the numerator equal to zero and solve.
- Plot these points on a number line (denominator is always open dot).
- Use test points between these values to determine the solution set.

↳ Must test every interval.

Note: If $\leq$ or $\geq$ is in the original inequality, any values from denominator must have (,) Try this: $\frac{x+3}{x-1} > 0$ → Greater than 0 (or Pos) is a "Yes"

Num:

$$x+3=0$$
$$\underline{\;-3\quad -3\;}$$
$$x=-3$$

Den:

$$x-1=0$$
$$\underline{\;+1\quad +1\;}$$
$$x=1$$

(1) Test $x=-5$: $\frac{-5+3}{-5-1} = \frac{(\text{Neg})}{(\text{Neg})} = \text{Pos} > 0$ Yes

(2) Test $x=0$: $\frac{0+3}{0-1} = \frac{(\text{Pos})}{(\text{Neg})} = \text{Neg} < 0$ No

(3) Test $x=2$: $\frac{2+3}{2-1} = \frac{(\text{Pos})}{(\text{Pos})} = \text{Pos} > 0$ Yes

$$\boxed{(-\infty, -3) \cup (1, \infty)}$$

Now try this: $\dfrac{x-5}{x+8} \geq 0$ → Positive → Yes

Num: → square bracket

$$\begin{aligned} x - 5 &= 0 \\ +5 \quad &+5 \\ \hline x &= 5 \end{aligned}$$

Test $x = -10$: $\dfrac{-10-5}{-10+8} = \dfrac{\text{Neg}}{\text{Neg}} = \text{Pos} > 0$ Yes

Den: → round parenthesis

$$\begin{aligned} x + 8 &= 0 \\ -8 \quad &-8 \\ \hline x &= -8 \end{aligned}$$

Test $x = 0$: $\dfrac{0-5}{0+8} = \dfrac{\text{Neg}}{\text{Pos}} = \text{Neg} < 0$ No

Test $x = 10$: $\dfrac{10-5}{10+8} = \dfrac{\text{Pos}}{\text{Pos}} = \text{Pos} > 0$ Yes

$$\boxed{(-\infty, -8) \cup [5, \infty)}$$

since $x = -8$ makes the fraction undefined, we round parenthesis

original question has endpoint included

Now, a tough one: $\frac{x \cdot 5}{x(x+2)} + \frac{1 (x+2)}{x(x+2)} \geq 0$ → positive = Yes

CD: $x(x+2)$

$$\frac{5x}{x(x+2)} + \frac{x+2}{x(x+2)} = 0$$

$$\frac{6x+2}{x(x+2)} = 0$$

No −2 Yes −1/3 No 0 Yes

Test $x = -5$: $\frac{6(-5)+2}{-5(-5+2)} = \frac{N}{N \cdot N} = \frac{N}{P} = N < 0$ No

Test $x = -1$: $\frac{6(-1)+2}{-1(-1+2)} = \frac{N}{N \cdot P} = \frac{N}{N} = P > 0$ Yes

Test $x = -1/6$: $\frac{6(-1/6)+2}{-1/6(-1/6+2)} = \frac{P}{N \cdot P} = \frac{P}{N} = N < 0$ No

Test $x = 1$: $\frac{6(1)+2}{1 \cdot (1+2)} = \frac{P}{P \cdot P} = \frac{P}{P} = P > 0$ Yes

Num:

$6x + \cancel{2} = 0$

$\quad - \cancel{2} \quad -2$

$\frac{\cancel{6}x}{\cancel{6}} = \frac{-2}{6}$

$x = -1/3$*

Den:

$x = 0$*

$x + \cancel{2} = 0$

$\quad -\cancel{2} \quad -2$

$x = -2$*

$\boxed{(-2, -1/3] \cup (0, \infty)}$

Popper #4:

1. $x^2 + x - 12 < 0$ $\rightarrow x^2+x-12=0 \rightarrow (x+4)(x-3)=0$

$x+4=0$ $\quad$ $x-3=0$

$x=-4$ $\quad$ $x=3$

No $\quad$ Yes $\quad$ No

-4 $\quad$ 3

Test $x=0$:

$0^2+0-12<0$

$-12<0$ True

a. (-∞, -4)U(3, ∞) $\qquad$ b. (-∞, -12)U(1, ∞)

c. (-4, 3) $\qquad$ d. (-3, 4)

2. $\dfrac{x+8}{x-2} > 0$

Num: $x+8=0$

$x=-8$

Den: $x-2=0$

$x=2$

Yes $\quad$ No $\quad$ Yes

-8 $\quad$ 2

a. (-∞, -8)U(2, ∞) $\qquad$ b. (-∞, -2)U(8, ∞)

c. (-8, 2) $\qquad$ d. (-2, 8)

Test $x=-10$: $\dfrac{-10+8}{-10-2} = \dfrac{N}{N} = P > 0$ Yes

Test $x=0$: $\dfrac{0+8}{0-2} = \dfrac{P}{N} = N < 0$ No

Test $x=5$: $\dfrac{5+8}{5-2} = \dfrac{P}{P} = P > 0$ Yes

Popper #4, continued

Neg → Yes

3. $\frac{x+2}{x-1} + 5 \le 0$

$\rightarrow \frac{x+2}{x-1} + \frac{5x-5}{x-1} = 0 \rightarrow \frac{6x-3}{x-1} = 0$

a. (-∞, ½]

b. [½ , 1) U (1, ∞)

c. [½ , 1)

d. [½ , 1]

Num: $6x-3=0 \rightarrow 6x=3 \rightarrow x = 1/2$

Den: $x-1=0 \rightarrow x=1$

Test $x=0$: $\frac{6(0)-3}{0-1} = \frac{N}{N} = P > 0$ No

Test $x=\frac{3}{4}$: $\frac{6(3/4)-3}{3/4-1} = \frac{P}{N} = N < 0$ Yes

Test $x=2$: $\frac{6(2)-3}{2-1} = \frac{P}{P} = P > 0$ No

$\frac{\cancel{6}^{3}}{1}\cdot\frac{3}{\cancel{4}_{2}} = \frac{9}{2} - \frac{3}{1} = \frac{9}{2} - \frac{6}{2} = \frac{3}{2}$