

# MATH 1314

Section 2.8

# Absolute Value Equations

In this lesson, you'll learn to solve absolute value equations and inequalities.

Definition: The **absolute value** of  $x$ , denoted  $|x|$ , is the distance  $x$  is from 0.

## Solving Absolute Value Equations

If  $C$  is positive, then  $|x| = C$  if and only if  $x = \pm C$ .

$|x| = 3$   
which numbers are a distance  
of 3 from zero.

$$x = 3$$

$$x = -3$$

$$|7| = 7 \rightarrow \text{Distance from zero}$$

$$|-7| = 7 \rightarrow \text{Distance from zero}$$

$$|x + 5| = 3$$

$$\begin{array}{r} x + 5 = 3 \\ -5 \quad -5 \\ \hline x = -2 \end{array}$$

$$\begin{array}{r} x + 5 = -3 \\ -5 \quad -5 \\ \hline x = -8 \end{array}$$

### Special Cases for $|x| = C$ :

**Case 1:** If  $C$  is negative then the equation  $|x| = C$  has no solution since absolute value cannot be negative.

**Case 2:** The solution of the equation  $|x| = 0$  is  $x = 0$ .

Case 1:  $|x| = \text{negative}$

$$|x+3| = -2 \quad \text{No Solution.}$$

Case 2:  $|x| = 0 \rightarrow$  single answer

$$|x+3| = 0$$

$$x+3 = 0$$

$$\begin{array}{r} x+3 = 0 \\ -3 \quad -3 \end{array} \quad x = -3$$

Solve the following:

- ① Isolate the Absolute Value
- ② Do Not Distribute over an absolute value.

a.  $|2x - 3| = 7$

$$\begin{array}{r} 2x - 3 = 7 \\ +3 \quad +3 \\ \hline 2x = 10 \\ \frac{2x}{2} = \frac{10}{2} \\ x = 5 \end{array}$$

$$\begin{array}{r} 2x - 3 = -7 \\ +3 \quad +3 \\ \hline 2x = -4 \\ \frac{2x}{2} = \frac{-4}{2} \\ x = -2 \end{array}$$

$$\boxed{\{-2, 5\}}$$

b.  $|6 - 2x| + 6 = 14$

$$\begin{array}{r} |6 - 2x| + 6 = 14 \\ -6 \quad -6 \\ \hline |6 - 2x| = 8 \end{array}$$

$$\boxed{\{-1, 7\}}$$

$$\begin{array}{r} 6 - 2x = 8 \\ -6 \quad -6 \\ \hline -2x = 2 \\ \frac{-2x}{-2} = \frac{2}{-2} \\ x = -1 \end{array}$$

$$\begin{array}{r} 6 - 2x = -8 \\ -6 \quad -6 \\ \hline -2x = -14 \\ \frac{-2x}{-2} = \frac{-14}{-2} \\ x = 7 \end{array}$$

$$c. 2|-3(2x-8)| + 4 = 30$$

$$\begin{array}{r} \underline{\quad -4 \quad -4} \\ 2|-3(2x-8)| = 26 \\ \underline{\quad \quad \quad 2} \\ |-3(2x-8)| = 13 \end{array}$$

$$-3(2x-8) = 13$$

$$\begin{array}{r} -6x + 24 = 13 \\ \underline{\quad -24 \quad -24} \end{array}$$

$$\begin{array}{r} -6x = -11 \\ \underline{\quad -6 \quad -6} \\ x = 11/6 \end{array}$$

$$-3(2x-8) = -13$$

$$\begin{array}{r} -6x + 24 = -13 \\ \underline{\quad -24 \quad -24} \end{array}$$

$$\begin{array}{r} -6x = -37 \\ \underline{\quad -6 \quad -6} \\ x = 37/6 \end{array}$$

$$\boxed{\left\{ \frac{11}{6}, \frac{37}{6} \right\}}$$

$$\text{d. } -4 \left| \frac{1}{2}x + 1 \right| + 3 = -11$$

$$\begin{array}{r} -4 \left| \frac{1}{2}x + 1 \right| + 3 = -11 \\ \hline -4 \left| \frac{1}{2}x + 1 \right| = -14 \\ \hline -4 \qquad \qquad -4 \end{array}$$

$$\left| \frac{1}{2}x + 1 \right| = \frac{14 \div 2}{4 \div 2} = \frac{7}{2}$$

$$2 \left( \frac{1}{2}x + 1 = \frac{7}{2} \right)$$

$$\begin{array}{r} x + 2 = 7 \\ \hline -2 \quad -2 \\ \hline x = 5 \end{array}$$

$$2 \left( \frac{1}{2}x + 1 = -\frac{7}{2} \right)$$

$$\begin{array}{r} x + 2 = -7 \\ \hline -2 \quad -2 \\ \hline x = -9 \end{array}$$

$$\boxed{\{-9, 5\}}$$

$$e. |2x - 1| = |x + 7|$$

$$\begin{array}{r} 2x - 1 = x + 7 \\ -x \quad \quad -x \\ \hline \end{array}$$

$$\begin{array}{r} x + 1 = 7 \\ +1 \quad +1 \\ \hline \end{array}$$

$$x = 8$$

check

$$|2(8) - 1| = |8 + 7|$$

$$|16 - 1| = |15|$$

$$15 = 15 \checkmark$$

Keep one side the same  
Change one side  
check answers (only if x is on both sides)

$$\begin{array}{r} 2x - 1 = -x - 7 \\ +x \quad \quad +x \\ \hline \end{array}$$

$$\begin{array}{r} 3x + 1 = -7 \\ +1 \quad \quad +1 \\ \hline \end{array}$$

$$\begin{array}{r} 3x = -6 \\ \frac{3x}{3} = \frac{-6}{3} \\ \hline \end{array}$$

$$x = -2$$

$$\boxed{\{-2, 8\}}$$

check

$$|2(-2) - 1| = |-2 + 7|$$

$$|-4 - 1| = |5|$$

$$|-5| = |5|$$

$$5 = 5 \checkmark$$

# Popper 5:

1.  $4 + |x + 8| = 12$   
~~-4~~                      ~~-4~~

$$|x + 8| = 8$$

$$\begin{array}{r} x + 8 = 8 \\ -8 \quad -8 \\ \hline x = 0 \end{array}$$

$$\begin{array}{r} x + 8 = -8 \\ -8 \quad -8 \\ \hline x = -16 \end{array}$$

- a.  $\{-8, 8\}$       b.  $\{0, 16\}$       c.  $\{-16, 0\}$       d. No Answer

2.  $|2x + 4| = 3$

- a.  $\{-0.5\}$       b.  $\{-3.5\}$       c.  $\{-3.5, -0.5\}$       d. No Answer

$$\begin{array}{r} 2x + 4 = 3 \\ -4 \quad -4 \\ \hline 2x = -1 \\ \frac{2x}{2} = \frac{-1}{2} \end{array}$$

$$x = -\frac{1}{2} = -0.5$$

$$\begin{array}{r} 2x + 4 = -3 \\ -4 \quad -4 \\ \hline 2x = -7 \\ \frac{2x}{2} = \frac{-7}{2} \end{array}$$

$$x = -\frac{7}{2} = -3.5$$



## Popper 5...continued

3.  $|3x - 2| + 1 = 4 \rightarrow |3x - 2| = 3$

a.  $\{-1/3, 5/3\}$

b.  $\{1/3, 5/3\}$

c.  $\{5/3\}$

d. No Answer

$$\begin{array}{r} 3x - 2 = 3 \\ +2 \quad +2 \\ \hline 3x = 5 \end{array}$$

$$\begin{array}{r} 3x = 5 \\ \hline x = 5/3 \end{array}$$

$$\begin{array}{r} 3x - 2 = -3 \\ +2 \quad +2 \\ \hline 3x = -1 \end{array}$$

$$\begin{array}{r} 3x = -1 \\ \hline x = -1/3 \end{array}$$

4.  $|x + 3| = -4$

a.  $\{-7, 7\}$

b.  $\{-7\}$

c.  $\{-7, -1\}$

d. No Answer

$|\text{whatever}| = \text{Neg} \rightarrow \text{No Answer}$

Try this one:

$$\cancel{6x} \frac{3|x^2 + 2x - 1| - 5}{\cancel{8}} = 2 \times \cancel{8}$$

$$3|x^2 + 2x - 1| - 5 = 16$$

$+5 \quad +5$

$$\frac{3|x^2 + 2x - 1|}{3} = \frac{21}{3}$$

$$|x^2 + 2x - 1| = 7$$

$$x^2 + 2x - 1 = 7$$

$-7 \quad -7$

$$x^2 + 2x - 8 = 0$$

$$(x+4)(x-2) = 0$$

$$x+4=0 \quad x-2=0$$

$$x = -4 \quad x = 2$$

$$\boxed{\{-4, 2, -1 \pm i\sqrt{5}\}}$$

$$x^2 + 2x - 1 = 7$$

$+7 \quad +7$

$$x^2 + 2x + 6 = 0$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot 6}}{2 \cdot 1}$$

$$x = \frac{-2 \pm \sqrt{4 - 24}}{2}$$

$$x = \frac{-2 \pm \sqrt{-20}}{2} = \frac{-2 \pm 2i\sqrt{5}}{2}$$

$$x = -1 \pm i\sqrt{5}$$