MATH 1314

Section 2.8

Absolute Value Equations

In this lesson, you'll learn to solve absolute value equations and inequalities.

Definition: The **absolute value** of x, denoted $|x|$, is the distance x s from 0.

Solving Absolute Value Equations
If C is positive, then $|x| = C$ if and only if $x = \pm C$.

Special Cases for $|x| = C$:

Case 1: If C is negative then the equation $|x| = C$ has no solution since absolute value cannot be negative.

Case 2: The solution of the equation $|x| = 0$ is $x = 0$.

Solve the following:

a. $|2x - 3| = 7$

b. $|6 - 2x| + 6 = 14$

c. $2|-3(2x - 8)| + 4 = 30$

d. $-4\left|\frac{1}{2}x+1\right|+3=-11$

e. $|2x - 1| = |x + 7|$

$4 + |x + 8| = 12$

$|2x + 4| = 3$

$|3x - 2| + 1 = 4$

$|x + 3| = -4$

Try this one:

$$\frac{3|x^2 + 2x - 1| - 5}{8} = 2$$

Next, we'll look at inequalities. The approach to these problems will depend on whether the problem is a "less than" problem or a "greater than" problem. If C is zero, then $x = 0$.

Solving Absolute Value Inequalities

If C is positive, then

a. $|x| < C$ if and only if $-C < x < C$.

b. $|x| \leq C$ if and only if $-C \leq x \leq C$.

c. $|x| > C$ if and only if $x > C$ or $x < -C$.

d. $|x| \geq C$ if and only if $x \geq C$ or $x \leq -C$

Example 2: Solve the inequality. Graph the solution on the real number line. Write the solution using interval notation:

a. $|x + 3| \leq 8$

b. $|4 - 2x| < 12$

c. $3|2x - 6| \leq 6$

d. $|-3x + 1| < 4$

e. $2|1 - 4x| + 1 > 7$

f. $-\frac{2}{3}|x - 4| \leq -\frac{4}{3}$

Special Cases:

Case 1:

If C is negative, then:
a) The inequalities $|x| < C$ and $|x| \leq C$ have no solution.
b) Every real number satisfies the inequalities $|x| > C$ and $|x| \geq C$

Case 2:

a) The inequality $|x| < 0$ has no solution.
b) The solution of the inequality $|x| \leq 0$ is $x = 0$.
c) Every real number satisfies the inequality $|x| \geq C$

$|2x + 6| \geq 8$

$-4|x - 3| + 5 > -7$

$|5x + 5| + 3 < 28$

$5|x - 12| + 8 \leq 8$

$|2x + 7| + 9 \geq 4$