MATH 1314

Section 3.1

Functions: Basic Ideas

Basic Definition (Nonformal):
Plug in one x-value you get one answer.

The rest of this course deals with **functions.**

Definition: A **function**, f, is a rule that assigns to each element x in a set A exactly one elements, called $f(x)$, in a set B.

Functions are so important that we use a special notation when working with them. We'll write $f(x)$ to denote the value of function f at x. We read this as "f of x." We can use letters other than f to denote a function, so you may see a function such as $g(x)$, $h(x)$ or $P(x)$.

- You <u>cannot</u> have one x with 2 answers.
- You can have two x with the same answer

$$y = 2x + 5$$

$$f(x) = 2x + 5$$

$f(x)$: "f of x"

- This is a function
- Ind. variable is x
- Named as "f"

Definition: The set A is called the **domain** and is the set of all valid inputs for the function.

Definition: The set B is called the **range** and is the set of all possible values of $f(x)$ as x varies throughout the domain.
Sets A and B will consist of real numbers.

Domain: All inputs (that evaluate to answers). Typically x-values

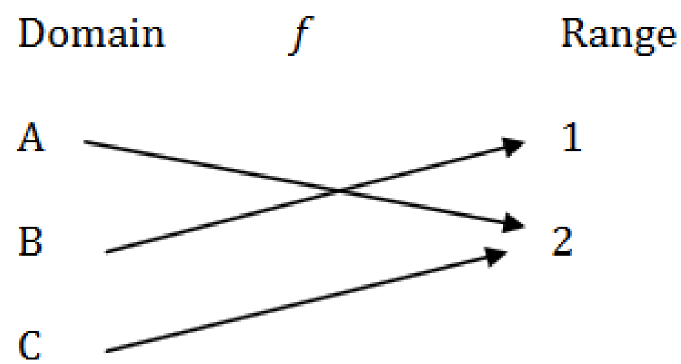
Range: All possible output of a a function. Typically, y-values

Sometimes, Domain and Range will be All Real Numbers

Sometime, they will be restrictions based on the nature of the function.

Example 1:

a. Given:



Is f a function?

$f(A) = 2$

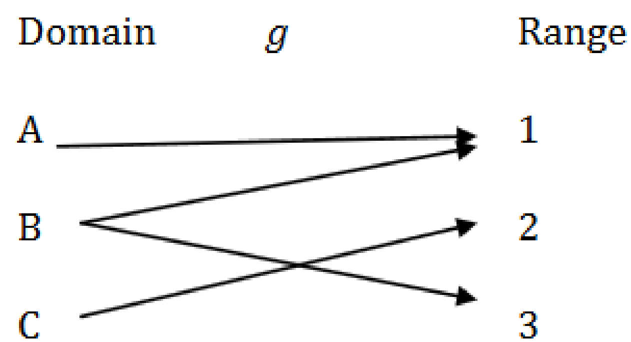
$f(B) = 1$

$f(C) = 2$

This is a function.

Every element of the domain has exactly 1 answer.

b. Given:



Is g a function?

$g(A)=1$

$g(B)=1$

$g(B)=3$

$g(C)=2$

B has 2 different answers.

g is Not a function.

(It only takes 1 part of the domain to make this Not a function)

Next we'll consider some things you'll need to be able to do when working with functions. First, you'll need to be able to evaluate all types of functions when given a specific value for the variable.

Example 2: Let $f(x) = x^2 - 4x$ Calculate

a. $f(-3)$ → Replace x with −3 in f(x)

$f(-3) = (-3)^2 - 4(-3)$

$= 9 + 12 = 21$ → $(-3, 21)$

x y

b. $-2f(x)$ (Notice: −2 is outside the function)

$-2f(x) = -2(x^2 - 4x)$

$= -2x^2 + 8x$

c. $f(3x) = (3x)^2 - 4(3x)$ → Replacing x with 3x

$= \boxed{9x^2 - 12x}$

d. $f(x+2) = (x+2)^2 - 4(x+2)$ → Replace x with x+2

$= (x+2)(x+2) - 4(x+2)$

$= x^2 + \cancel{2x} + \cancel{2x} + 4 - \cancel{4x} - 8$

$= \boxed{x^2 - 4}$

Example 3: Suppose $g(x) = \begin{cases} 2x - 6, & \boxed{x < -2} \\ x^2 + 2x + 3, & \boxed{x \geq -2} \end{cases}$. Calculate the following

piecewise function

a. $g(-5)$

x

Is $-5 < -2$? $\rightarrow 2x-6$

or

$-5 \geq -2$

$g(-5) = 2(-5) - 6 = -10 - 6 = \boxed{-16}$

$(-5, -16)$

y

Popper #10:

1. $g(-2) = (-2)^2 + 2(-2) + 3 = \cancel{4} - \cancel{4} + 3 = \boxed{3}$

$-2 \geq -2$

2. $g(3) = (3)^2 + 2(3) + 3$

$3 \geq -2$

$= 9 + 6 + 3 = \boxed{18}$

a. 18

b. 3

c. -10

d. 0

1B

2A

Finding the Domain of a Function

Recall: The domain is the set of all real numbers for which the expression is defined as a real number. Exclude from a function's domain real numbers that cause division by zero or real numbers that result in an even root of a negative number.

We express the set of real numbers as $(-\infty, \infty)$.
The domain of any polynomial function is $(-\infty, \infty)$.

2 Things we look for to eliminate:

- Zeros in the denominator
- Negatives inside even-powered roots.

(If neither of those two occur, then the domain is $(-\infty, \infty)$).

Example 4: Find the domain of each function below and express your answer in interval notation.

a. $f(x) = -17$

Domain: $(-\infty, \infty)$

- No fractions
- No Roots

b. $f(x) = 3x - 4$

Domain: $(-\infty, \infty)$

- No fractions
- No Roots

c. $h(x) = \dfrac{5x}{2x-8}$

Fraction

Denominator $\neq 0$

$$2x - \cancel{8} \neq 0$$
$$\quad \cancel{+8} \quad +8$$
$$\frac{\cancel{2}x}{\cancel{2}} \neq \frac{8}{2}$$
$$x \neq 4$$

Domain: $(-\infty, 4) \cup (4, \infty)$

d. $f(x) = \dfrac{x-1}{2x-6}$

Denominator $\neq 0$

$$2x - \cancel{6} \neq 0$$
$$\quad \cancel{+6} \quad +6$$
$$\frac{\cancel{2}x}{\cancel{2}} \neq \frac{6}{2}$$
$$x \neq 3$$

Domain: $(-\infty, 3) \cup (3, \infty)$

e. $p(x) = \frac{x^2-16}{x^2-4x-12}$

Denominator $\neq 0$

$x^2-4x-12 \neq 0$

$(x-6)(x+2) \neq 0$

$x - 6 \neq 0$ $\quad$ $x+2 \neq 0$

$+6 \quad +6$ $\quad$ $-2 \quad -2$

$x \neq 6$ $\quad$ $x \neq -2$

Domain:

$(-\infty,-2) \cup (-2,6) \cup (6,\infty)$

f. $h(x) = \sqrt{x}$

Root inside ≥ 0

$x \geq 0$

$[0,\infty)$

g. $j(x) = \frac{\sqrt{x-5}}{x+2}$

• Root: inside ≥ 0

$x - 5 \geq 0$

$+5 \quad +5$

$x \geq 5$

• Denominator $\neq 0$

$x+2 \neq 0$

$-2 \quad -2$

$x \neq -2$

$-2 \quad 5$

Domain: $\boxed{[5,\infty)}$

h. $k(x) = \frac{\sqrt{x-5}}{x-8}$

• Root: $x-5 \geq 0$

$x \geq 5$

• Den: $x-8 \neq 0$

$x \neq 8$

$5 \quad 8$

$\boxed{[5,8) \cup (8,\infty)}$

Note: odd Roots have domain: $(-\infty,\infty)$

Popper #10…continued

3. $q(x) = \sqrt{x-4}$

D

$x - 4 \geq 0$

$+4 \quad +4$

$x \geq 4$

$[4, \infty)$

4. $f(x) = \sqrt[3]{2x+4}$

C

Cube Root

odd roots have domains of $(-\infty, \infty)$

5. $g(x) = \sqrt[10]{42-2x}$

Even Root

A

a. (-∞, 21]

b. [21, ∞)

c. (-∞, ∞)

d. [4, ∞)

$42 - 2x \geq 0$

$-42 \quad -42$

$-2x \geq -42$

$\overline{-2} \quad \overline{-2}$

$x \leq 21$

$(-\infty, 21]$