# MATH 1314

Section 3.2

# Functions and Graphs

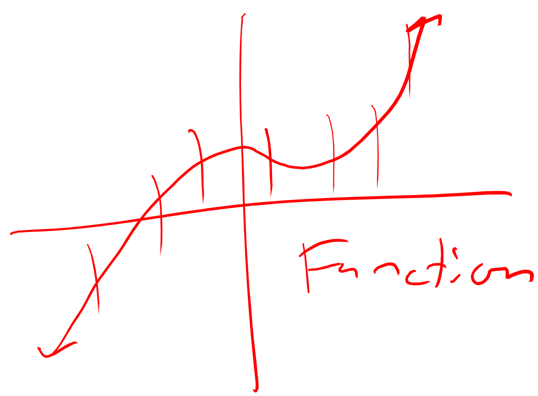
You can answer many questions given a graph.

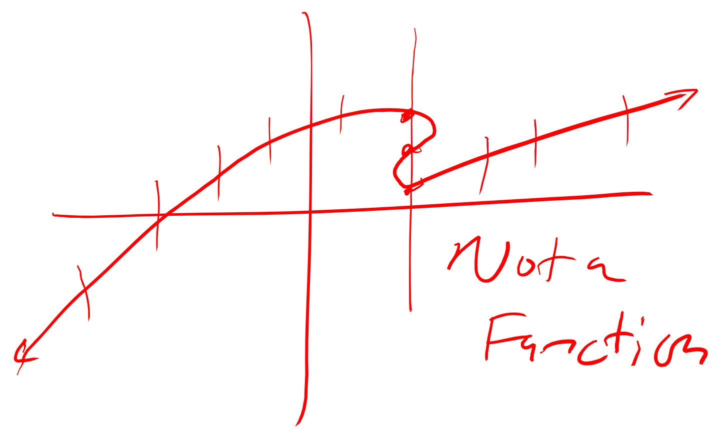
Every x-value corresponds to exactly 1 y-value.

**Definition:** The graph of a function $f(x)$ is the set of points $(x, y)$ whose $x$ coordinates are in the domain of $f$ and whose $y$ coordinates are given by $y = f(x)$.

First, does the graph represent a function? To answer this, you will need to use the **vertical line test (VLT)**.

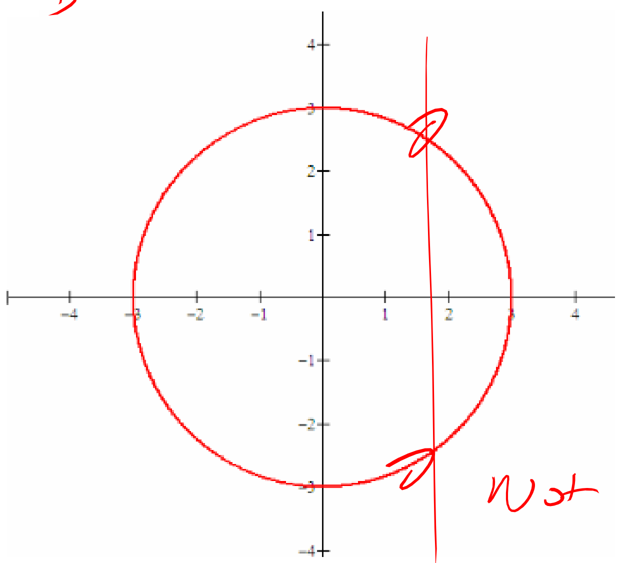
**The Vertical Line Test:**
If you can draw a vertical line that crosses the graph more than once, it is NOT the graph of a function.



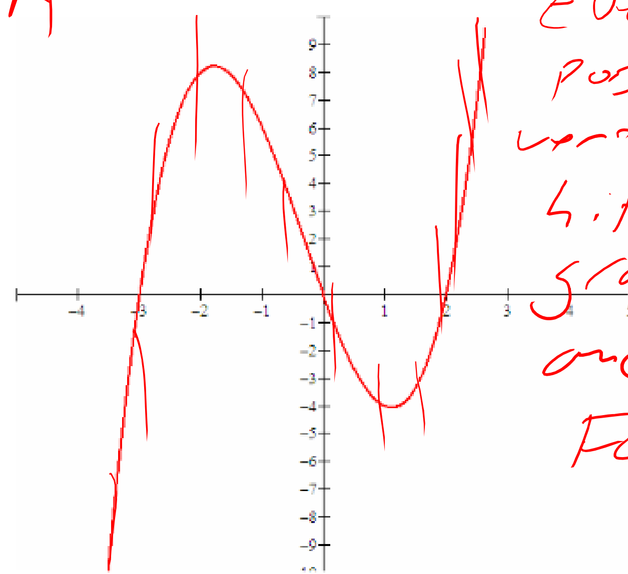
# Popper 11: Does the graph represent a function? a. Yes b. No

1. B



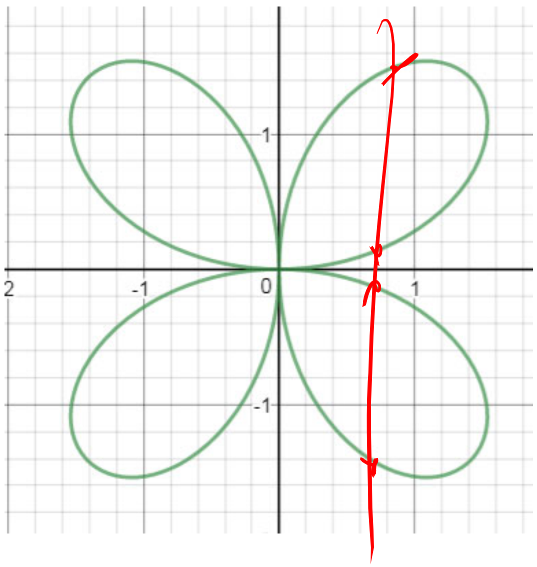
Not a Function.

2. A



Every possible vertical line hits the graph once → Function.

3. B




Not a Function

4. A


Function

Definition: An **equation defines** $y$ **as a function of** $x$ if when one value for $x$ is substituted in the equation, **exactly one value for** $y$ **is returned.**

**Example 2**: Does the following equation define $y$ as a function of $x$?

$$y - x^2 = 4$$

1. Solve for $y$.
2. For each value $x$, do we get exactly one value for $y$ back?

$$y - \cancel{x^2} = 4$$
$$\cancel{+x^2} \quad +x^2$$
$$y = x^2 + 4$$

No ± appeared in the solving.

Function

$y = |x|$

Example: $x = 5 \rightarrow y = 5$
$x = -5 \rightarrow y = 5$

Function

$x = |y| \rightarrow y = \pm x$

$x = 5 < \begin{matrix} y = 5 \\ y = -5 \end{matrix}$

Not a Function.

b. $x^2 + y^2 = 9$

1. Solve for $y$.

2. For each value $x$, do we get exactly one value for $y$ back?

$$\cancel{x^2} + y^2 = 9$$
$$\cancel{-x^2} \qquad -x^2$$
$$\sqrt{y^2} = \sqrt{9 - x^2}$$
$$y = \pm\sqrt{9 - x^2}$$

A single $x$ gives 2 $y$-values. (Not a function)

General Rule:

If there are any $y$ raised to an even power $\rightarrow$ Not a function.

$2x^2 + 3y^2 + 2x + 4y = 10$

$\rightarrow$ Not a Function.

$2x^2 + 2x + 4y = 10$

Is a function.

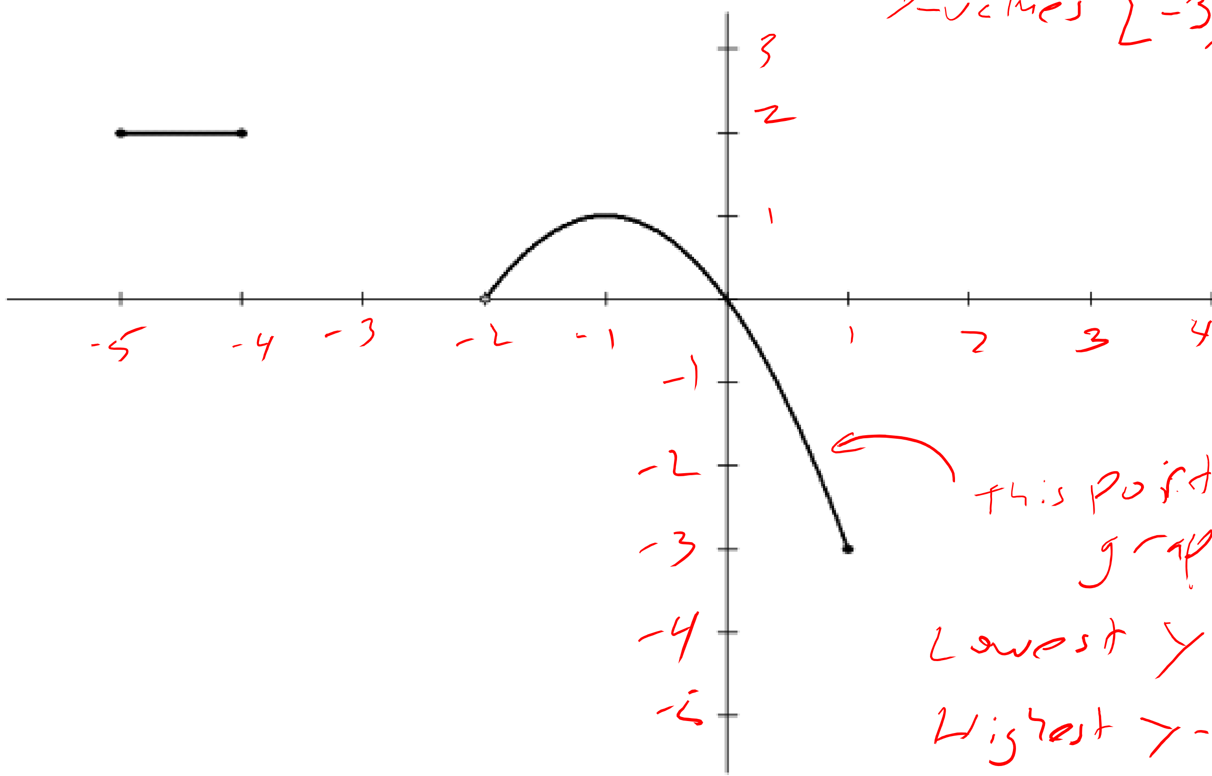
**Example 3:** Find the domain and range of the function whose graph is shown.

x-values

Domain: $[-5, -4] \cup (-2, 1]$

Range: $[-3, 1] \cup [2, 2]$

y-values $[-3, 1] \cup \{2\}$ or




This portion of graph:

Lowest y-value: -3

Highest y-value: +1

You'll also need to be able to graph functions. For now, you can do so by plotting points. But...
YOU MUST KNOW THESE FUNCTIONS AND GRAPHS

Library of Functions

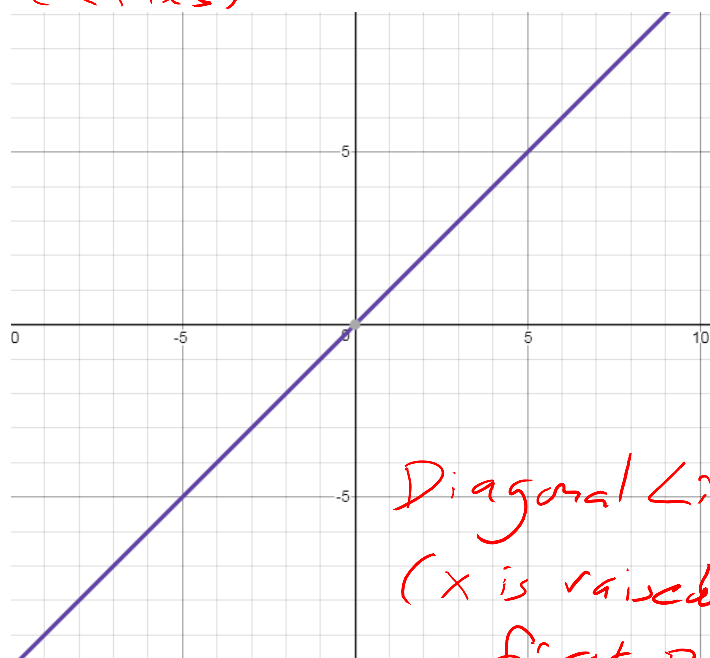
Constant Function
$y = k$

$y = 5$


Horizontal Line

Identity Function (Linear)
$y = x$
(Lines)

$y = 2x+5$



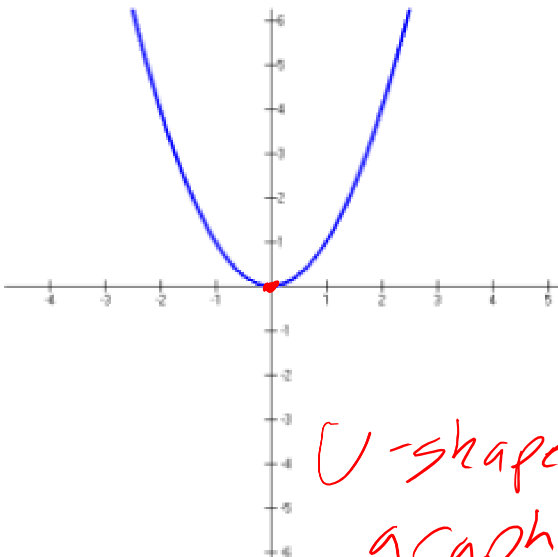

Diagonal Lines
(x is raised to first power)

Key Point: Central point on the graph that is unaltered during transformations (0,0)

Quadratic Function

(highest)
x is raised to a 2.

$f(x) = x^2$



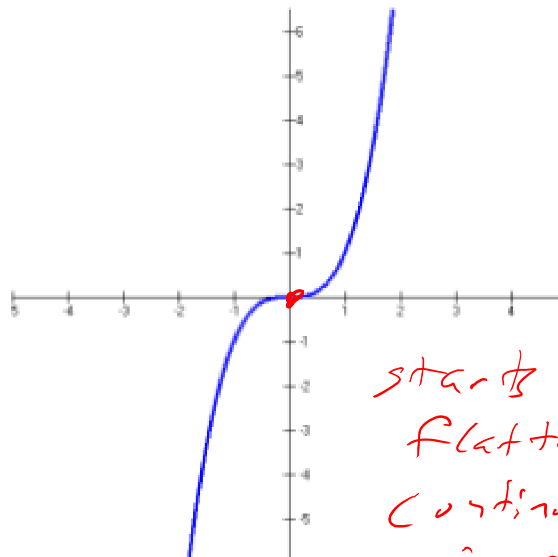
U-shaped graph

$f(x) = 2x^2 + 5x - 1 \rightarrow$ Quadratic

Cubic Function

x raised to 3 as highest exponent.

$f(x) = x^3$

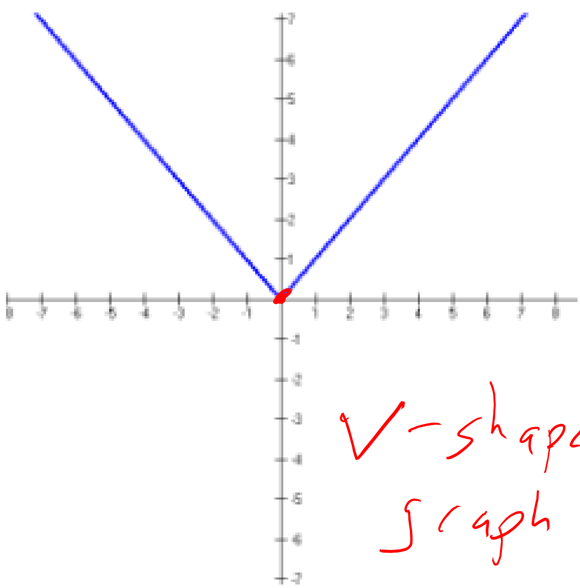


starts low, flattens, continues to increase

Absolute Value Function

$f(x) = |x|$

Absolute value symbols

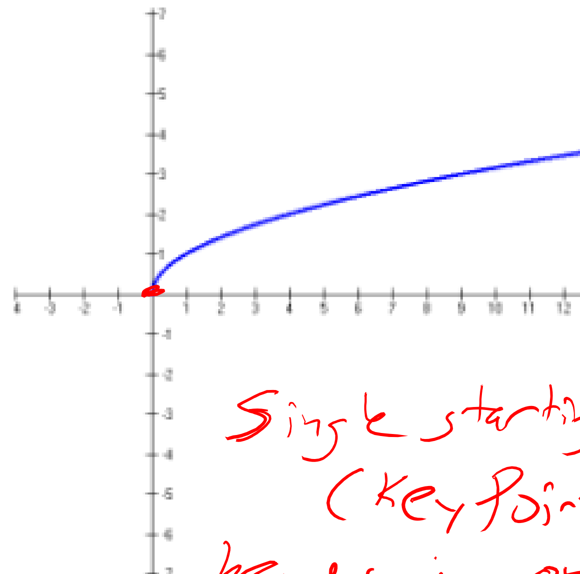


V-shaped graph

Radical Function

$f(x) = \sqrt{x}$

square root in the equation.



Single starting point (key point)

heads in one direction

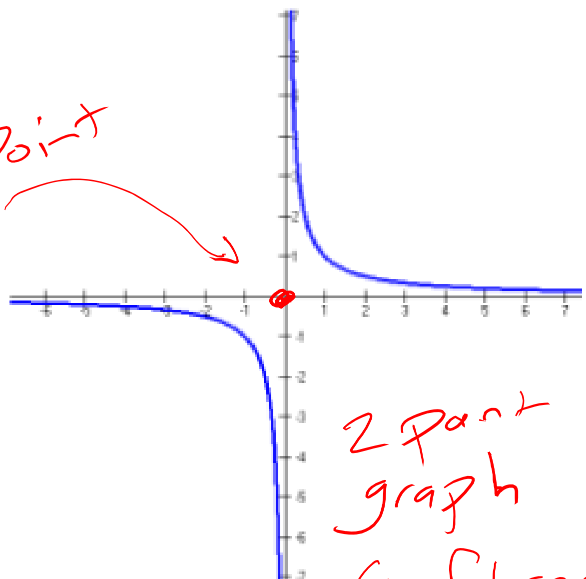
Know all by their ① Graph
② Equation
③ Name

Rational Function

$$f(x) = \frac{1}{x}$$

x in the denominator

Key Point

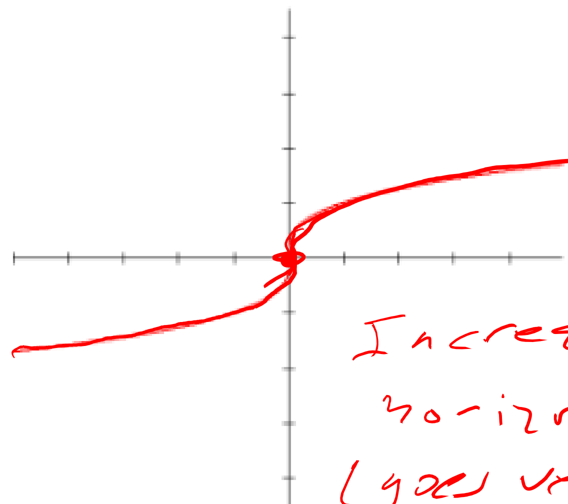



2 part graph (left and right)

Cube Root Function

$$f(x) = \sqrt[3]{x}$$

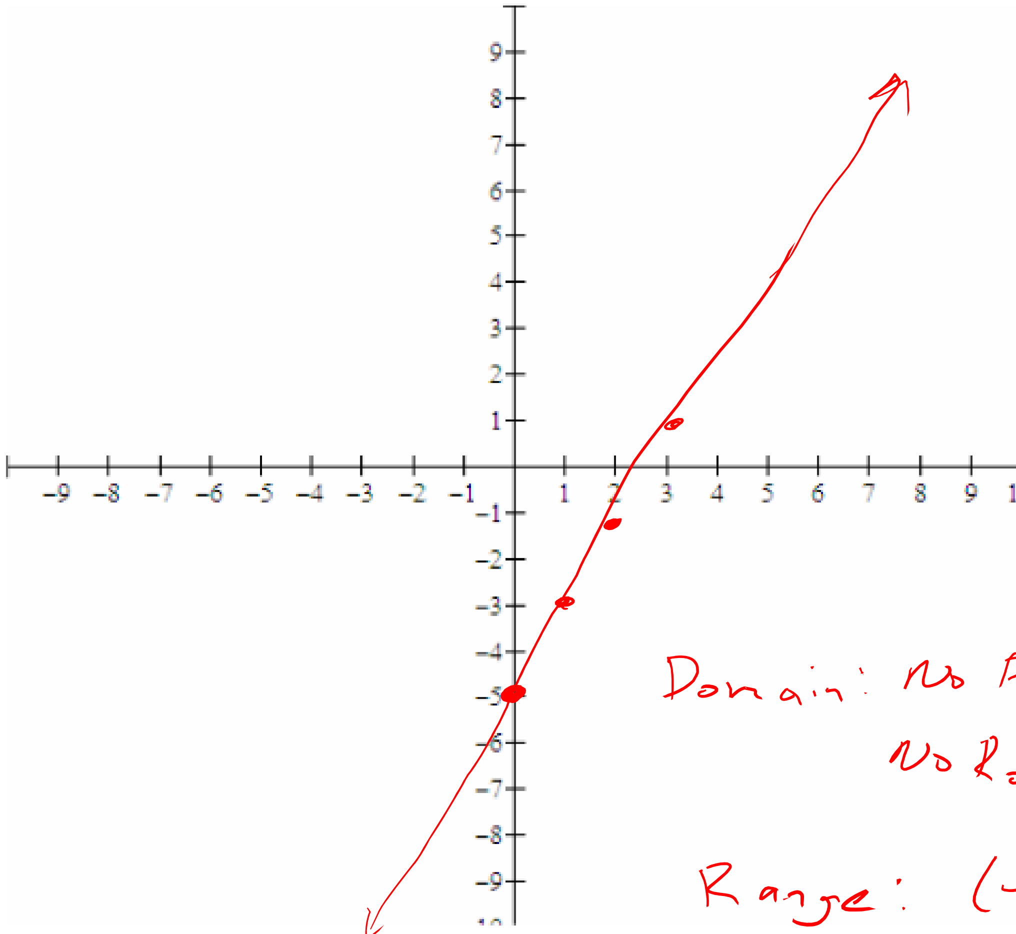
Cube root in the equation.



Increasing horizontally (goes vertical at x=0)

Linear

**Example 4:** Suppose $f(x) = 2x - 5$. State the domain of the function and graph it.



Linear:
x is raised to the first power

$y = 2x - 5$

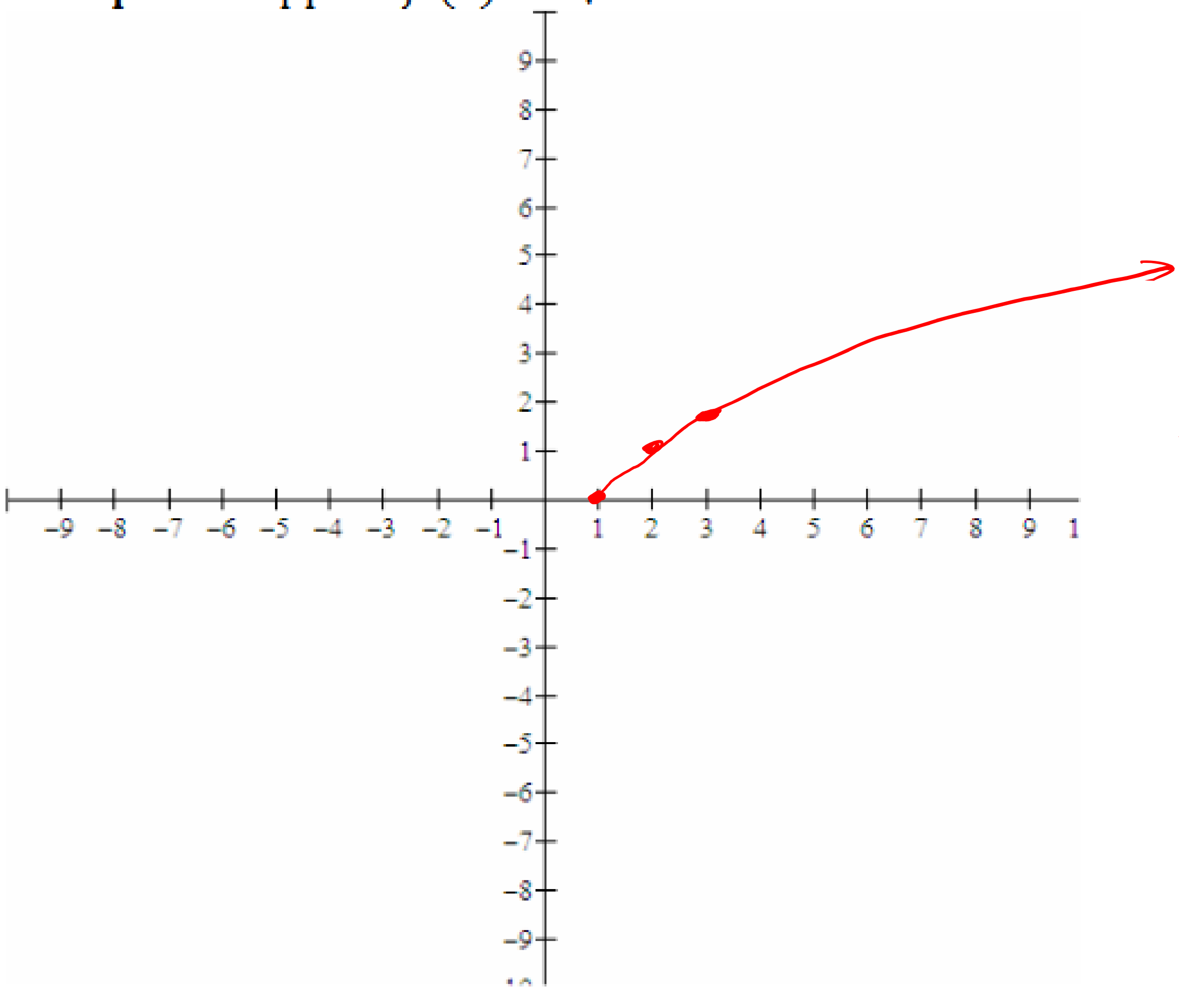
slope: $m = \frac{2}{1}$

y-intercept: $b = -5$

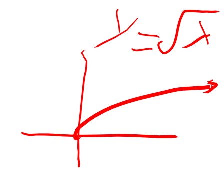
Domain: No Fractions
No Roots
$(-\infty, \infty)$

Range: $(-\infty, \infty)$
↳ From the Graph

**Example 6:** Suppose $f\ (x) = \sqrt{x - 1}$. State the domain of the function and graph it.




Radical Function:




(Parent function)

Domain: $x - 1 \geq 0$

$\quad\ \ +1 \quad +1$

$x \geq 1 \to [1, \infty)$

| x | y | |
|---|---|---|
| 1 | $\sqrt{1-1} = \sqrt{0} = 0$ | $(1,0)$ |
| 2 | $\sqrt{2-1} = \sqrt{1} = 1$ | $(2,1)$ |
| 3 | $\sqrt{3-1} = \sqrt{2} \approx 1.4$ | $(3,\sqrt{2})$ |

For $f(x) = \frac{5}{2x+4}$ evaluate $f\left(\frac{a+1}{a-1}\right)$

$$f\left(\frac{a+1}{a-1}\right) = \frac{5}{2\left(\frac{a+1}{a-1}\right)+4} = \frac{5\,(a-1)}{\cancel{(a-1)}\frac{2a+2}{\cancel{a-1}}+4(a-1)} = \frac{5a-5}{2a+2+4a-4}$$

$$= \boxed{\frac{5a-5}{6a-2}}$$

For $g(x) = x^2 + 2x - 1$ evaluate $g\left(\frac{5}{b}\right)$

$$g\left(\frac{5}{b}\right) = \left(\frac{5}{b}\right)^2 + \frac{2}{1}\left(\frac{5}{b}\right) - 1$$

$$= \frac{25}{b^2} + \frac{10 \cdot b}{b \cdot b} - \frac{1}{1} \cdot \frac{b^2}{b^2} = \frac{25}{b^2} + \frac{10b}{b^2} - \frac{b^2}{b^2}$$

(CD: $b^2$)

$$= \frac{25 + 10b - b^2}{b^2} = \boxed{-\frac{b^2 - 10b - 25}{b^2}}$$

negative is factored out

# Popper 11….continued:

constant

D: $(-\infty, \infty)$

**Example 8:** Let $P(x) = \begin{cases} -3, & x < 2 \\ x^2, & x > 2 \\ 2, & x = 2 \end{cases}$ State the domain of the function and graph it.

$\longrightarrow (2,2)$

Quadratic

Find $p(-2)$, $p(2)$ and $p(3)$.

5. P(-2) $= -3$

[$-2 < 2$] : $y = -3$

5C

6. P(2) $= 2$

[$2 = 2$] : $(2,2)$

6A

7. P(3) $= 3^2 = 9$

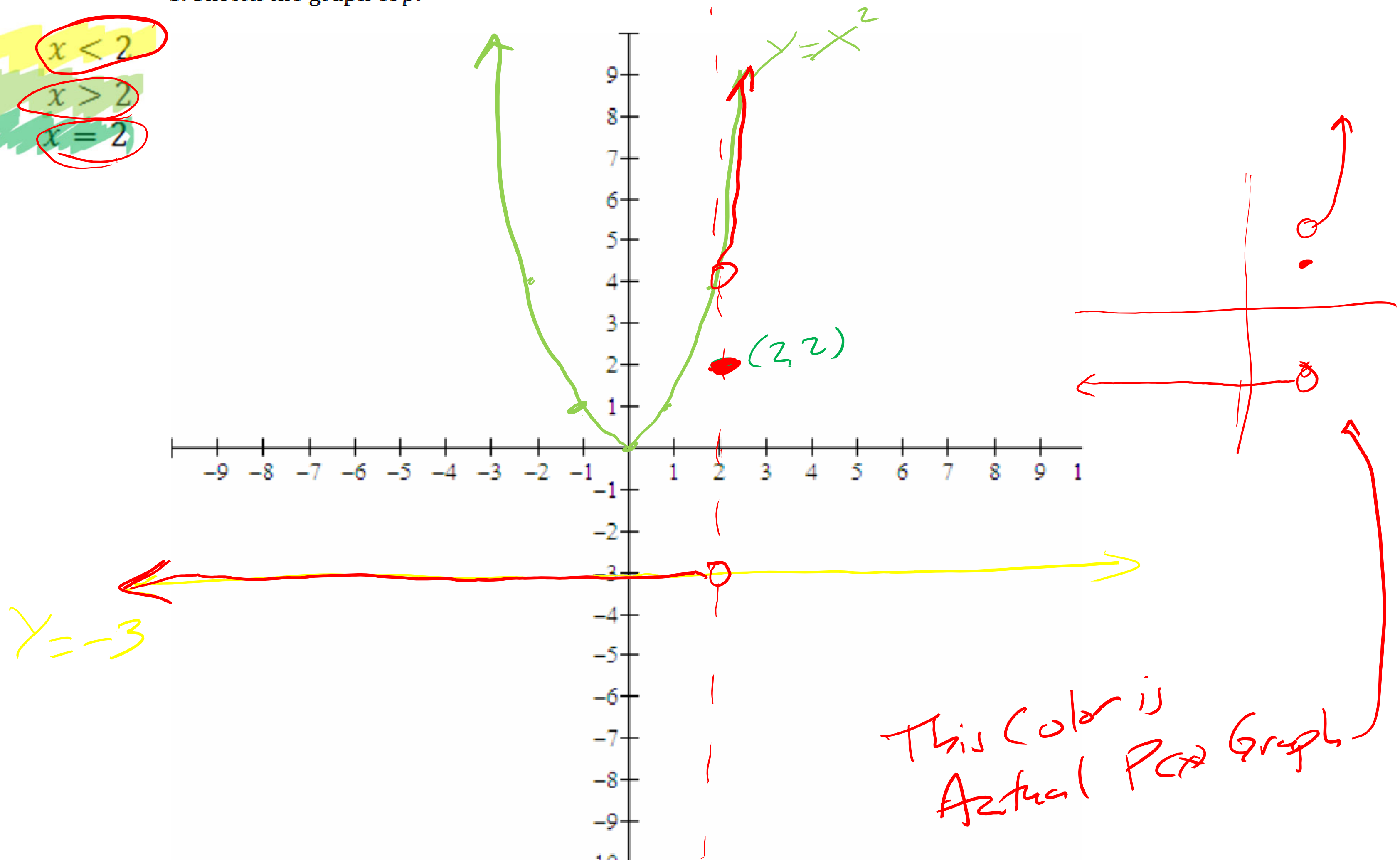
[$3 > 2$] : $y = x^2$

7D

a. 2
b. 4
c. -3
d. 9

$$P(x) = \begin{cases} -3, & x < 2 \\ x^2, & x > 2 \\ 2, & x = 2 \end{cases}$$

b. Sketch the graph of $p$.



# Odd and Even Functions:

Odd Functions have only odd exponents, such as $f(x) = 2x^3 + 8x^{1}$.

They satisfy the formula: $f(-x) = -f(x)$

They are symmetric about the origin.

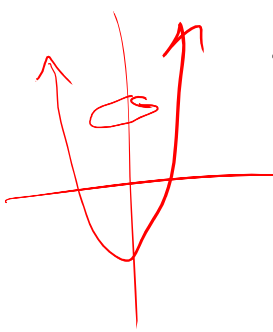
If they contain the point (a, b) they also contain (-a, -b).

$x = 2$ $\quad f(2) = 2(2)^3 + 8(2)$
$2(8) + 16$
$16 + 16 = 32$
$f(-2) = 2(-2)^3 + 8(-2)$
$2(-8) + 8(-2) = -32$

(2, 32) $\quad$ (-2, -32)

Constants → Even Exponents

Even Functions only have even exponents, such as $g(x) = 3x^4 + 2x^2 + 5$.



They satisfy the formula: $g(-x) = g(x)$

They are symmetric about the y-axis.

If they contain the point (a, b), they also contain (-a, b).

$x = 2$ $\quad g(2) = 3(2)^4 + 2(2)^2 + 5$ → $5x^0$
$3(16) + 2(4) + 5$
$48 + 8 + 5 = 61$

(2, 61)

(-2, 61)

$g(-2) = 3(-2)^4 + 2(-2)^2 + 5$
$3(16) + 2(4) + 5$
$48 + 8 + 5 = 61$

# An even function contains the point (-5, -2).

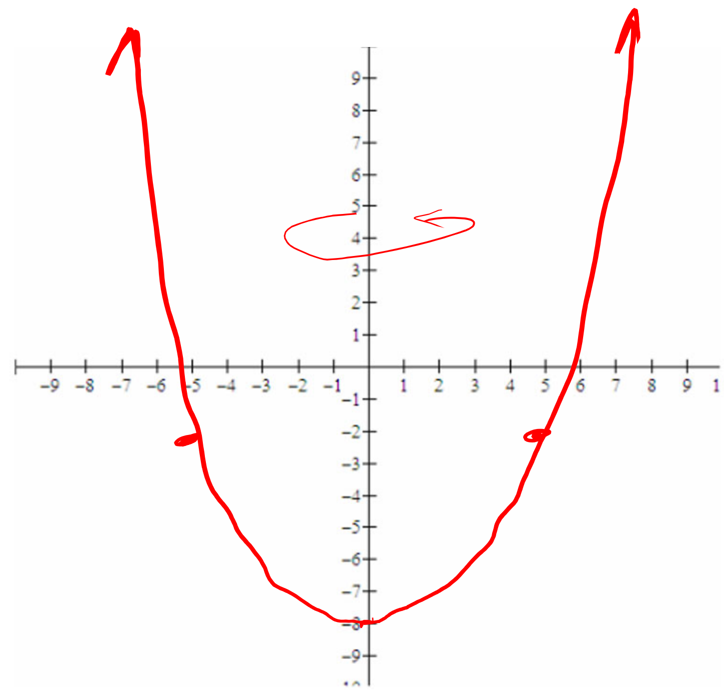
What point must it also contain?

$(-5, -2) \to (a, b)$

$(5, -2) \to (-a, b)$

(change x-sign, keep y-sign)

What is a possible graph of the function?

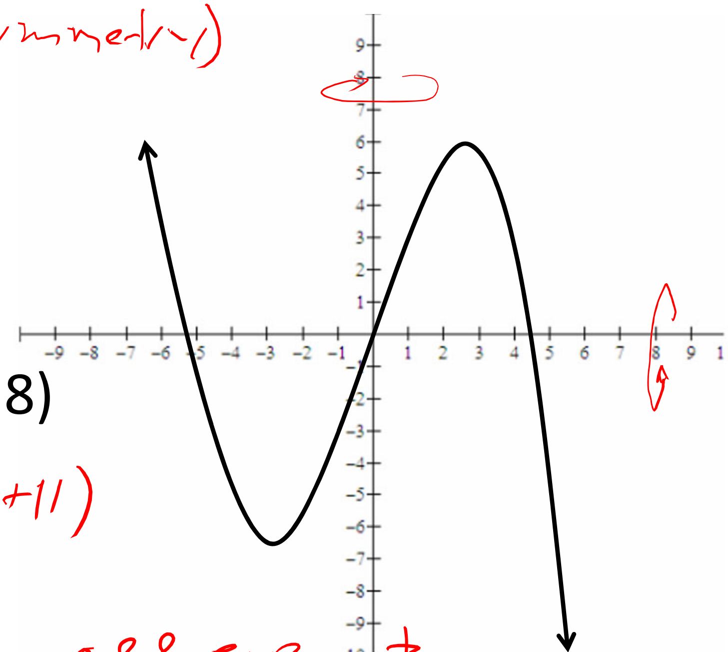


Anything with
y-axis
symmetry.

Popper 11, continued
The following function passes through the point (8, -11).

8. Is the function even or odd? (origin symmetry)

a. even  b. odd  c. neither



9. What other point must it contain?

a. (-8, -11)  b. (-8, 11)  c. (8, 11)  d. (-11, 8)

Negate x and y  $(8,-11) \to (-8,+11)$

10. What is a possible equation?

(assume all letters represent constants) only odd exponents

~~a.~~ $f(x) = ax^3 + bx^2 + cx + d$  c. $h(x) = ax^3 + bx^1$

b. ~~g(x)~~ $= ax^2 + b$  ~~d.~~ $j(x) = ax^2 + bx$

Determine the value of the difference quotient for $\boxed{f(x) = -4x + 5}$

The difference quotient is: (Not Memorize)

$$\frac{f(x+h) - f(x)}{h}$$

① $f(x+h) = -4(x+h) + 5 = \underline{-4x - 4h + 5}$

② $\underline{f(x+h)} - f(x) = \cancel{-4x} - 4h \cancel{+5} \cancel{+4x} \cancel{-5} = -4h$

↳ change all signs of f(x)

③ $\dfrac{f(x+h) - f(x)}{h} = \dfrac{-4\cancel{h}}{\cancel{h}} = \boxed{-4}$

should always cancel

✓ Check #1: only terms with h will remain.

✓ Check #2: −f(x) will cancel completely

# Determine the value of the difference quotient for f(x) = 2x² + 3x – 1

$(x+h)^2$

$(x+h)(x+h)$

$x^2+xh+xh+h^2$

$x^2+2xh+h^2$

The difference quotient is:

$$\frac{f(x+h)-f(x)}{h}$$

$$f(x+h) = 2\underbrace{(x+h)^2} + 3(x+h) - 1 = 2(x^2+2xh+h^2) + 3(x+h) - 1$$

$$= 2x^2 + 4xh + 2h^2 + 3x + 3h - 1$$

$$f(x+h) - f(x) = \underbrace{\cancel{2x^2} + 4xh + 2h^2 \cancel{+3x} + 3h \cancel{-1}}_{f(x+h)} \underbrace{\cancel{-2x^2} \cancel{-3x} \cancel{+1}}_{-f(x)} = 4xh + 2h^2 + 3h$$

$$\frac{f(x+h)-f(x)}{h} = \frac{4xh+2h^2+3h}{h} = \frac{4x\cancel{h}}{\cancel{h}} + \frac{2h^{\cancel{2}}}{\cancel{h}} + \frac{3\cancel{h}}{\cancel{h}} = \boxed{4x+2h+3}$$