

MATH 1314

Section 3.2

Functions and Graphs

You can answer many questions given a graph.

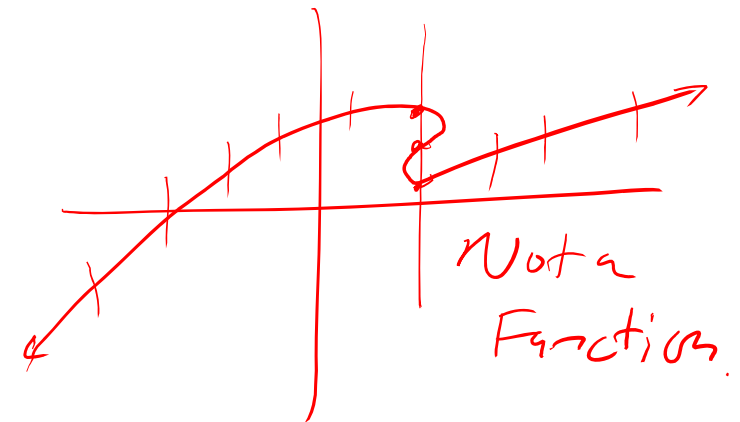
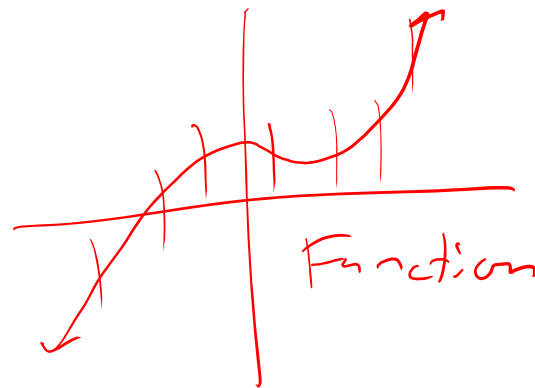
Every x -value corresponds to exactly 1 y -value.

Definition: The graph of a function $f(x)$ is the set of points (x, y) whose x coordinates are in the domain of f and whose y coordinates are given by $y = f(x)$.

First, does the graph represent a function? To answer this, you will need to use the vertical line test (VLT).

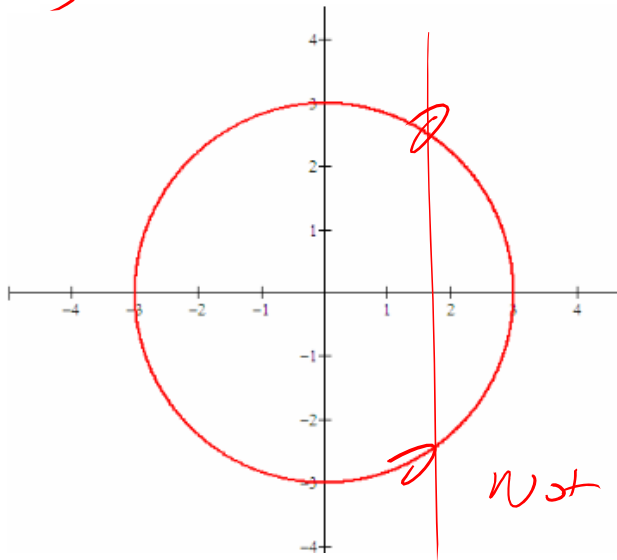
The Vertical Line Test:

If you can draw a vertical line that crosses the graph more than once, it is NOT the graph of a function.



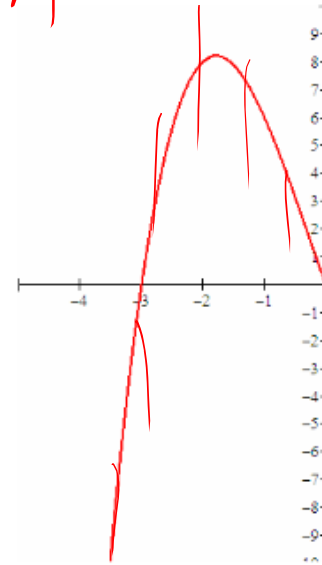
Popper 10: Does the graph represent a function? a. Yes b. No

1. B



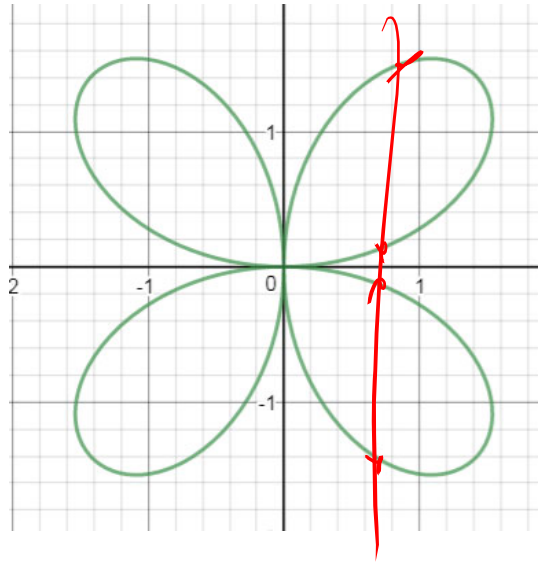
Not
a
Function.

2. A



Every
possible
vertical line
hits the
graph
once →
Function.

3. B



Not a
Function

4. A



Function

Definition: An **equation defines y as a function of x** if when one value for x is substituted in the equation, **exactly one value for y is returned.**

Example 2: Does the following equation define y as a function of x ?

$$y - x^2 = 4$$

1. Solve for y .
2. For each value x , do we get exactly one value for y back?

$$\begin{array}{r} y - x^2 = 4 \\ +x^2 \quad +x^2 \\ \hline \end{array}$$

$$y = x^2 + 4$$

no \pm appeared
in the solving.

Function

$$y = |x|$$

Example: $x = 5 \rightarrow y = 5$
 $x = -5 \rightarrow y = 5$

Function

$$x = |y| \rightarrow y = \pm x$$

not a
Function.

$$x = 5 \begin{cases} y = 5 \\ y = -5 \end{cases}$$

b. $x^2 + y^2 = 9$

1. Solve for y .

2. For each value x , do we get exactly one value for y back?

$$\begin{array}{r} x^2 + y^2 = 9 \\ -x^2 \quad \quad -x^2 \\ \hline y^2 = 9 - x^2 \end{array}$$

$$y = \pm \sqrt{9 - x^2}$$

A single x gives 2

y -values. (not a function)

General Rule:

If there are any y -raised to an even power \rightarrow Not a function.

$$2x^2 + 3x^2 + 2x + 4y = 10$$

\rightarrow Not a Function.

$$2x^2 + 2x + 4y = 10$$

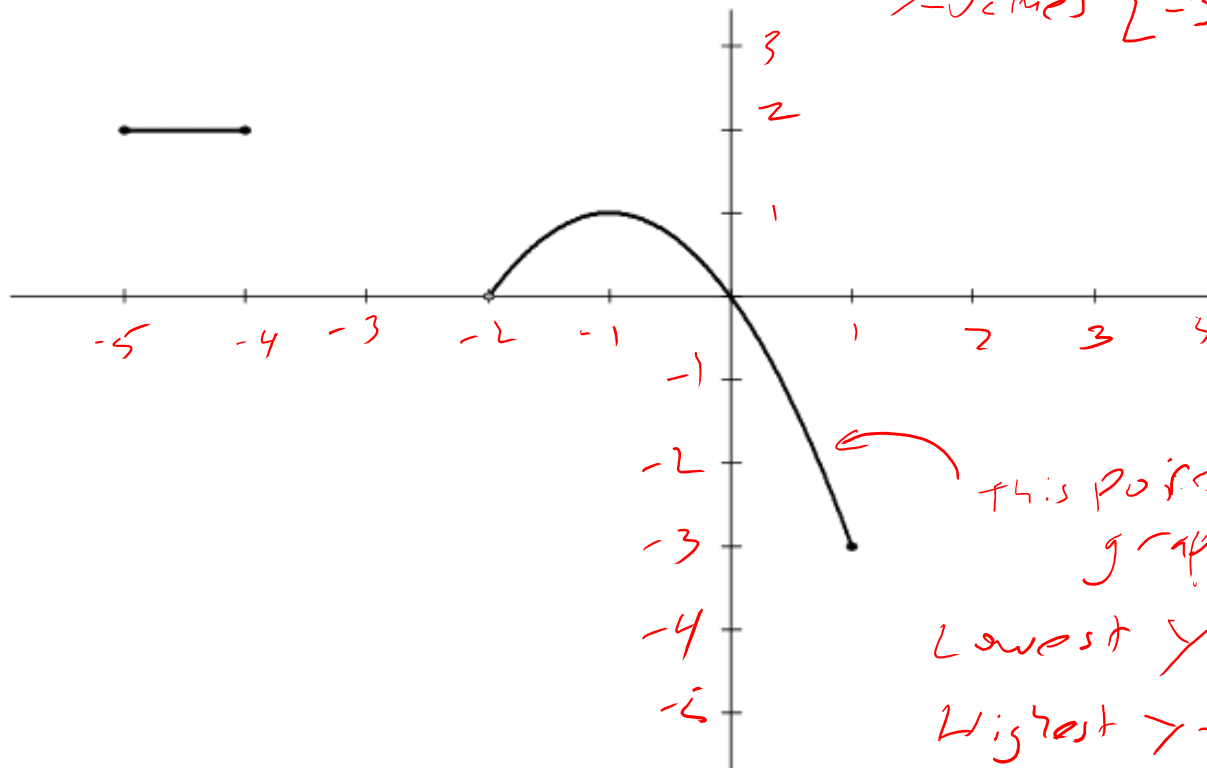
Is a function.

Example 3: Find the domain and range of the function whose graph is shown.

x-values
Domain: $[-5, -4] \cup [-2, 1]$

Range: $[-3, 1] \cup [2, 2]$

y-values $[-3, 1] \cup \{2\}$ or



← this portion of graph:
Lowest y -value: -3
Highest y -value: $+1$

You'll also need to be able to graph functions. For now, you can do so by plotting points. But...
YOU MUST KNOW THESE FUNCTIONS AND GRAPHS

Library of Functions

Constant Function
 $y = k$

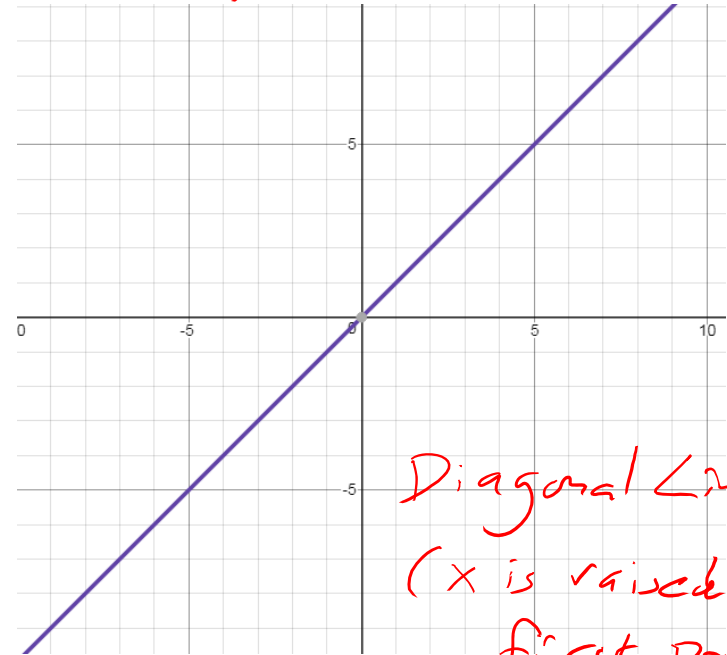
$y = 5$



Horizontal Line

Identity Function (Linear)
 $y = x$
(Lines)

$y = 2x + 5$



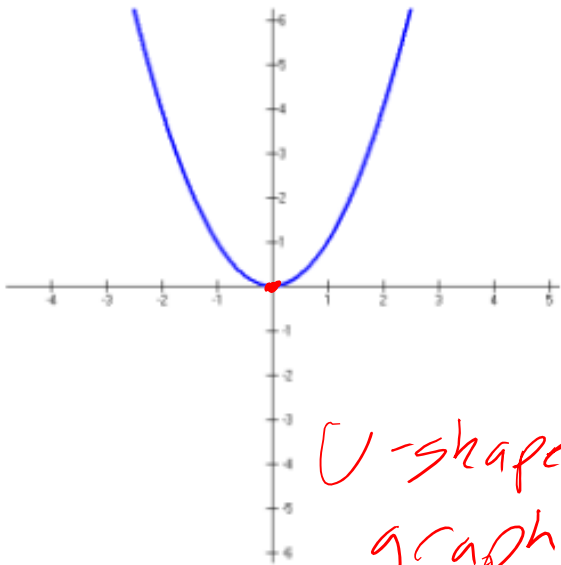
Diagonal Lines
(x is raised to first power)

Quadratic Function

(highest)

$$f(x) = x^2$$

x is raised to a 2.



U-shaped graph

$$f(x) = 2x^2 + 5x - 1 \rightarrow \text{Quadratic}$$

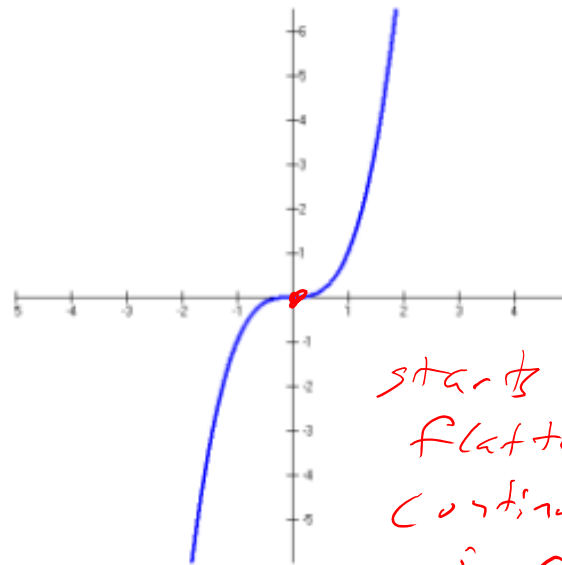
Key Point: Central point on the graph that is monitored during transformations

(0,0)

Cubic Function

$$f(x) = x^3$$

x raised to 3 as highest exponent.

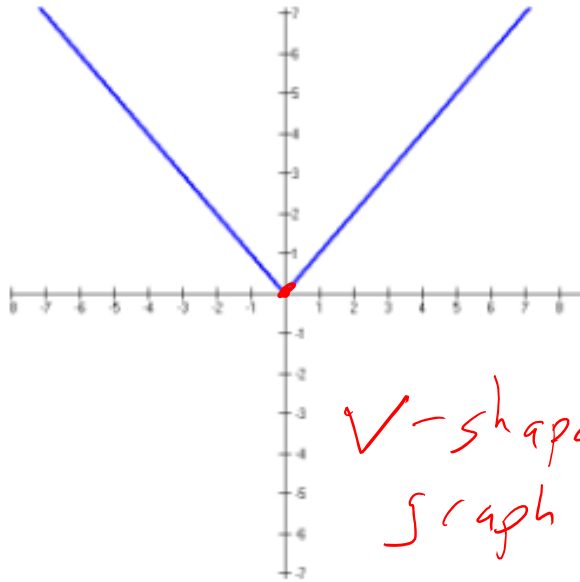


starts low, flattens, continues to increase

Absolute Value Function

$$f(x) = |x|$$

Absolute value symbols

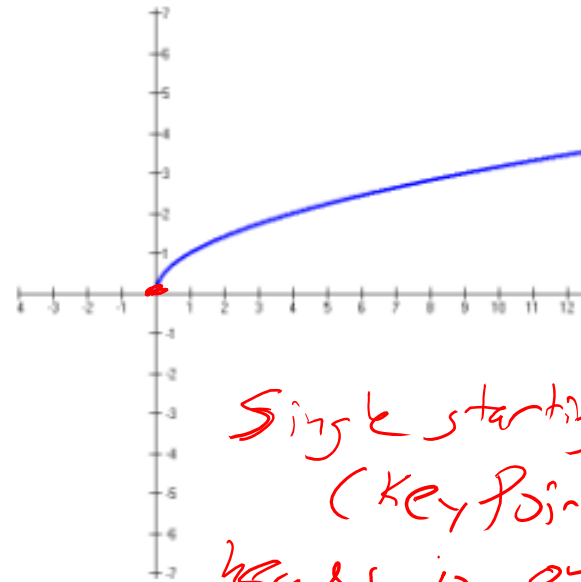


V-shaped graph

Radical Function

$$f(x) = \sqrt{x}$$

square root in the equation.



Single starting point (key point)
heads in one direction

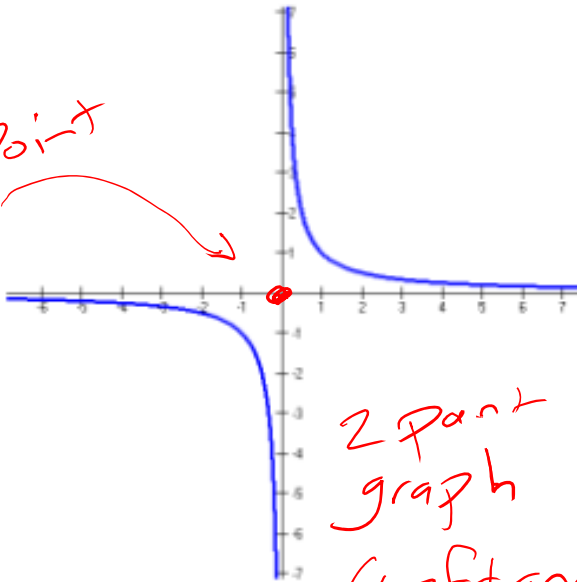
know all by the: ① Graph
② Equation
③ Name

Rational Function

$$f(x) = \frac{1}{x}$$

x in the denominator

Key Point

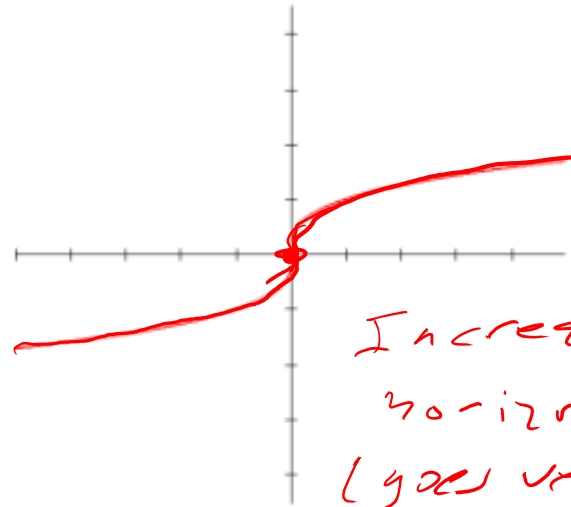


2 part graph
(left and right)

Cube Root Function

$$f(x) = \sqrt[3]{x}$$

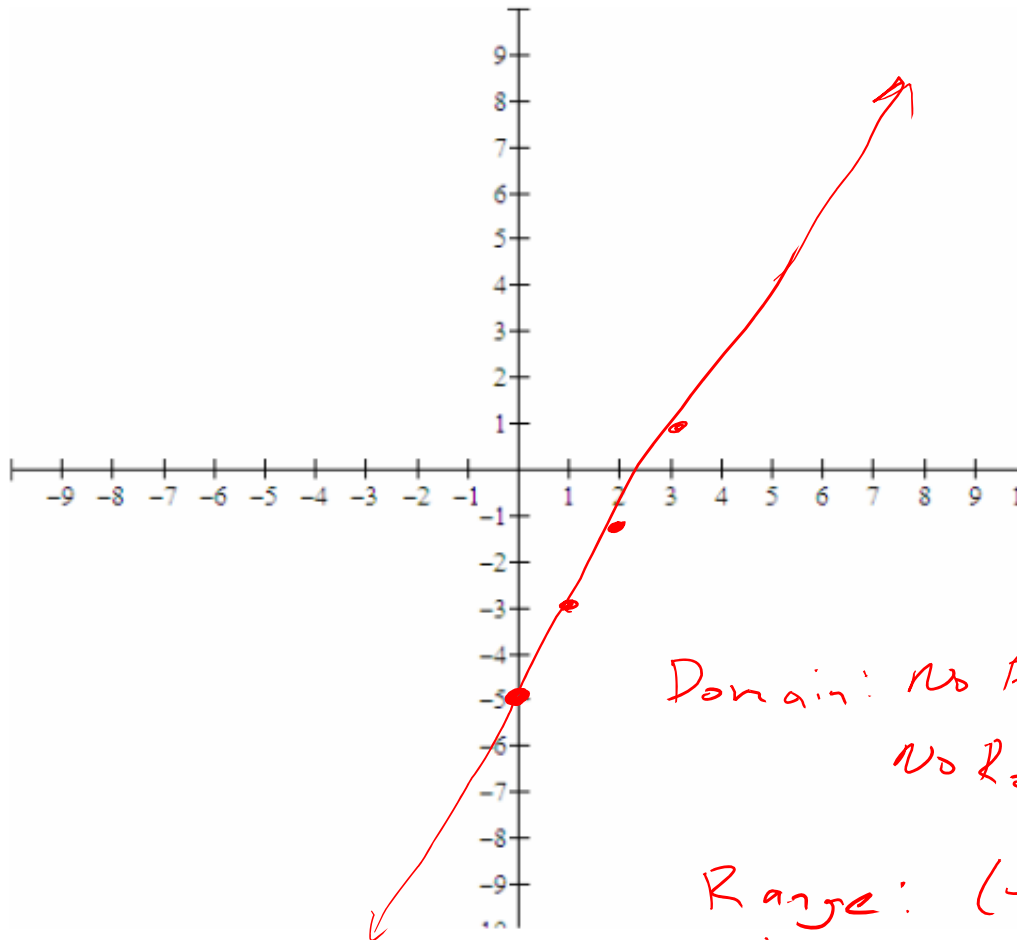
← cube root in the equation.



Increasing horizontally
(goes vertical at $x=0$)

Linear

Example 4: Suppose $f(x) = 2x - 5$. State the domain of the function and graph it.



Linear:

x is raised to the first power

$$y = 2x - 5$$

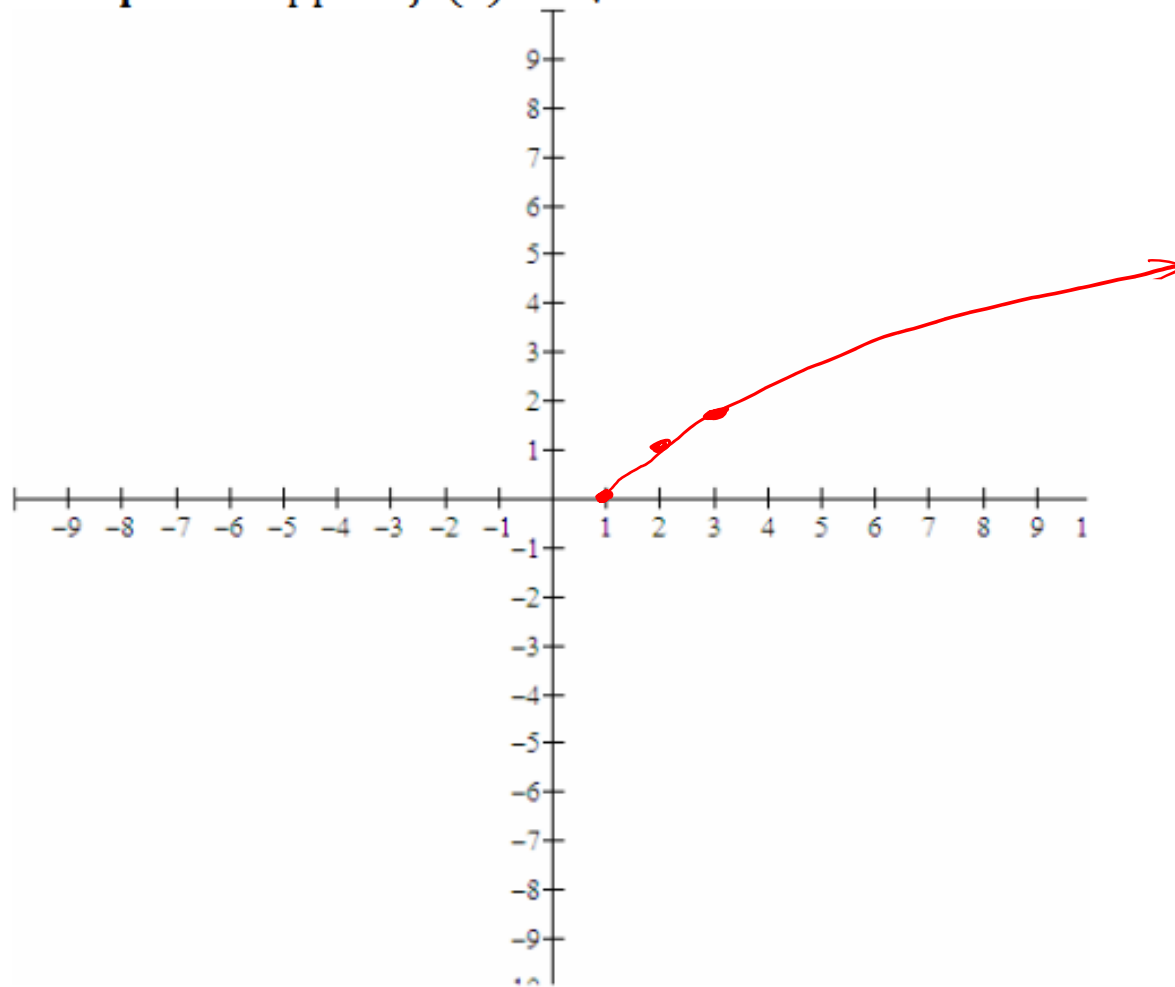
↑
slope: $m = \frac{2}{1}$

y -intercept: $b = -5$

Domain: No Fractions
No Roots $(-\infty, \infty)$

Range: $(-\infty, \infty)$
↳ From the Graph

Example 6: Suppose $f(x) = \sqrt{x-1}$. State the domain of the function and graph it.



Radical Function: $y = \sqrt{x}$
(Parent function)

Domain: $x - 1 \geq 0$
 $x \geq 1 \rightarrow [1, \infty)$

x	y	Point
1	$\sqrt{1-1} = \sqrt{0} = 0$	(1, 0)
2	$\sqrt{2-1} = \sqrt{1} = 1$	(2, 1)
3	$\sqrt{3-1} = \sqrt{2} \approx 1.4$	(3, $\sqrt{2}$)

For $f(x) = \frac{5}{2x+4}$ evaluate $f\left(\frac{a+1}{a-1}\right)$

$$\begin{aligned} f\left(\frac{a+1}{a-1}\right) &= \frac{5}{2\left(\frac{a+1}{a-1}\right)+4} = \frac{5(a-1)}{\cancel{2(a-1)} + 4(a-1)} = \frac{5a-5}{2a+2+4a-4} \\ &= \boxed{\frac{5a-5}{6a-2}} \end{aligned}$$

For $g(x) = x^2 + 2x - 1$ evaluate $g\left(\frac{5}{b}\right)$

$$g\left(\frac{5}{b}\right) = \left(\frac{5}{b}\right)^2 + 2\left(\frac{5}{b}\right) - 1$$

(CD: b^2)

$$= \frac{25}{b^2} + \frac{10 \cdot b}{b \cdot b} - \frac{1 \cdot b^2}{b^2} = \frac{25}{b^2} + \frac{10b}{b^2} - \frac{b^2}{b^2}$$

$$= \frac{25 + 10b - b^2}{b^2} = -\frac{b^2 - 10b - 25}{b^2}$$

negative is factored out

Popper 10....continued:

Example 8: Let $P(x) = \begin{cases} -3, & x < 2 \\ x^2, & x > 2 \\ 2, & x = 2 \end{cases}$ State the domain of the function and graph it.

Handwritten notes: \rightarrow constant (above $x < 2$), \rightarrow Quadratic (below x^2), $D: (-\infty, \infty)$ (top right), $\rightarrow (2, 2)$ (next to $x = 2$)

Find $p(-2)$, $p(2)$ and $p(3)$.

5. $P(-2) = -3$
 $[-2 < 2]: y = -3$

5C

6. $P(2) = 2$
 $[2 = 2]: (2, 2)$

6A

7. $P(3) = 3^2 = 9$
 $[3 > 2]: y = x^2$

7D

- a. 2
- b. 4
- c. -3
- d. 9

b. Sketch the graph of p .

$$P(x) = \begin{cases} -3, & x < 2 \\ x^2, & x > 2 \\ 2, & x = 2 \end{cases}$$

