MATH 1314

Section 3.2

Functions and Graphs

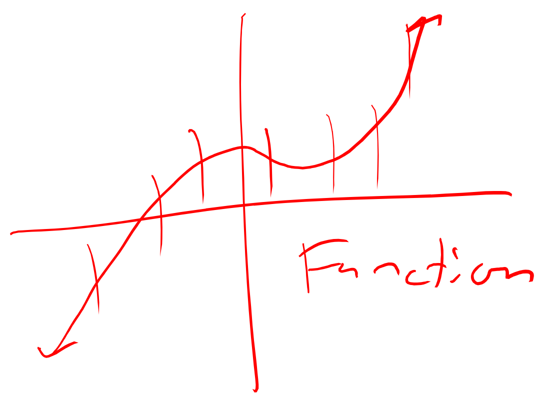
You can answer many questions given a graph.

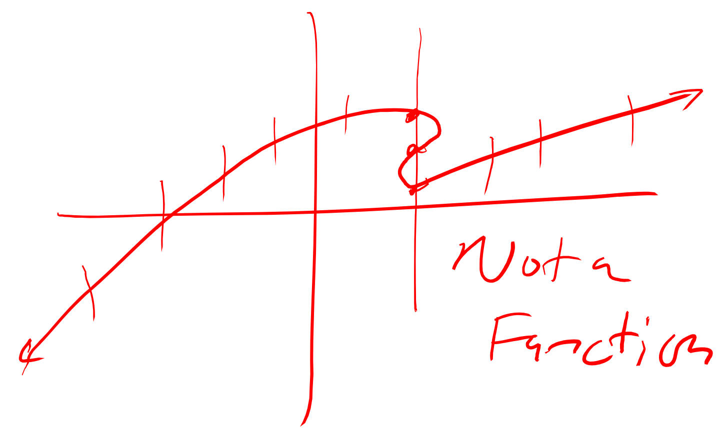
Every x-value corresponds to exactly 1 y-value.

Definition: The graph of a function $f(x)$ is the set of points (x, y) whose x coordinates are in the domain of f and whose y coordinates are given by $y = f(x)$.

First, does the graph represent a function? To answer this, you will need to use the **vertical line test (VLT)**.

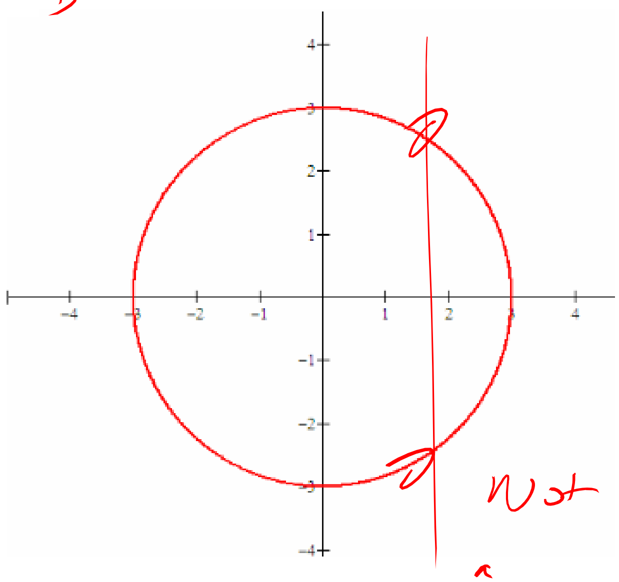
The Vertical Line Test:
If you can draw a vertical line that crosses the graph more than once, it is NOT the graph of a function.





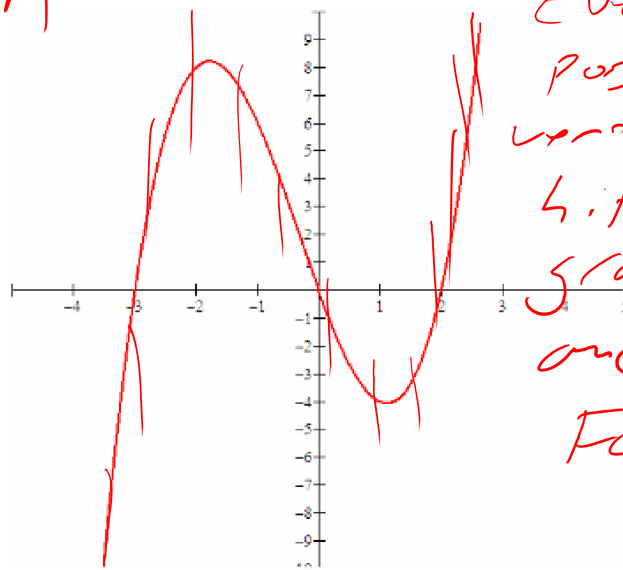
Popper 11: Does the graph represent a function? a. Yes b. No

1. B



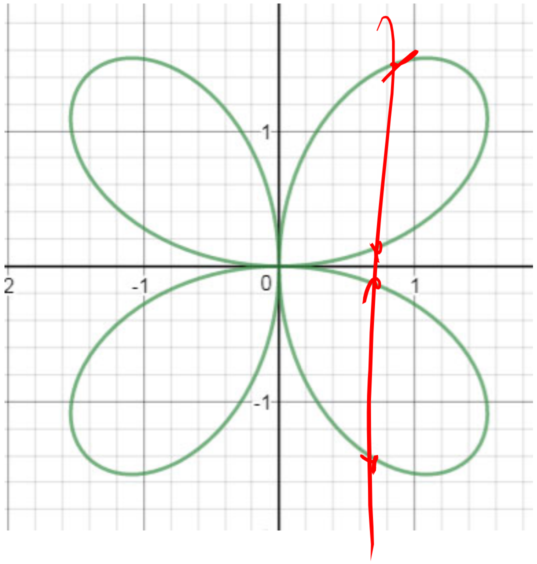
Not a Function.

2. A



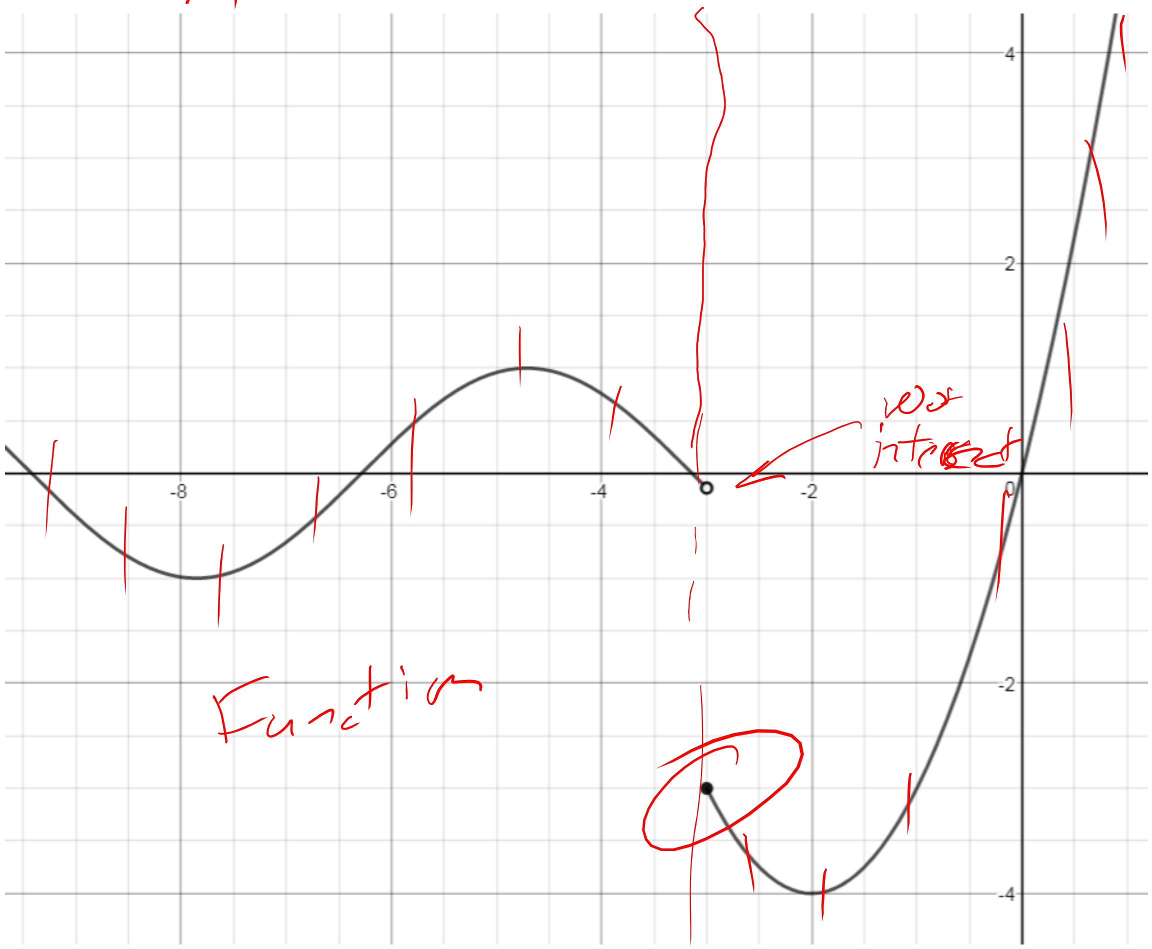
Every possible vertical line hits the graph once → Function.

3. B



Not a Function

4. A



Function

Definition: An **equation defines *y* as a function of *x*** if when one value for *x* is substituted in the equation, **exactly one value for *y* is returned.**

Example 2: Does the following equation define *y* as a function of *x*?

$y - x^2 = 4$

1. Solve for *y*.
2. For each value *x*, do we get exactly one value for *y* back?

$y - \cancel{x^2} = 4$

$\cancel{+x^2} \quad +x^2$

$y = x^2 + 4$

Function

No $\pm$ appeared in the solving.

$y = |x|$

Example: $x = 5 \to y = 5$

$x = -5 \to y = 5$

Function

$x = |y| \to y = \pm x$

Not a Function.

$x = 5 < \begin{matrix} y = 5 \\ y = -5 \end{matrix}$

b. $x^2 + y^2 = 9$

1. Solve for y.

2. For each value x, do we get exactly one value for y back?

$$\cancel{x^2} + y^2 = 9$$
$$\cancel{-x^2} \qquad -x^2$$
$$\sqrt{y^2} = \sqrt{9-x^2}$$
$$y = \pm\sqrt{9-x^2}$$

A single x gives 2 y-values (not a function)

General Rule:

If there are any y raised to an even power → Not a function.

$2x^2 + 3y^2 + 2x + 4y = 10$

→ Not a Function.

$2x^2 + 2x + 4y = 10$

Is a function.

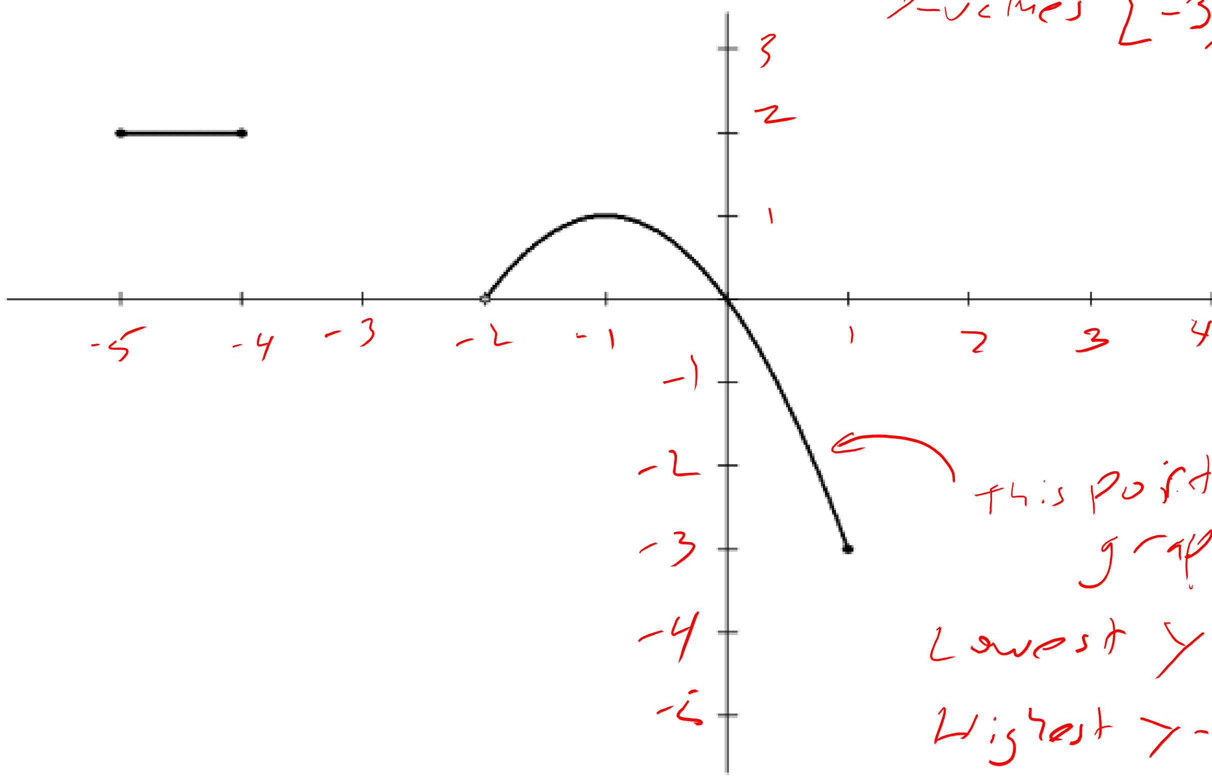
Example 3: Find the domain and range of the function whose graph is shown.

x-values

Domain: $[-5, -4] \cup (-2, 1]$

Range: $[-3, 1] \cup [2, 2]$ or

y-values $[-3, 1] \cup \{2\}$



This portion of graph:

Lowest y-value: -3

Highest y-value: +1

You'll also need to be able to graph functions. For now, you can do so by plotting points. But...

YOU MUST KNOW THESE FUNCTIONS AND GRAPHS

Library of Functions

Constant Function

y = k

$y = 5$

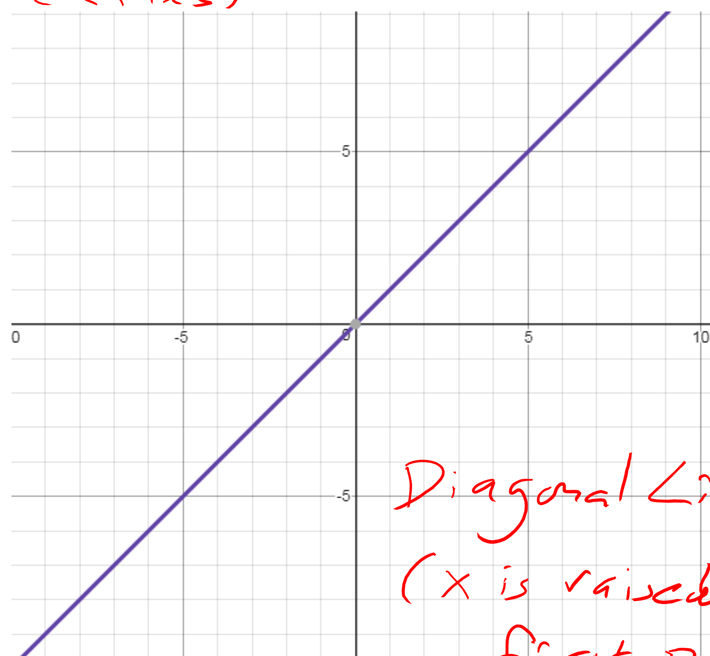
Horizontal Line

Identity Function (Linear)

y = x

(Lines)

$y = 2x+5$



Diagonal Lines

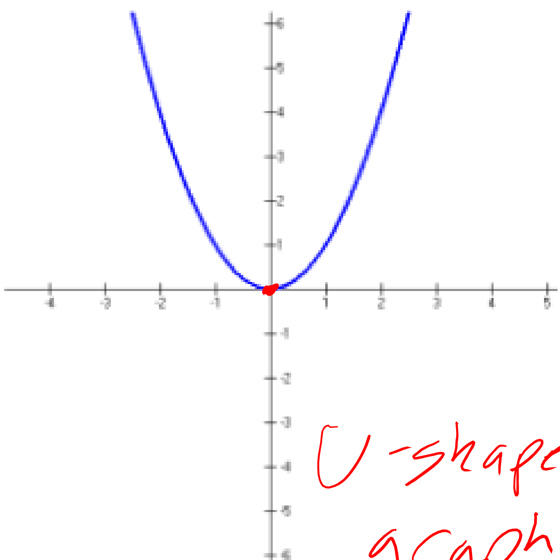
(x is raised to first power)

Key Point: Central point on the graph that is unaltered during transformations (0,0)

Quadratic Function

(highest) x is raised to a 2.

$f(x) = x^2$



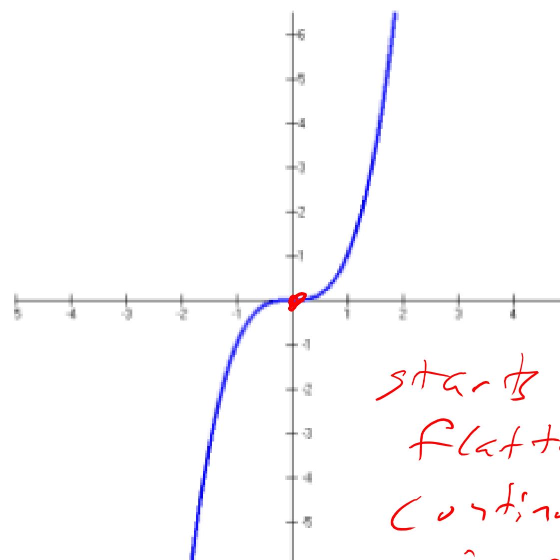
U-shaped graph

$f(x) = 2x^2 + 5x - 1 \rightarrow$ Quadratic

Cubic Function

x raised to 3 as highest exponent.

$f(x) = x^3$

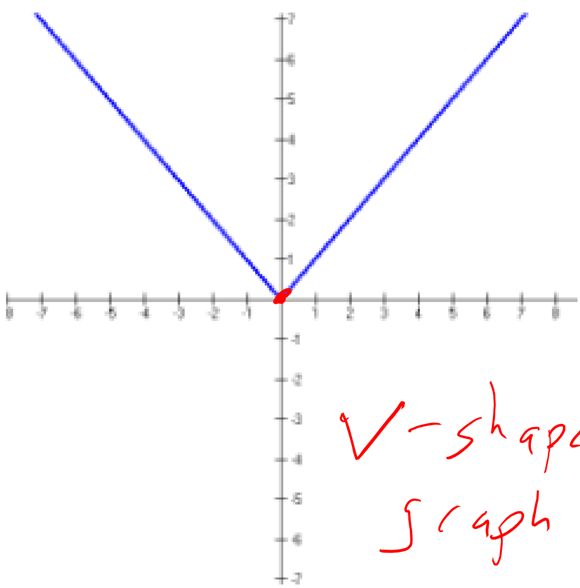


starts low, flattens, continues to increase

Absolute Value Function

$f(x) = |x|$

Absolute value symbols

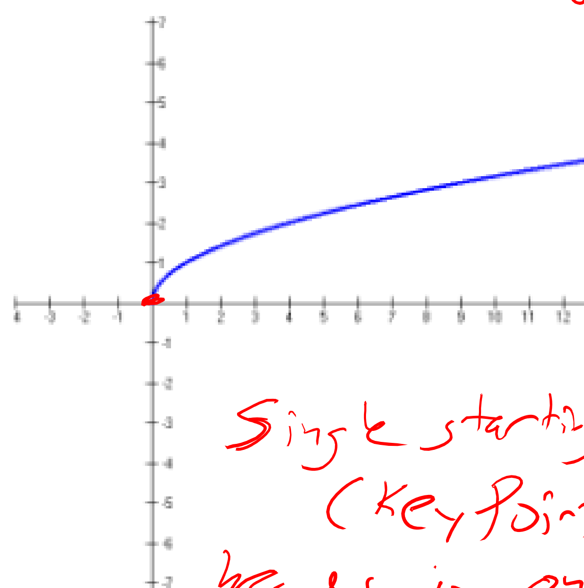


V-shaped graph

Radical Function

$f(x) = \sqrt{x}$

square root in the equation.



Single starting point (key Point) heads in one direction

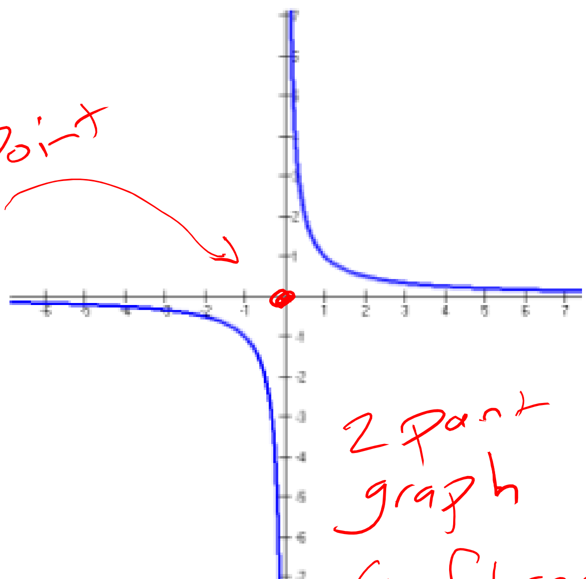
Know all by their ① Graph
② Equation
③ Name

Rational Function

$$f(x) = \frac{1}{x}$$

x in the denominator

Key Point

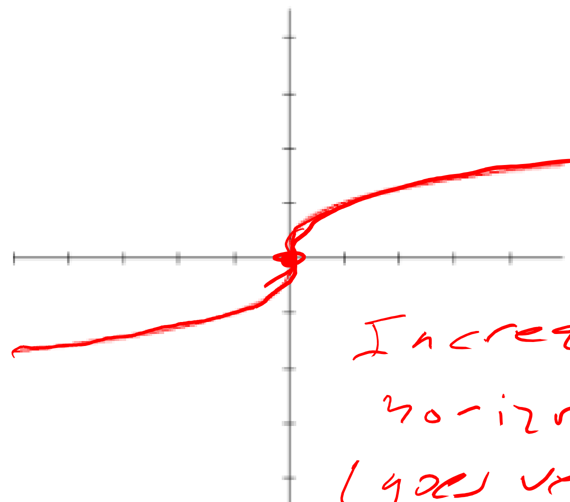


2 part graph (left and right)

Cube Root Function

$$f(x) = \sqrt[3]{x}$$

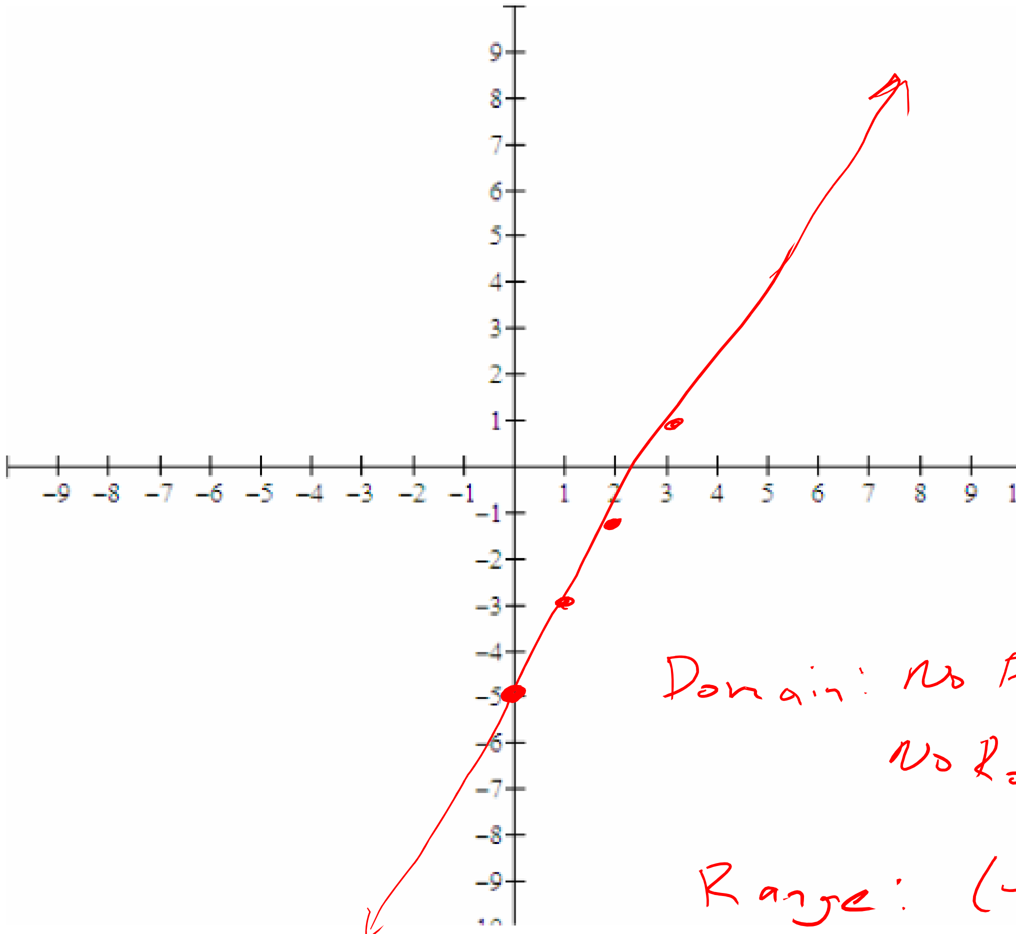
Cube root in the equation.



Increasing horizontally (goes vertical at x=0)

Linear

Example 4: Suppose $f(x) = 2x - 5$. State the domain of the function and graph it.



Linear:
x is raised to the first power

$y = 2x - 5$

slope: $m = \frac{2}{1}$

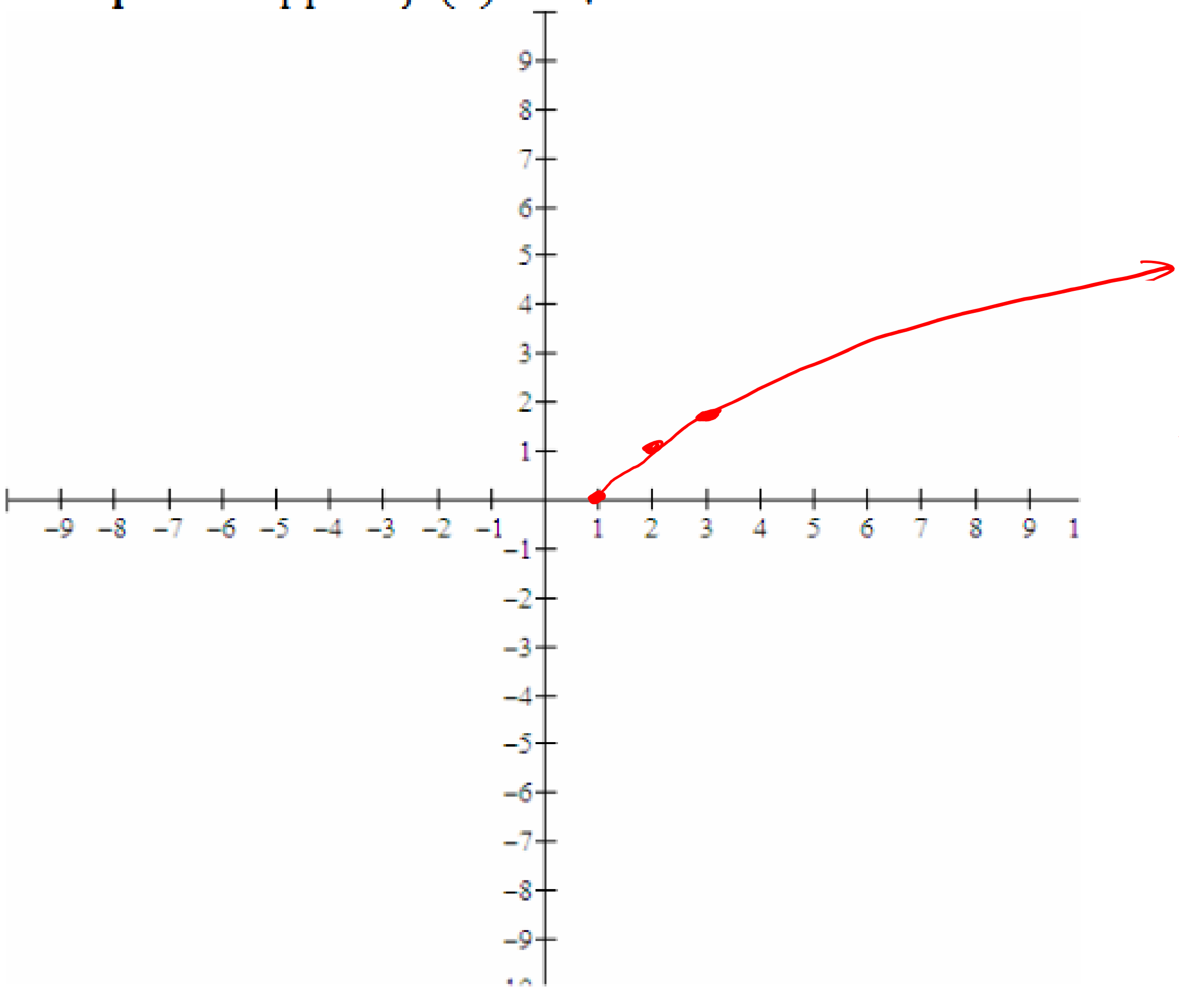
y-intercept: $b = -5$

Domain: No Fractions
No Roots

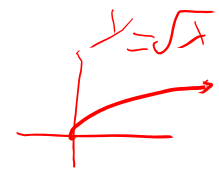
$(-\infty, \infty)$

Range: $(-\infty, \infty)$
↳ From the Graph

Example 6: Suppose $f\,(x) = \sqrt{x-1}$. State the domain of the function and graph it.



Radical Function:



(Parent function)

Domain: $x-1 \geq 0$
$\quad\;\; +1 \quad +1$
$x \geq 1 \rightarrow [1, \infty)$

x	y	
1	$\sqrt{1-1} = \sqrt{0} = 0$	(1,0)
2	$\sqrt{2-1} = \sqrt{1} = 1$	(2,1)
3	$\sqrt{3-1} = \sqrt{2} \approx 1.4$	$(3, \sqrt{2})$

For $f(x) = \frac{5}{2x+4}$ evaluate $f\left(\frac{a+1}{a-1}\right)$

$$f\left(\frac{a+1}{a-1}\right) = \frac{5}{2\left(\frac{a+1}{a-1}\right)+4} = \frac{5\,(a-1)}{\frac{2a+2}{\cancel{a-1}}\cancel{(a-1)}+4(a-1)} = \frac{5a-5}{2a+2+4a-4}$$

$$= \boxed{\frac{5a-5}{6a-2}}$$

For $g(x) = x^2 + 2x - 1$ evaluate $g\left(\frac{5}{b}\right)$

$$g\left(\frac{5}{b}\right) = \left(\frac{5}{b}\right)^2 + \frac{2}{1}\left(\frac{5}{b}\right) - 1$$

$$= \frac{25}{b^2} + \frac{10 \cdot b}{b \cdot b} - \frac{1}{1} \cdot \frac{b^2}{b^2} = \frac{25}{b^2} + \frac{10b}{b^2} - \frac{b^2}{b^2}$$

(CD: b^2)

$$= \frac{25 + 10b - b^2}{b^2} = \boxed{-\frac{b^2 - 10b - 25}{b^2}}$$

negative is factored out

Popper 11….continued:

constant

$D: (-\infty, \infty)$

Example 8: Let $P(x) = \begin{cases} -3, & x < 2 \\ x^2, & x > 2 \\ 2, & x = 2 \end{cases}$ State the domain of the function and graph it. $\longrightarrow (2,2)$

Quadratic

Find $p(-2)$, $p(2)$ and $p(3)$.

5. P(-2) $= -3$ 5C

[$-2<2$] : $y=-3$

6. P(2) $= 2$ 6A

[$2=2$] : $(2,2)$

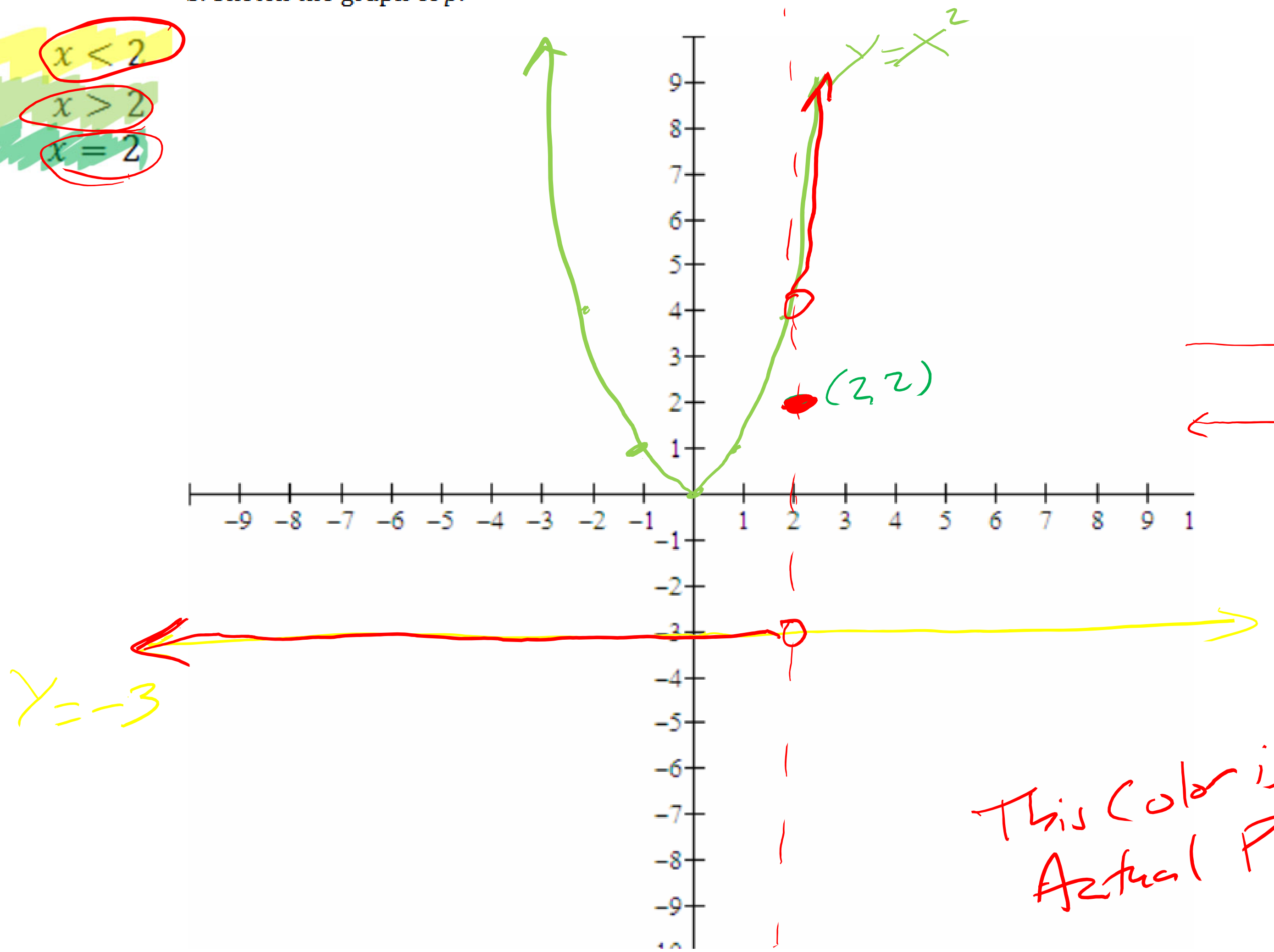
7. P(3) $= 3^2 = 9$ 7D

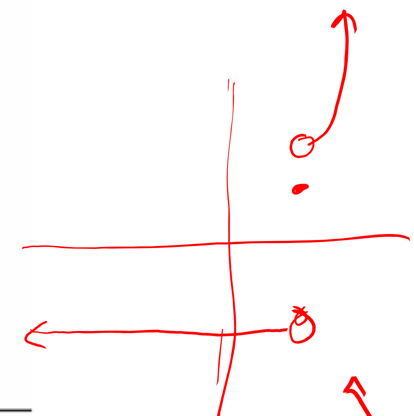
[$3>2$] : $y=x^2$

a. 2
b. 4
c. -3
d. 9

$$P(x) = \begin{cases} -3, & x < 2 \\ x^2, & x > 2 \\ 2, & x = 2 \end{cases}$$

b. Sketch the graph of p.





This Color is Actual P(x) Graph