MATH 1314

Section 3.2

Functions and Graphs

You can answer many questions given a graph.

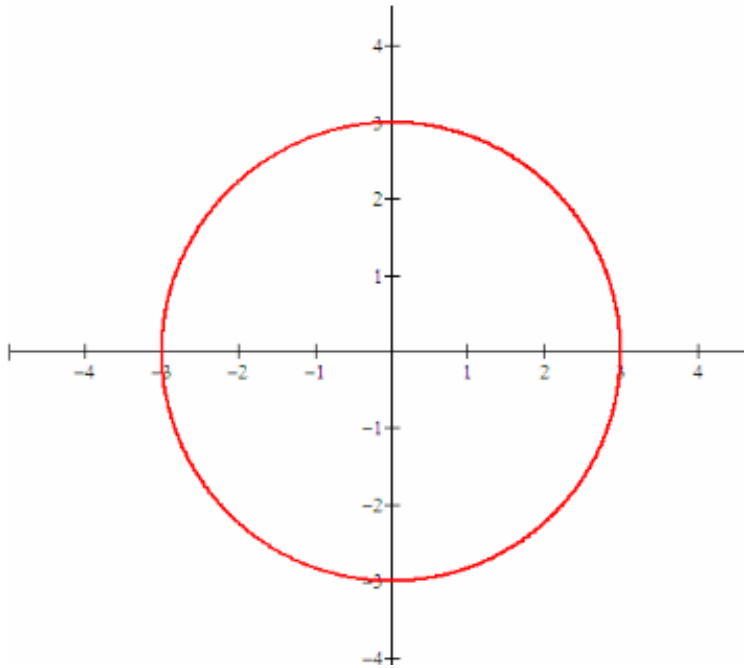
Definition: The graph of a function $f(x)$ is the set of points (x, y) whose x coordinates are in the domain of f and whose y coordinates are given by $y = f(x)$.

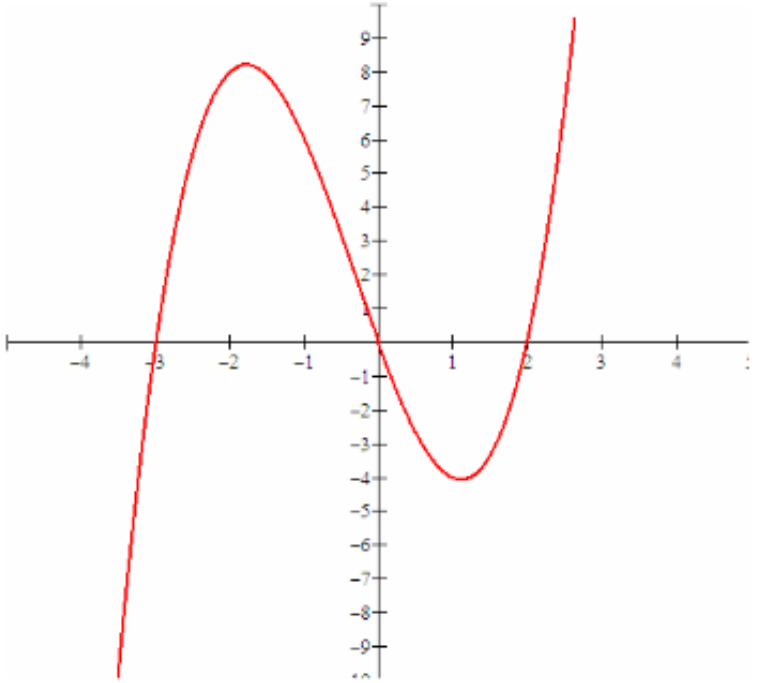
First, does the graph represent a function? To answer this, you will need to use the **vertical line test (VLT)**.

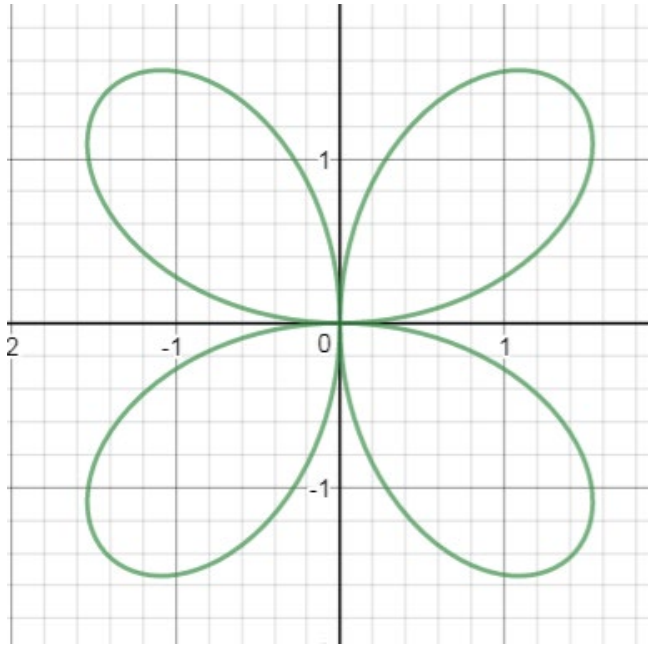
The Vertical Line Test:
If you can draw a vertical line that crosses the graph more than once, it is NOT the graph of a function.

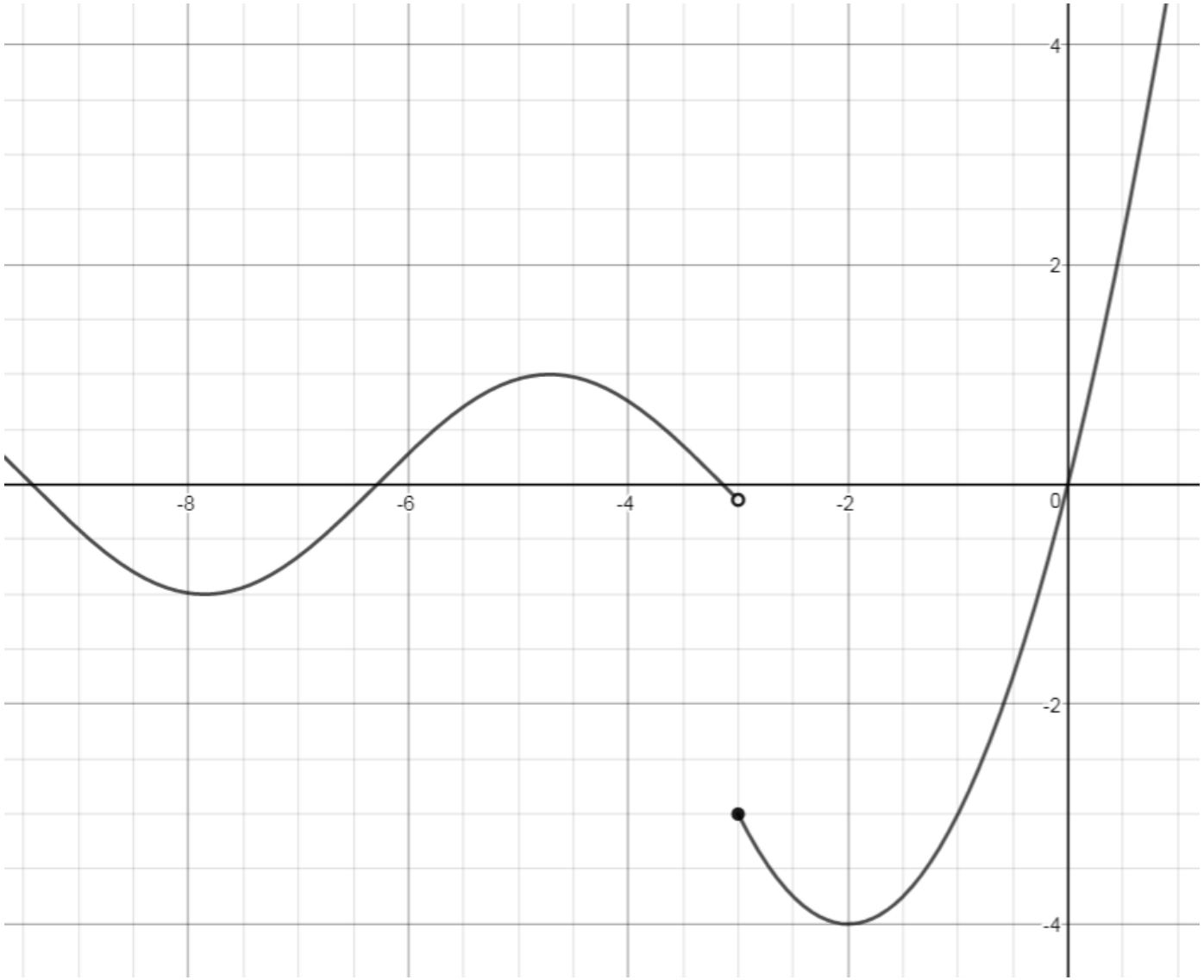
Does the graph represent a function?

a.






2
-1
0
1
1
-1


-8
-6
-4
-2
0
4
2
-2
-4

Definition: An **equation defines *y* as a function of *x*** if when one value for x is substituted in the equation, **exactly one value for *y* is returned.**

Example 2: Does the following equation define y as a function of x?

$y - x^2 = 4$

1. Solve for y.
2. For each value x, do we get exactly one value for y back?

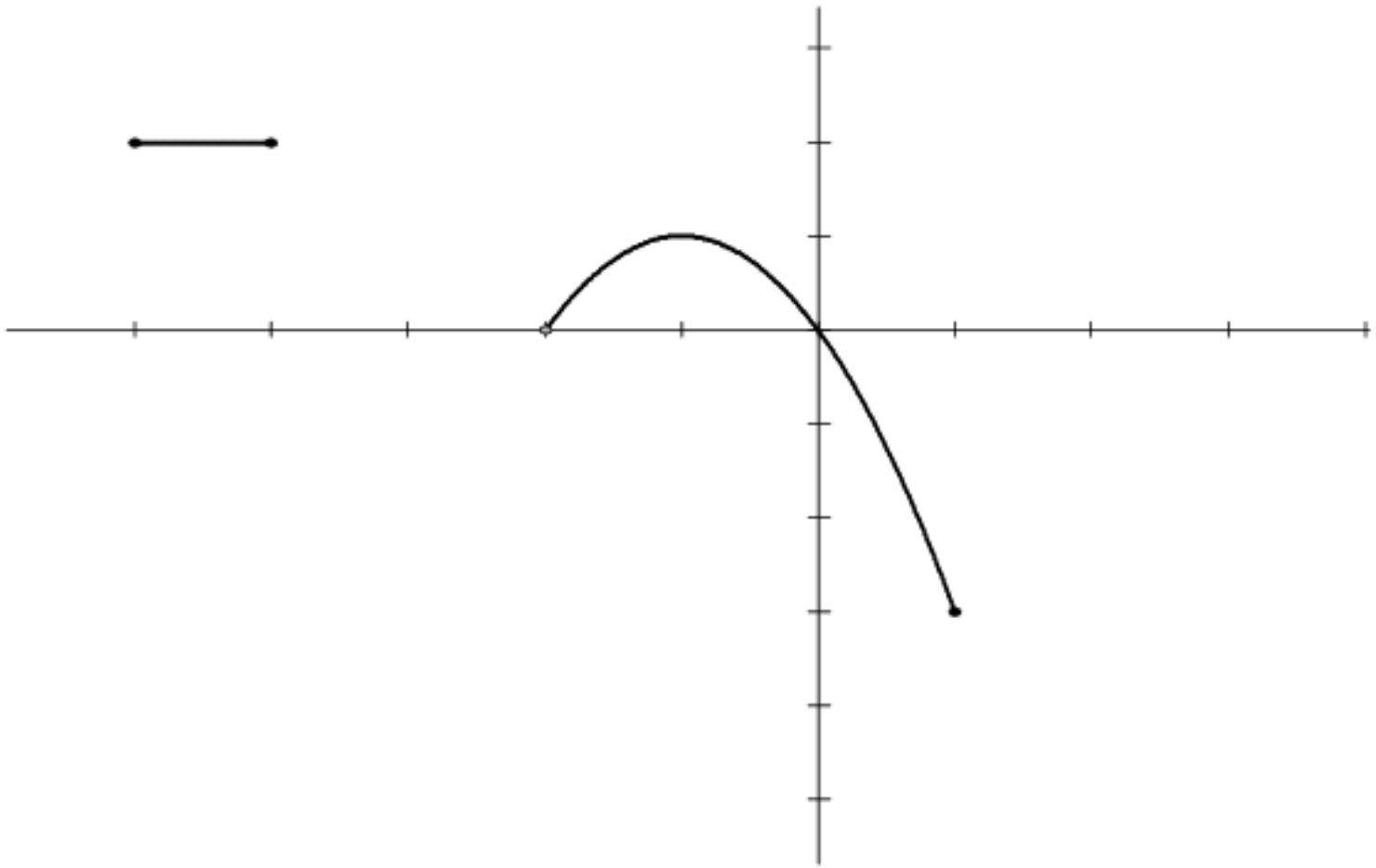
b. $x^2 + y^2 = 9$

1. Solve for y.
2. For each value x, do we get exactly one value for y back?

Example 3: Find the domain and range of the function whose graph is shown.

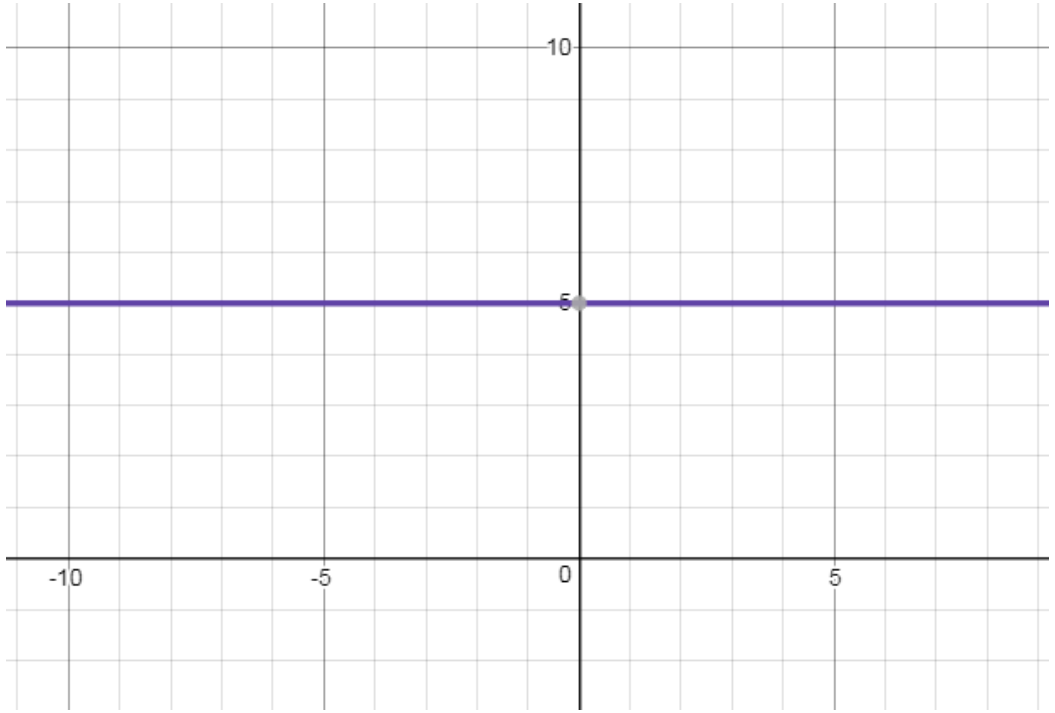
Domain: _____

Range: _____

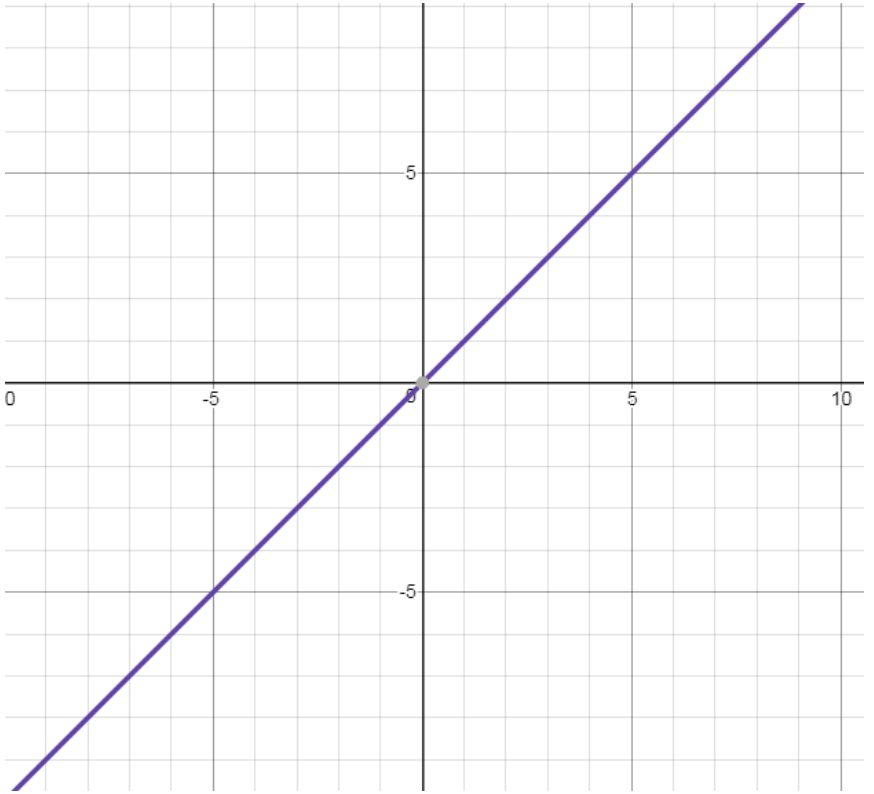


You'll also need to be able to graph functions. For now, you can do so by plotting points. But…
YOU MUST KNOW THESE FUNCTIONS AND GRAPHS

Constant Function
y = k

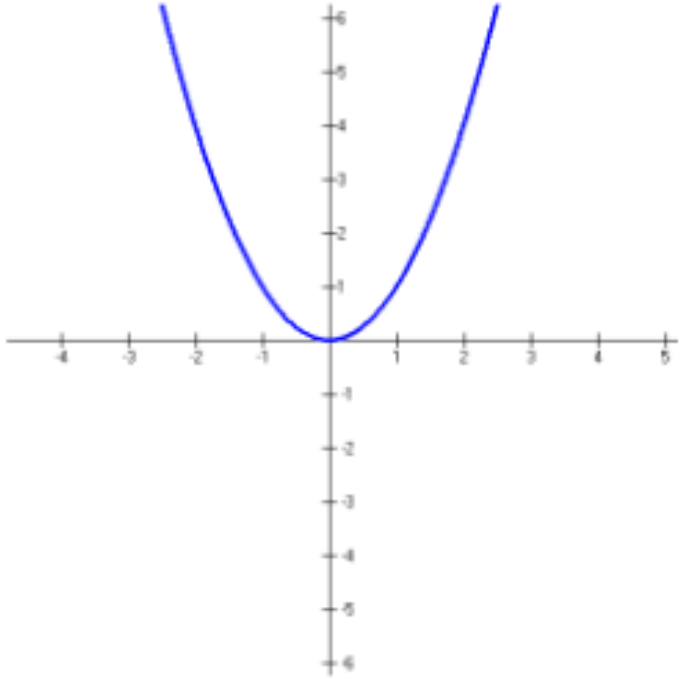


Identity Function
y = x



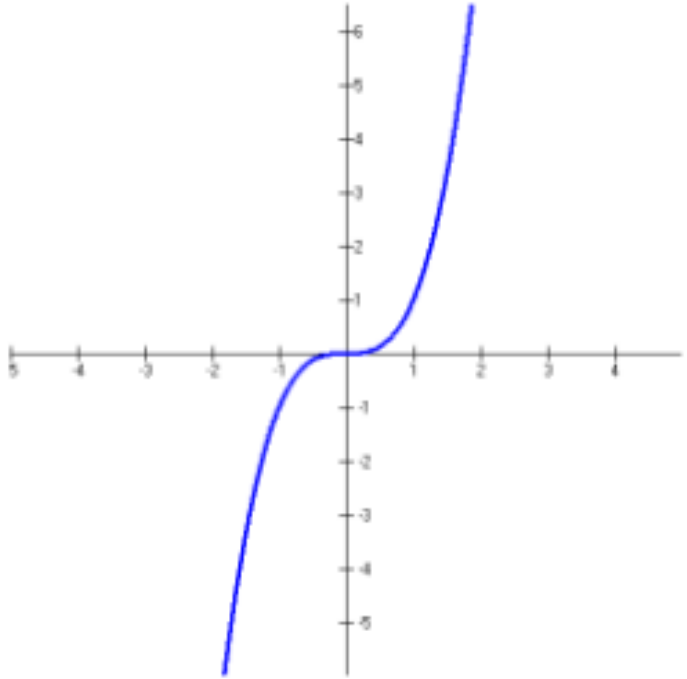
Quadratic Function

$f(x) = x^2$



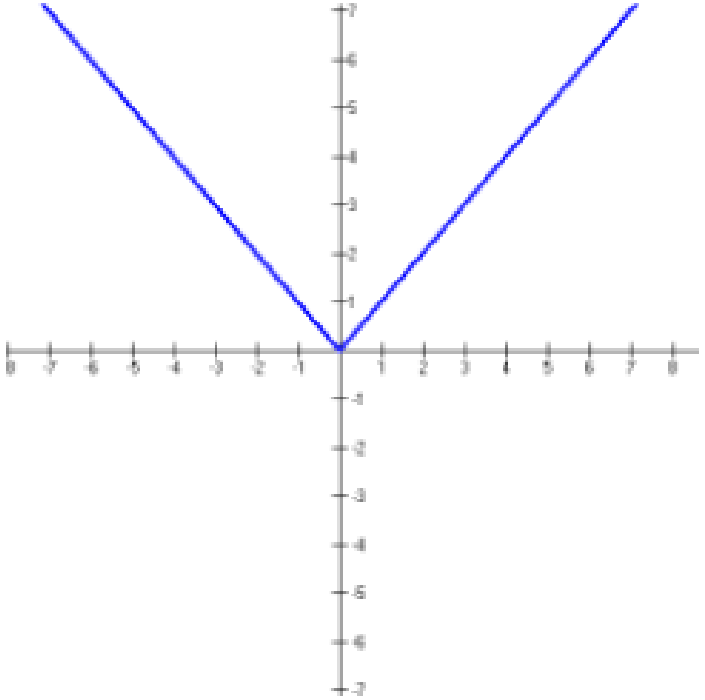
Cubic Function

$f(x) = x^3$



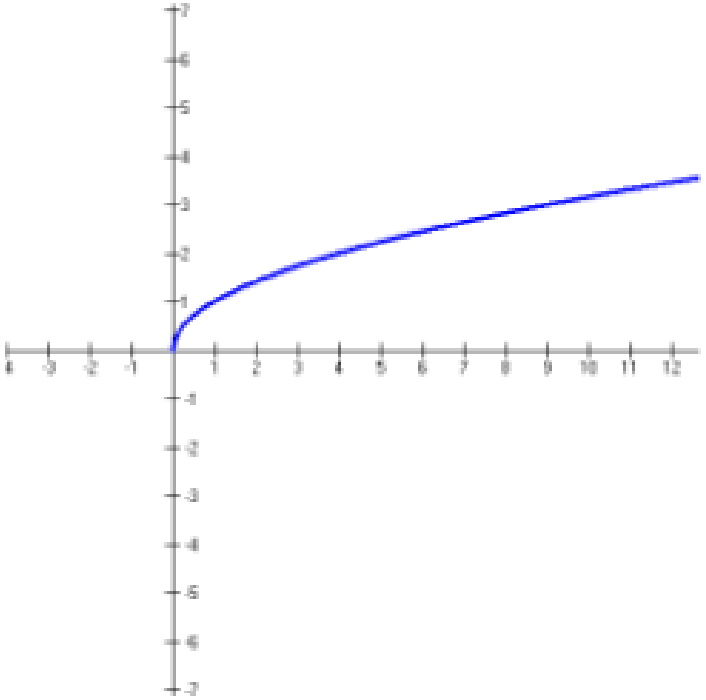
Absolute Value Function

$f(x) = |x|$



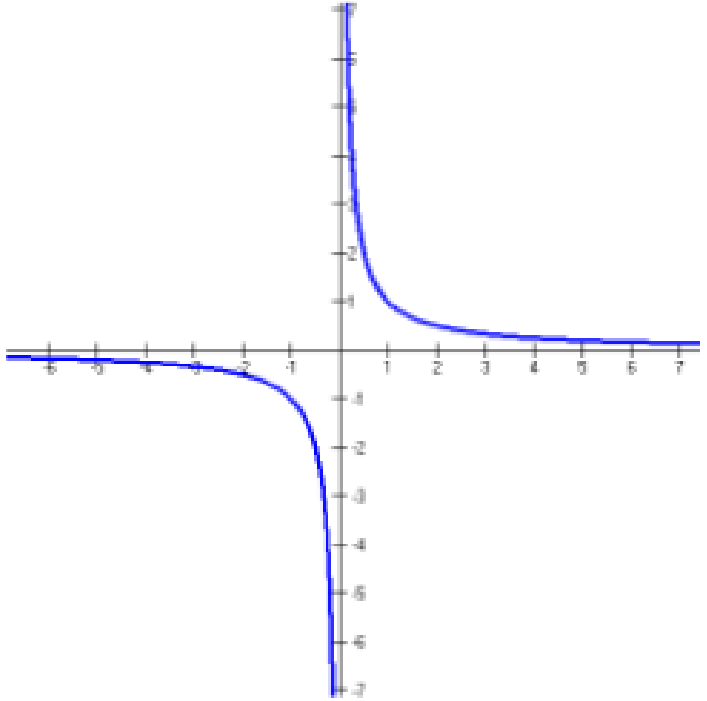
Radical Function

$f(x) = \sqrt{x}$



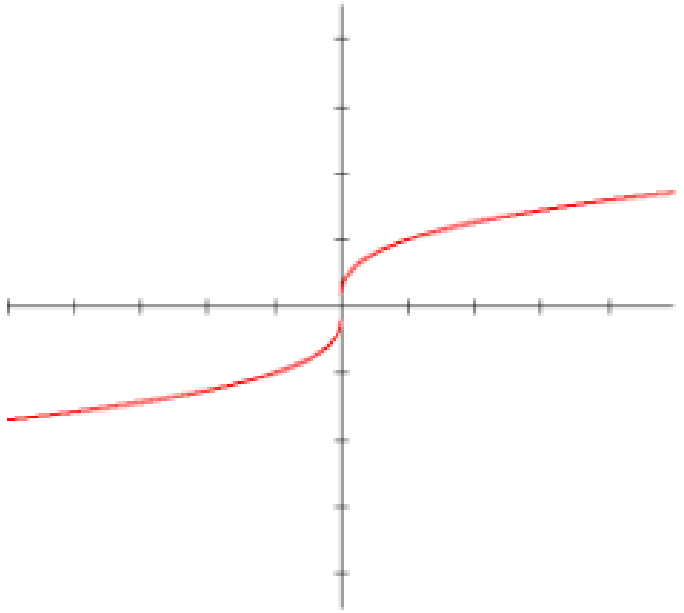
Rational Function

$$f(x) = \frac{1}{x}$$

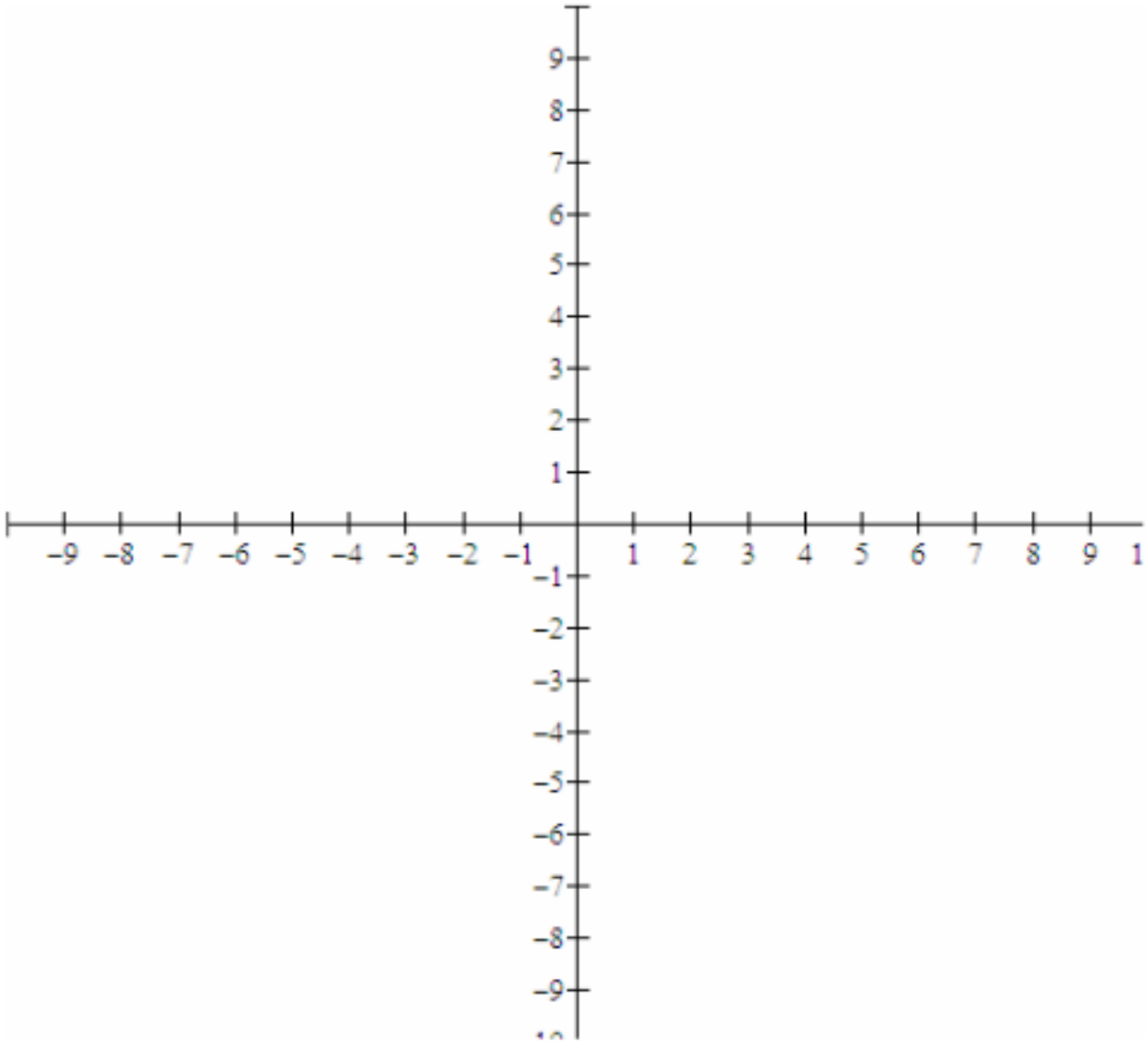


Cube Root Function

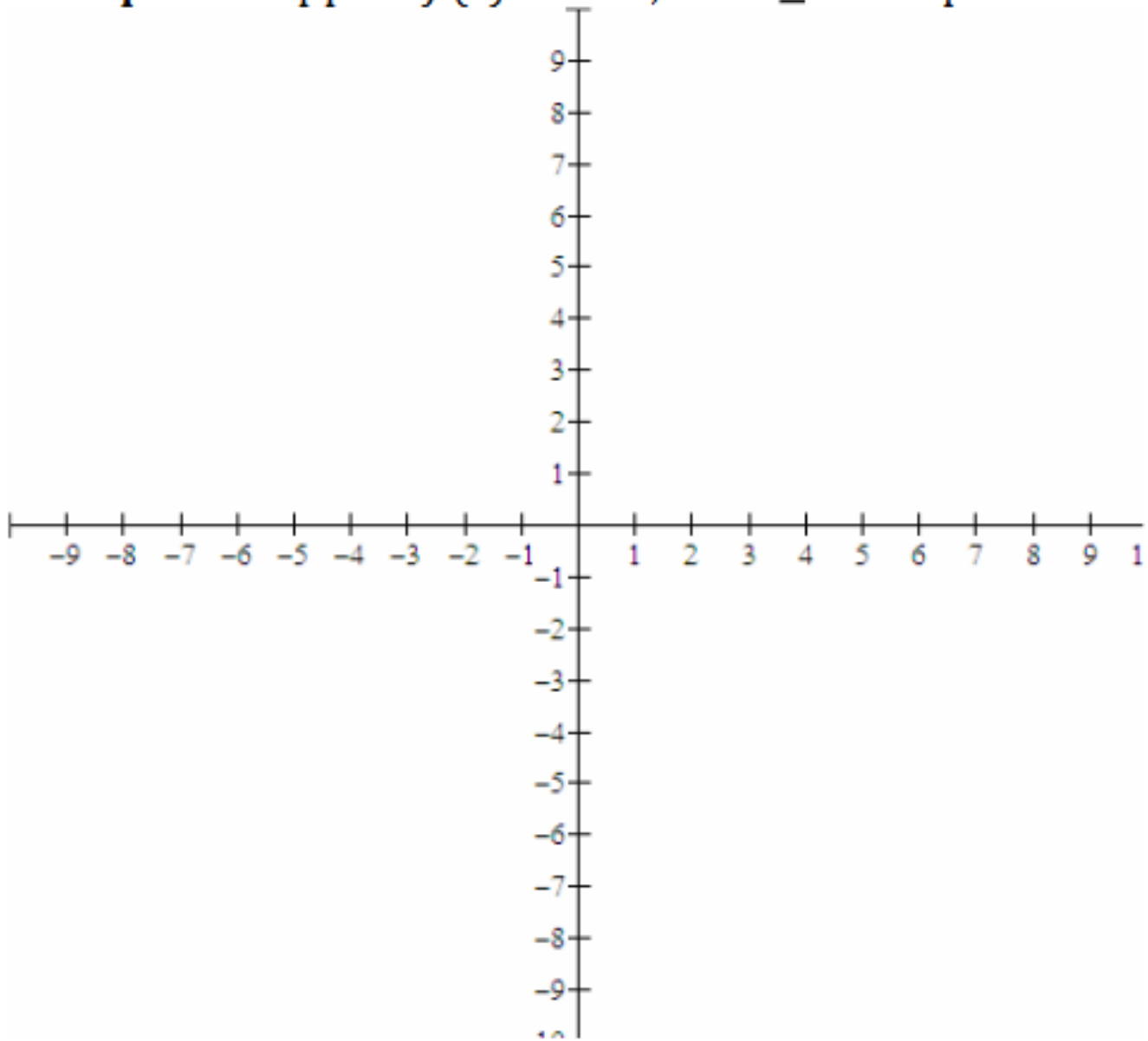
$$f(x) = \sqrt[3]{x}$$



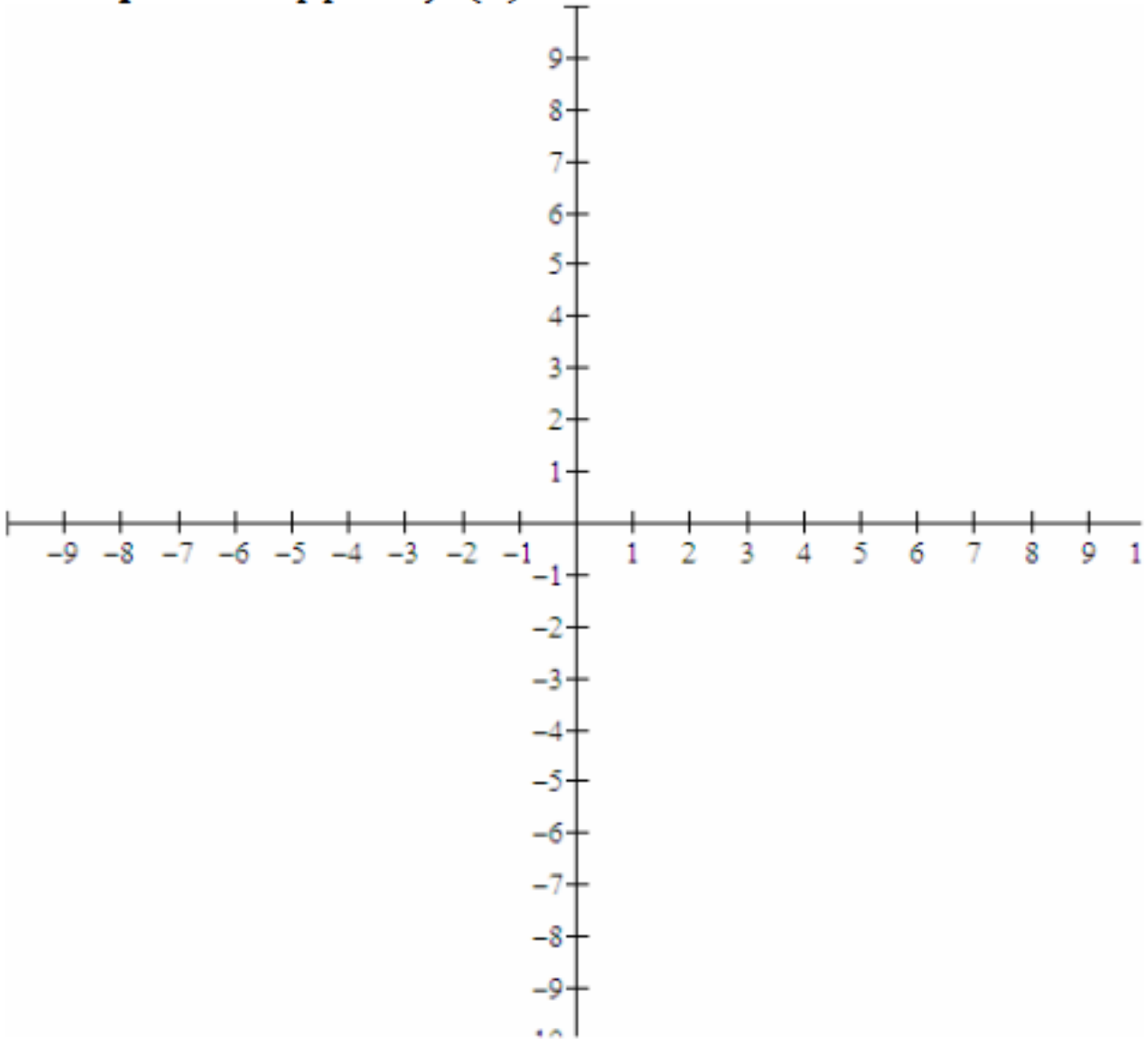
Example 4: Suppose $f(x) = 2x - 5$. State the domain of the function and graph it.



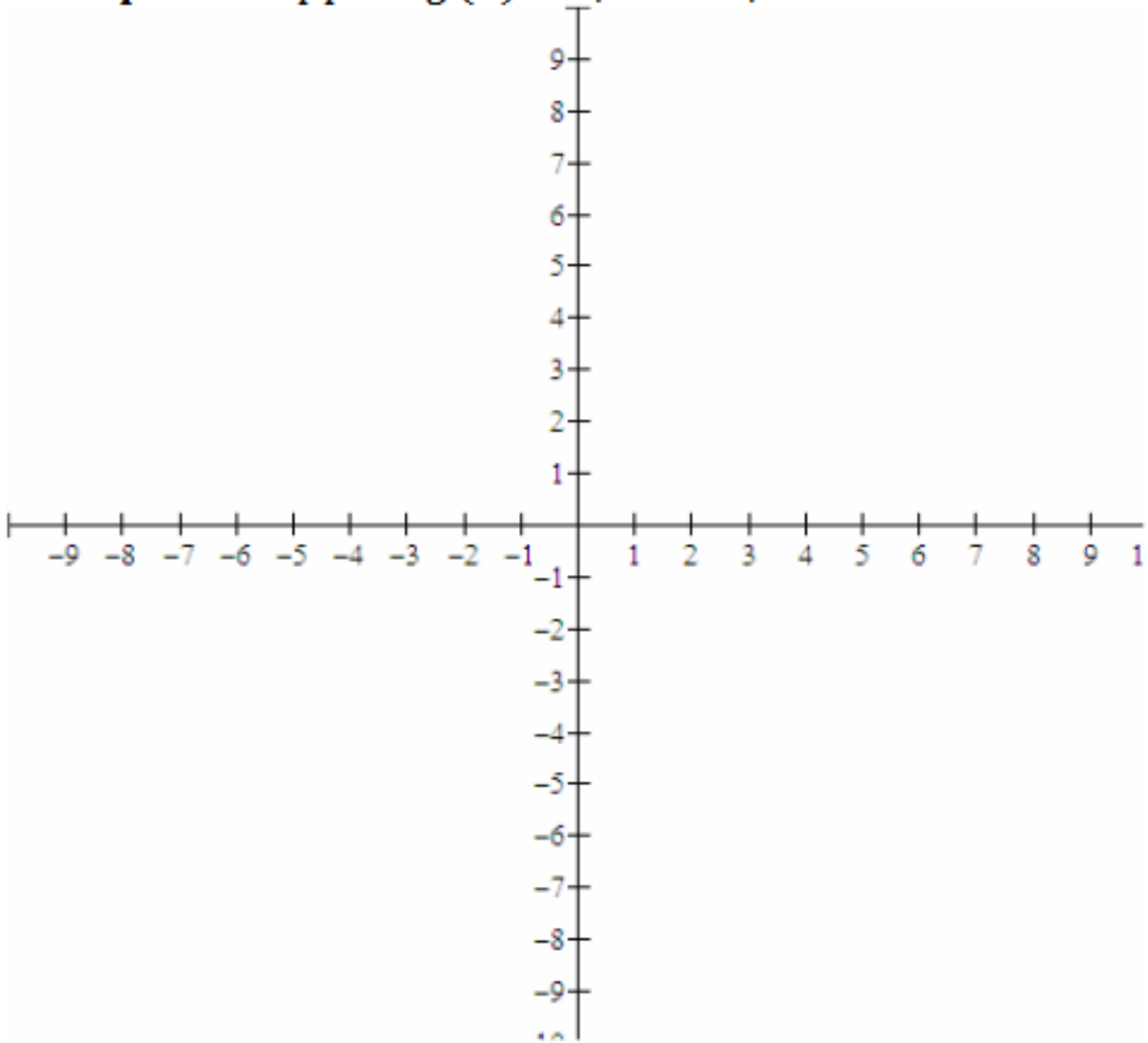
Example 5: Suppose $f(x) = 4x - 1, -1 < x \leq 2$. Graph the function.



Example 6: Suppose $f\ (x)\ =\ \sqrt{x\ -1}$. State the domain of the function and graph it.



Example 7: Suppose $g(x) = |x + 2| + 1$. State the domain of the function and graph it.



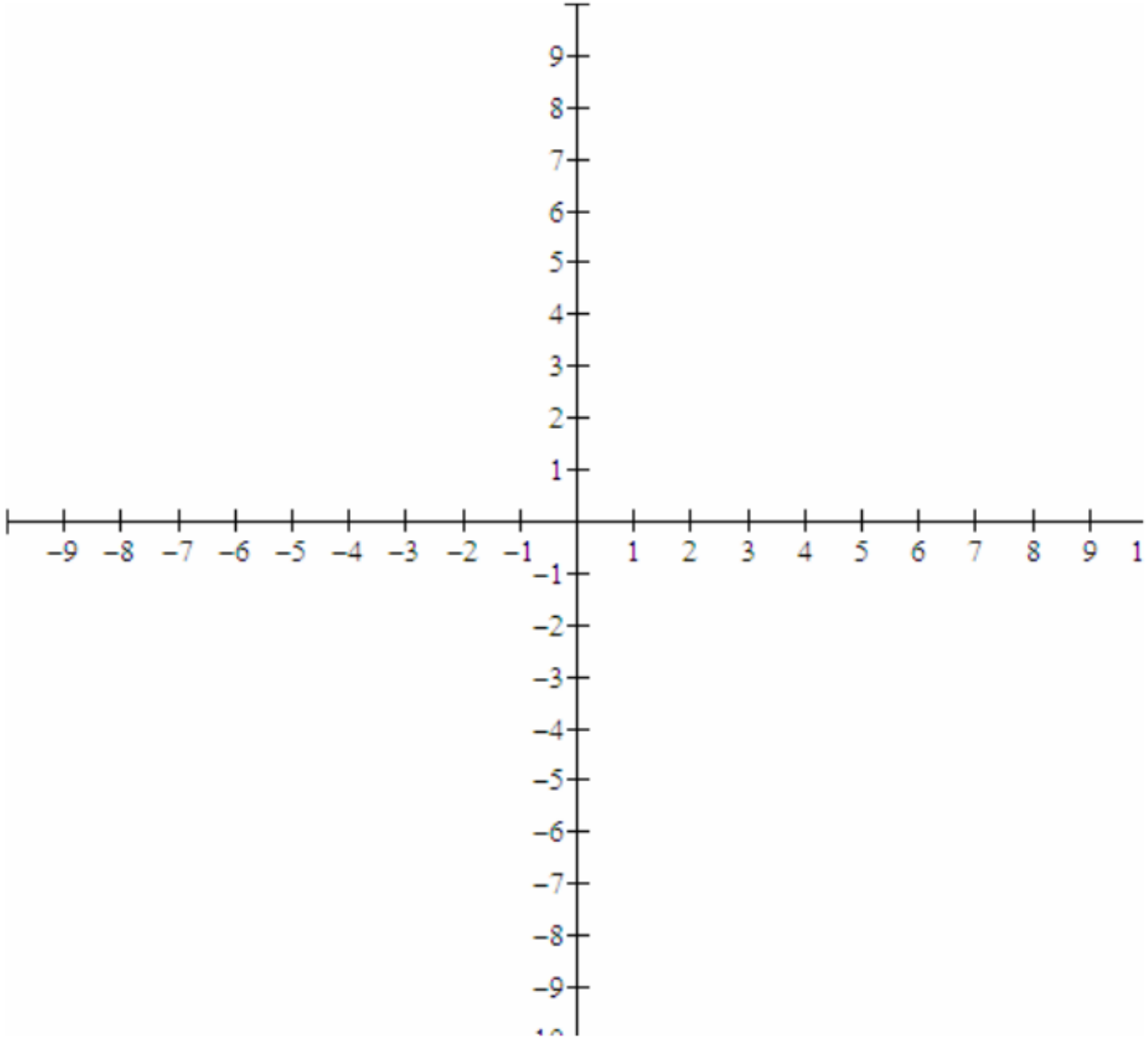
For $f(x) = \frac{5}{2x+4}$ evaluate $f\left(\frac{a+1}{a-1}\right)$

For $g(x) = x^2 + 2x - 1$ evaluate $g\left(\frac{5}{b}\right)$

Example 8: Let $P(x) = \begin{cases} -3, & x < 2 \\ x^2, & x > 2 \\ 2, & x = 2 \end{cases}$ State the domain of the function and graph it.

a. Find $p(-2)$, $p(2)$ and $p(3)$.

b. Sketch the graph of p.



Odd and Even Functions:

Odd Functions have only odd exponents, such as $f(x) = 2x^3 + 8x$.

They satisfy the formula: $f(-x) = -f(x)$

They are symmetric about the origin.

If they contain the point (a, b) they also contain (-a, -b).

Even Functions only have even exponents, such as $g(x) = 3x^4 + 2x^2 + 5$.

They satisfy the formula: $g(-x) = g(x)$

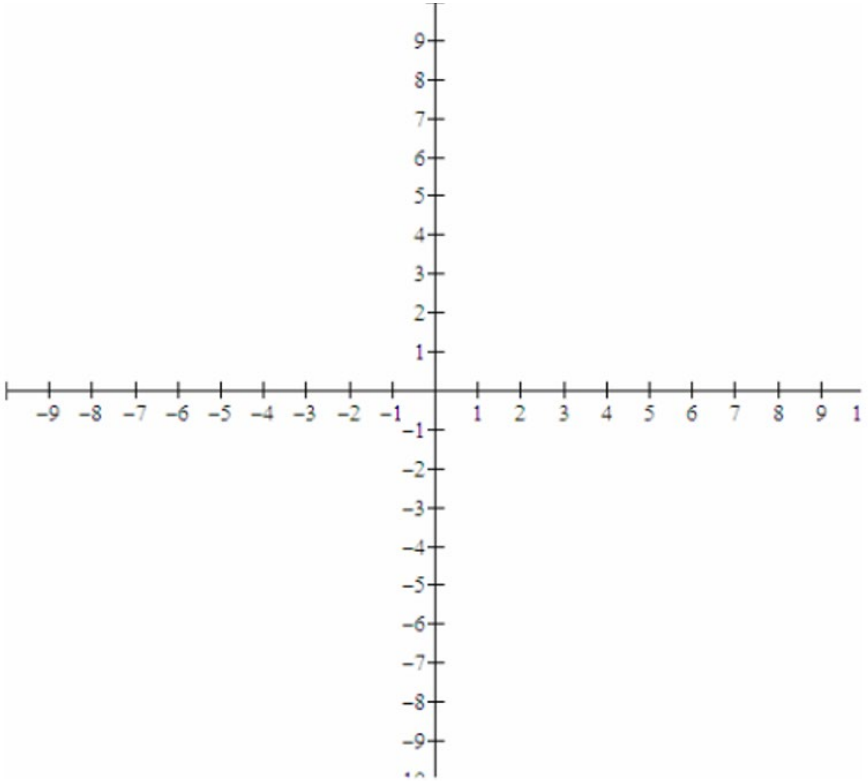
They are symmetric about the y-axis.

If they contain the point (a, b), they also contain (-a, b).

An even function contains the point (-5, -2).

What point must it also contain?

What is a possible graph of the function?

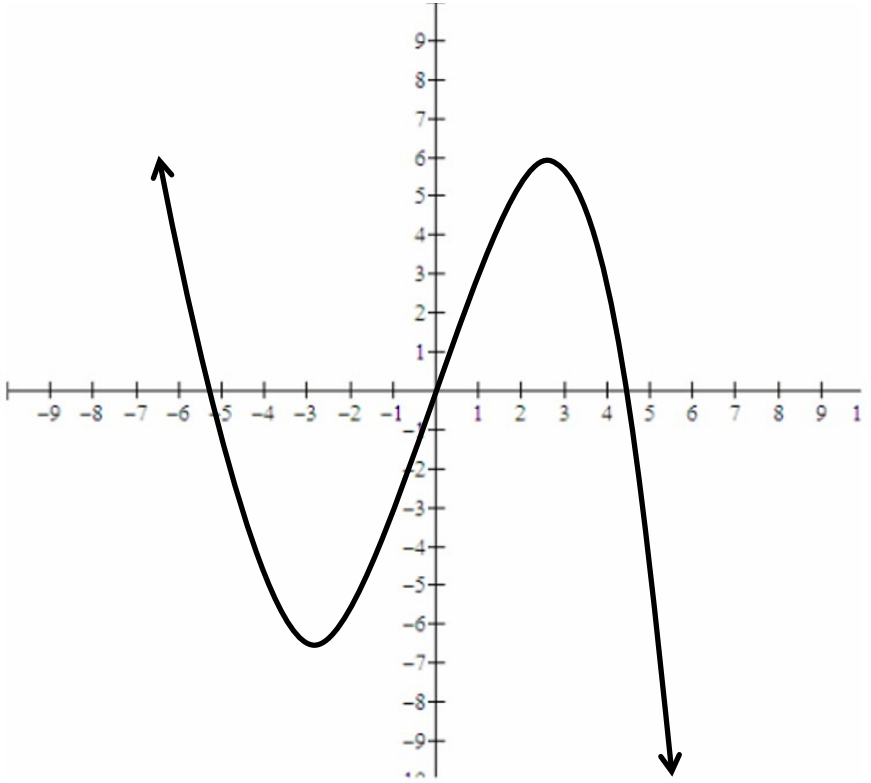


The following function passes through the point (8, -11).

Is the function even or odd?

What other point must it contain?

What is a possible equation?



Determine the value of the difference quotient for f(x) = -4x + 5

The difference quotient is:

$$\frac{f(x+h)-f(x)}{h}$$

Determine the value of the difference quotient for $f(x) = 2x^2 + 3x - 1$

The difference quotient is:

$$\frac{f(x+h) - f(x)}{h}$$