MATH 1314

Section 3.4

(We do Not cover Section 3.3)

Transforming Functions

In future courses, you will need to be able to sketch the graph of a function quickly and accurately. You can use transformations to do this. There are two types of transformations:

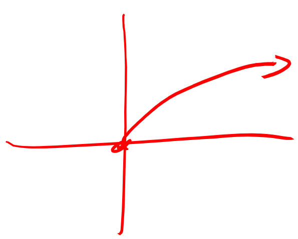
- Translation
- Reflections

We'll start with **translations**. To **translate** a graph means to shift it horizontally, vertically or both.

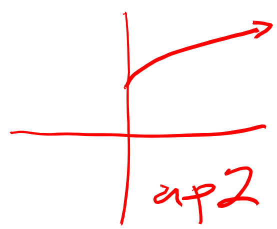
Vertical shifting:

To graph $y = f(x) + c, c > 0$, start with the graph of $f(x)$ and shift it upward c units.
To graph $y = f(x) - c, c > 0$, start with the graph of $f(x)$ and shift it downward c units.

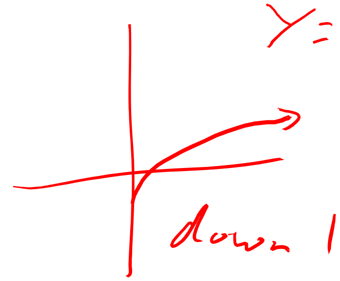
$y = \sqrt{x}$



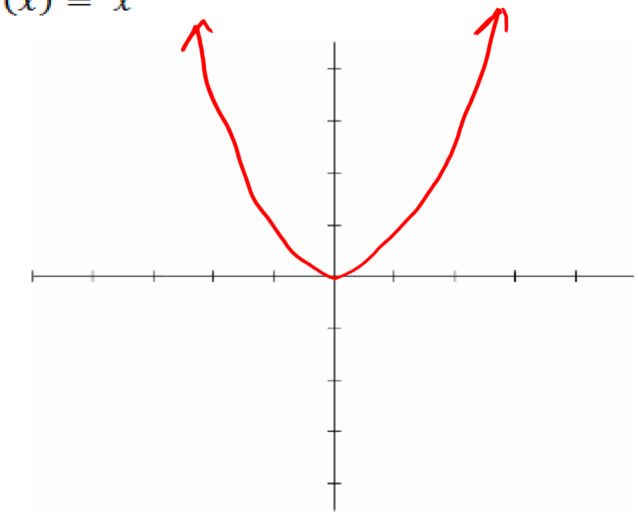
$y = \sqrt{x} + 2$



$y = \sqrt{x} - 1$



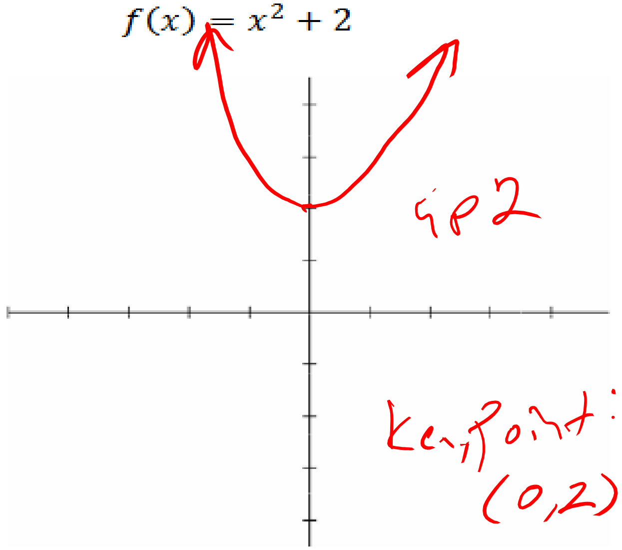
Example 1: Sketch $f(x) = x^2$



Key Point:

(0, 0)

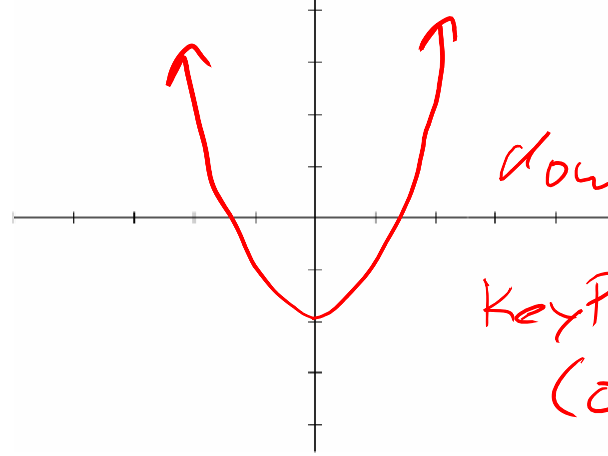
$f(x) = x^2 + 2$



up 2

Key Point:
(0,2)

$f(x) = x^2 - 2$

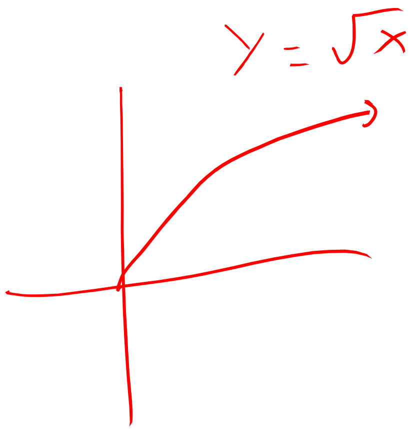


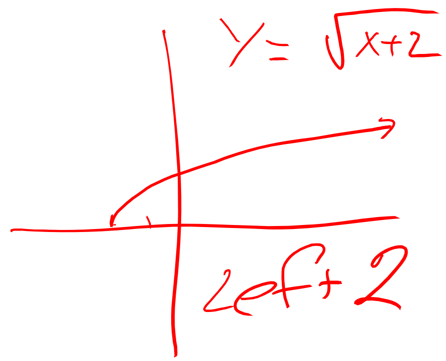
down

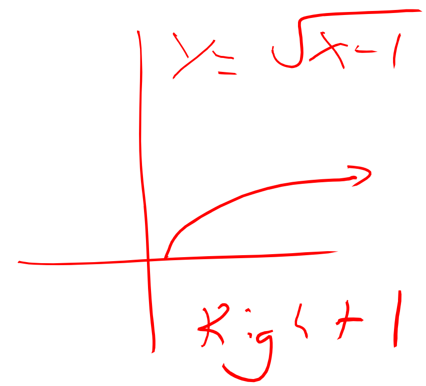
Key Point:
(0, -2)

Horizontal shifting:

To graph $y = f(x + c)$, $c > 0$, start with the graph of $f(x)$ and shift it left c units.
To graph $y = f(x - c)$, $c > 0$, start with the graph of $f(x)$ and shift it right c units.







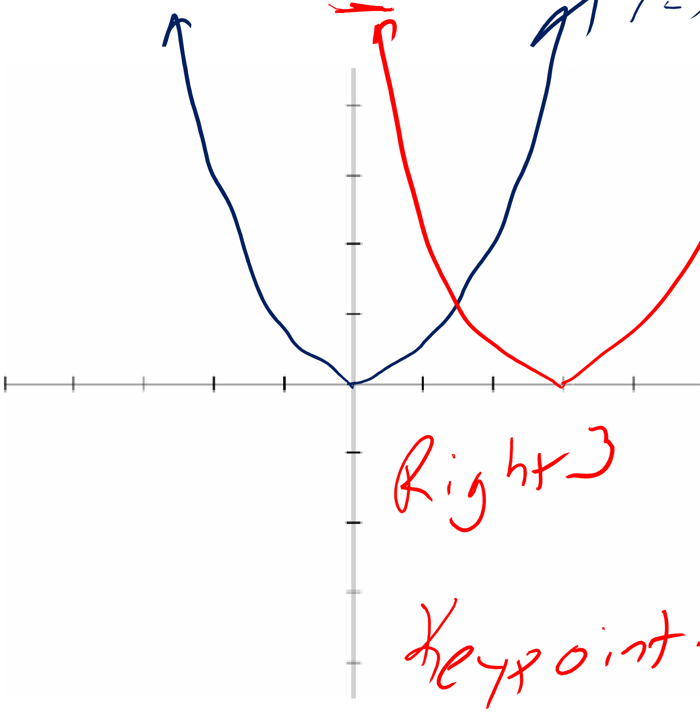
* Note: Horizontal shifting is the opposite of the direction it appears to be.
(+ → left; − → right)

Example 2: Sketch the following graphs

parent Function

$f(x) = (x-3)^2$

$y = x^2$



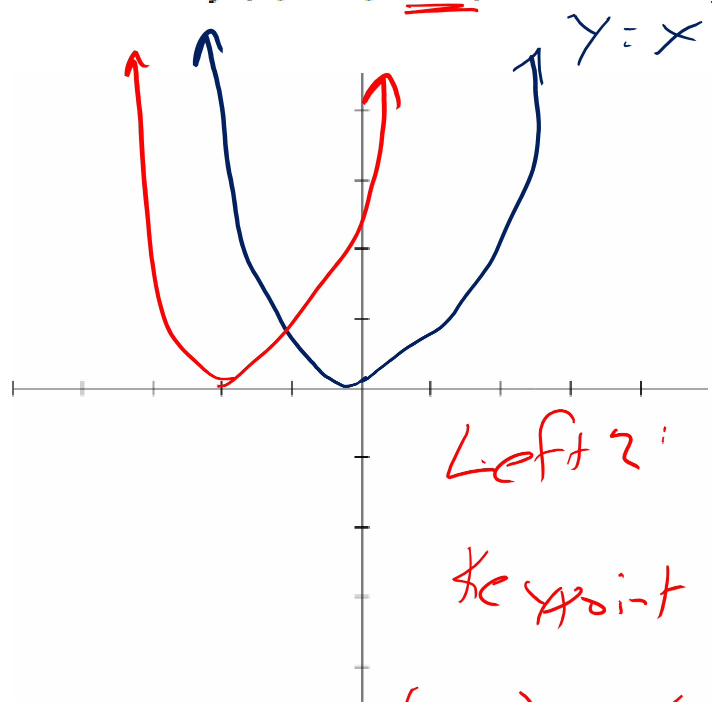
Right 3

Keypoint:

$(0,0) \to (3,0)$

$f(x) = (x+2)^2$

$y = x^2$

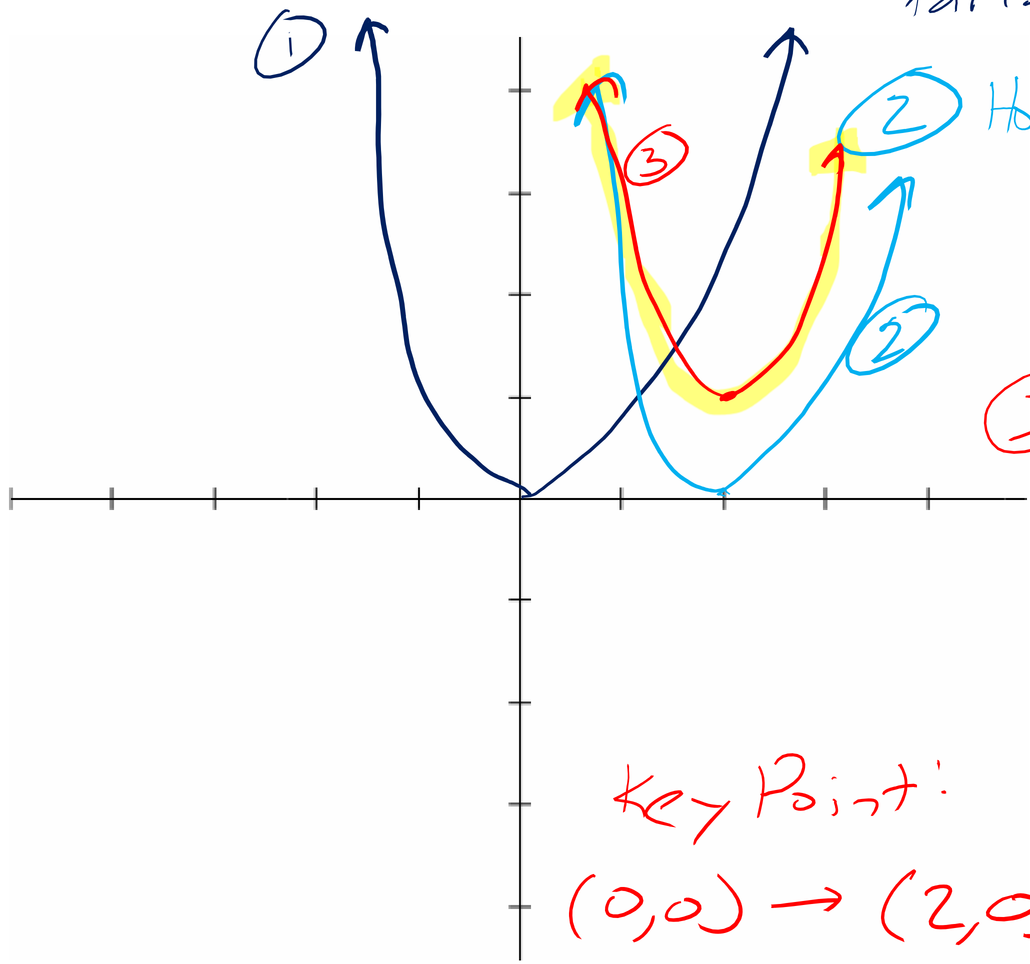


Left 2:

Keypoint:

$(0,0) \to (-2,0)$

Example 3: Sketch a graph of $f(x) = (x - 2)^2 + 1$.



① Parent Function: $y = x^2$

② Horizontal Shift: $y = (x-2)^2$ Right 2

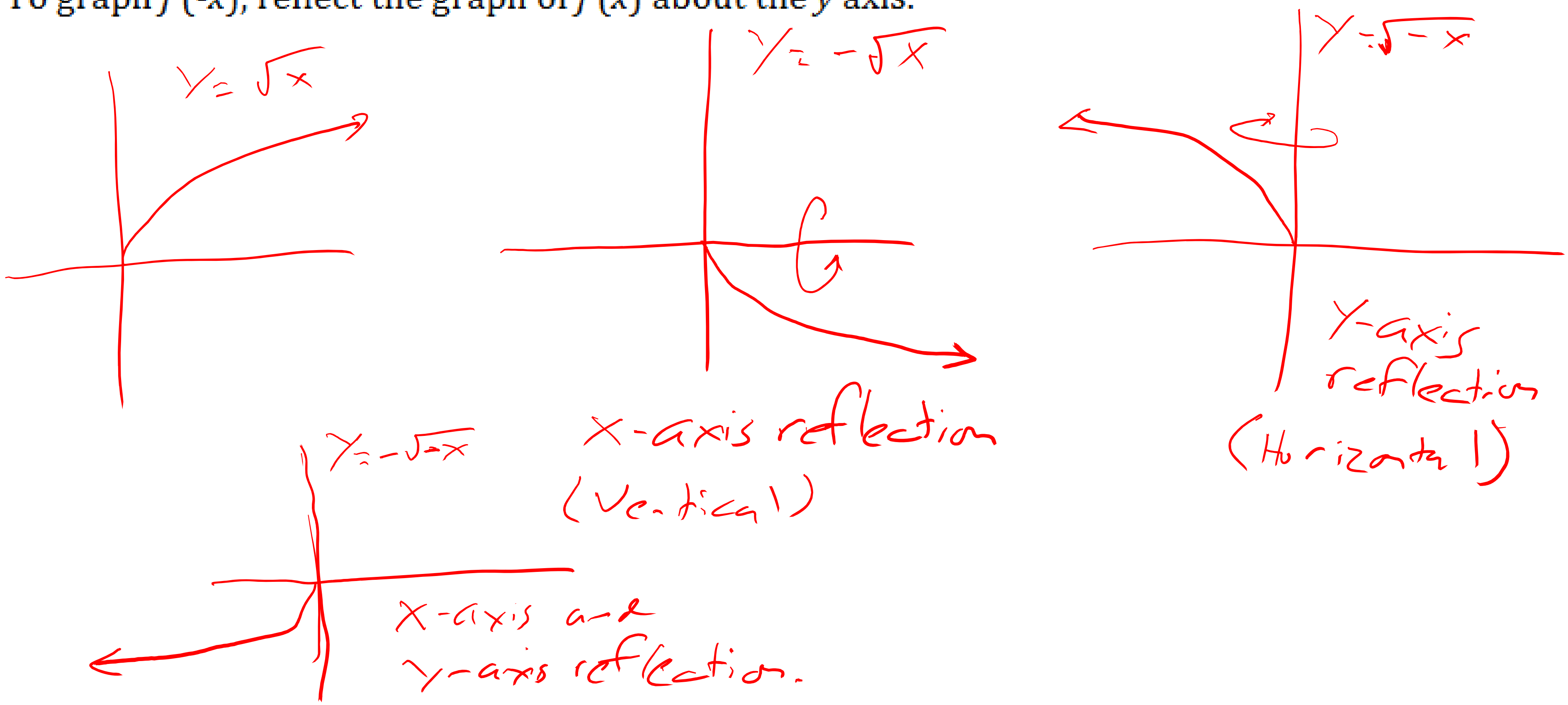
③ Vertical Shift $f(x) = (x-2)^2 + 1$ up 1

Key Point:

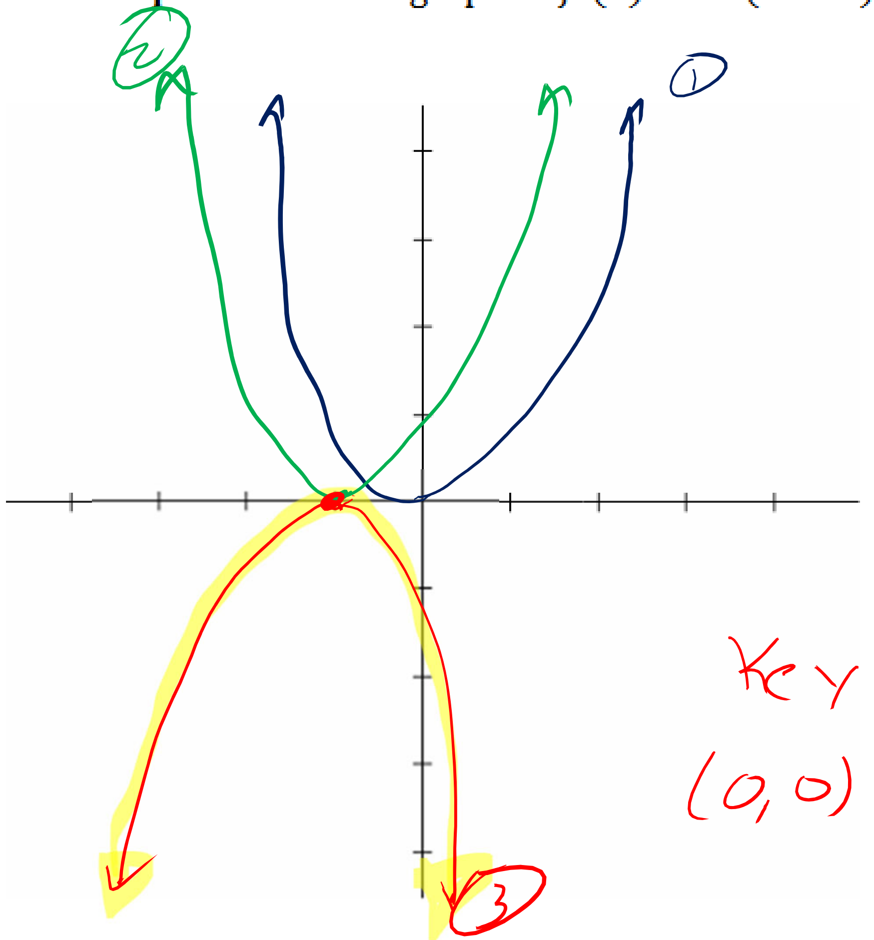
$(0,0) \rightarrow (2,0) \rightarrow (2,1)$

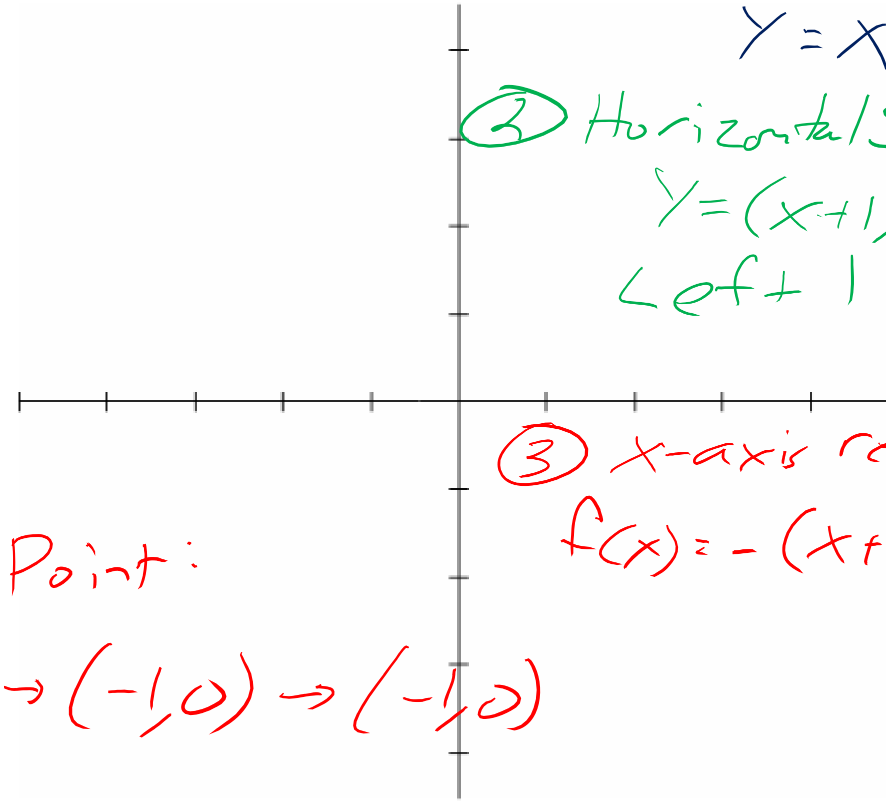
We can also reflect a function. A reflection of a function is its mirror image about the x axis or the y axis.

To graph $-f(x)$, reflect the graph of $f(x)$ about the x axis.
To graph $f(-x)$, reflect the graph of $f(x)$ about the y axis.



Example 4: Sketch a graph of $f\,(x) = -(x+1)^2$.





1 Parent Function:

$Y = X^2$

2 Horizontal Shift

$Y = (x+1)^2$

Left 1

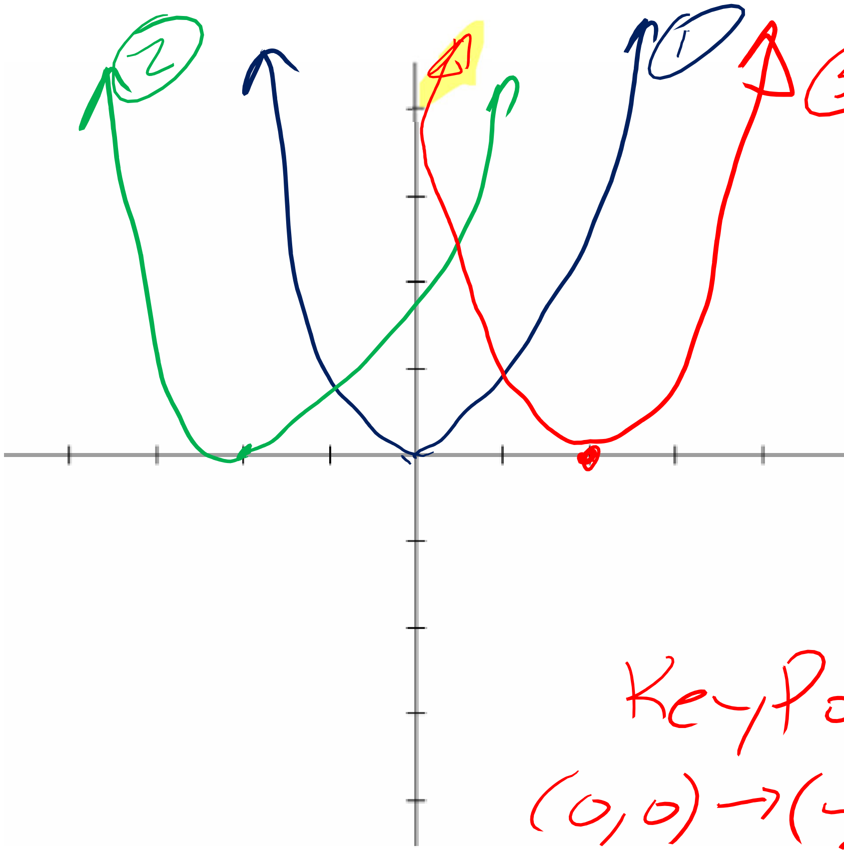
3 x-axis reflection

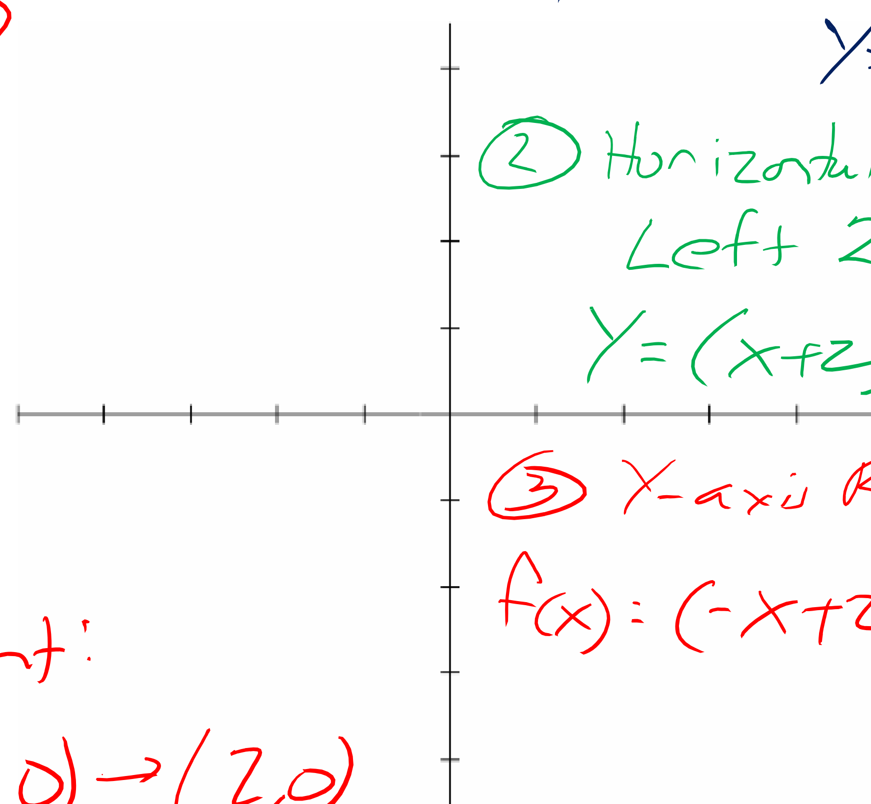
$f(x) = -(x+1)^2$

Key Point:

$(0,0) \to (-1,0) \to (-1,0)$

Example 5: Sketch a graph of $f\ (x) = \ (-x\ +\ 2)^2$.





1 Parent Function

$y = x^2$

2 Horizontal shift

Left 2

$y = (x+2)^2$

3 X-axis Reflection

$f(x) = (-x+2)^2$

Key Point:

$(0,0) \to (-2,0) \to (2,0)$

Popper 12: $f(x) = -\sqrt{x+2} - 5$

vertical

1. Identify the Parent Function: $y = \sqrt{x}$

   a. Linear $y = mx + b$ b. Quadratic $y = (x+k)^2 + 1$ c. Rational $y = \frac{1}{x+2} + 1$ d. Radical

2. Is there a vertical shift? $y = \sqrt{x} - 5$

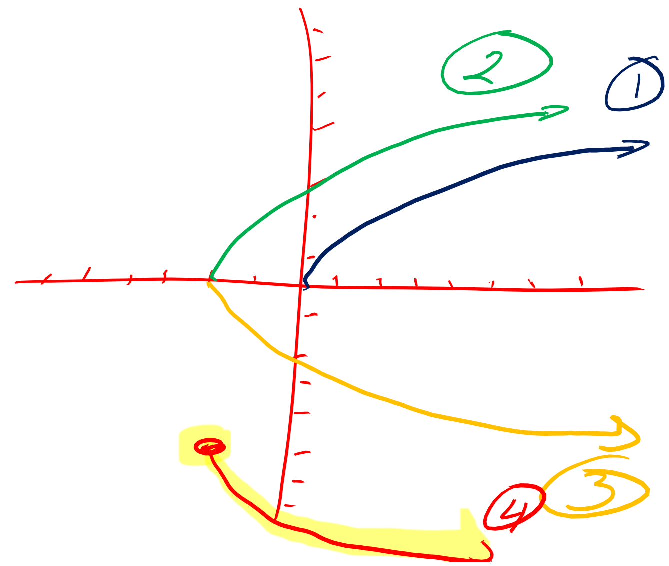
   a. up 5 b. up 2 c. down 5 d. down 2

3. Is there a horizontal (y-axis) reflection? $y = \sqrt{-x}$

   a. Yes b. No

4. Is there a vertical (x-axis) reflection? $y = -\sqrt{x}$

   a. Yes b. No

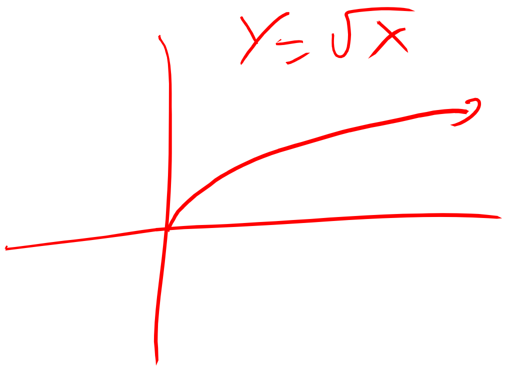


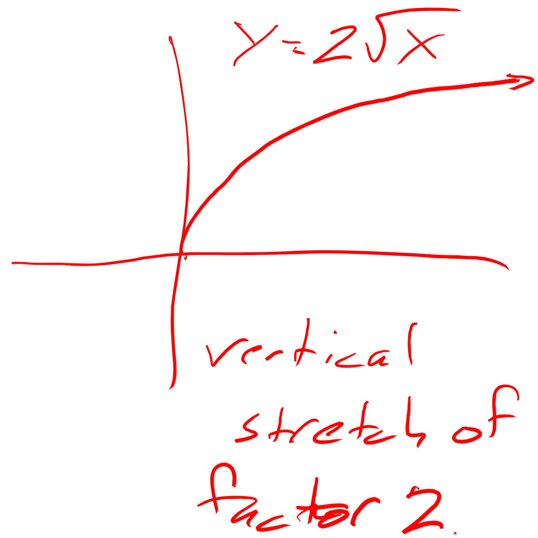
Finally, you can stretch or shrink your graph vertically. A vertical stretch will move your graph closer to the y axis, while a vertical shrink will move it closer to the x axis. It may be helpful to graph one or two points when your problem has a stretch or shrink.

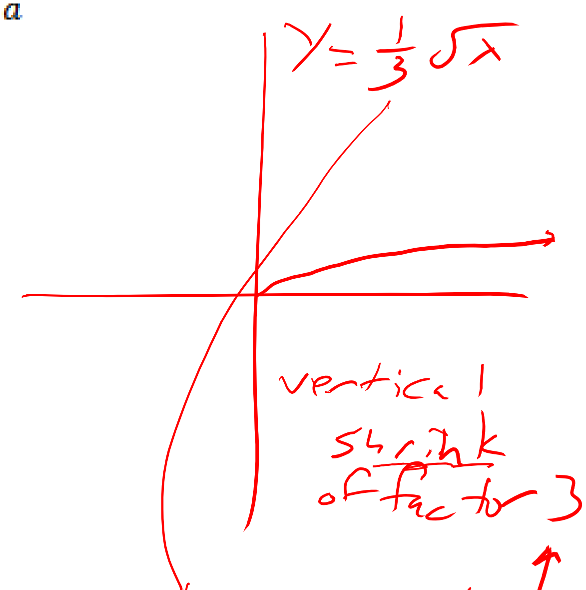
Non-Rigid Transformations

To graph $y = af(x)$, $a > 1$, stretch the graph of $f(x)$ by a factor of a.

To graph $y = af(x)$, $0 < a < 1$, shrink the graph of $f(x)$ by a factor of $\frac{1}{a}$





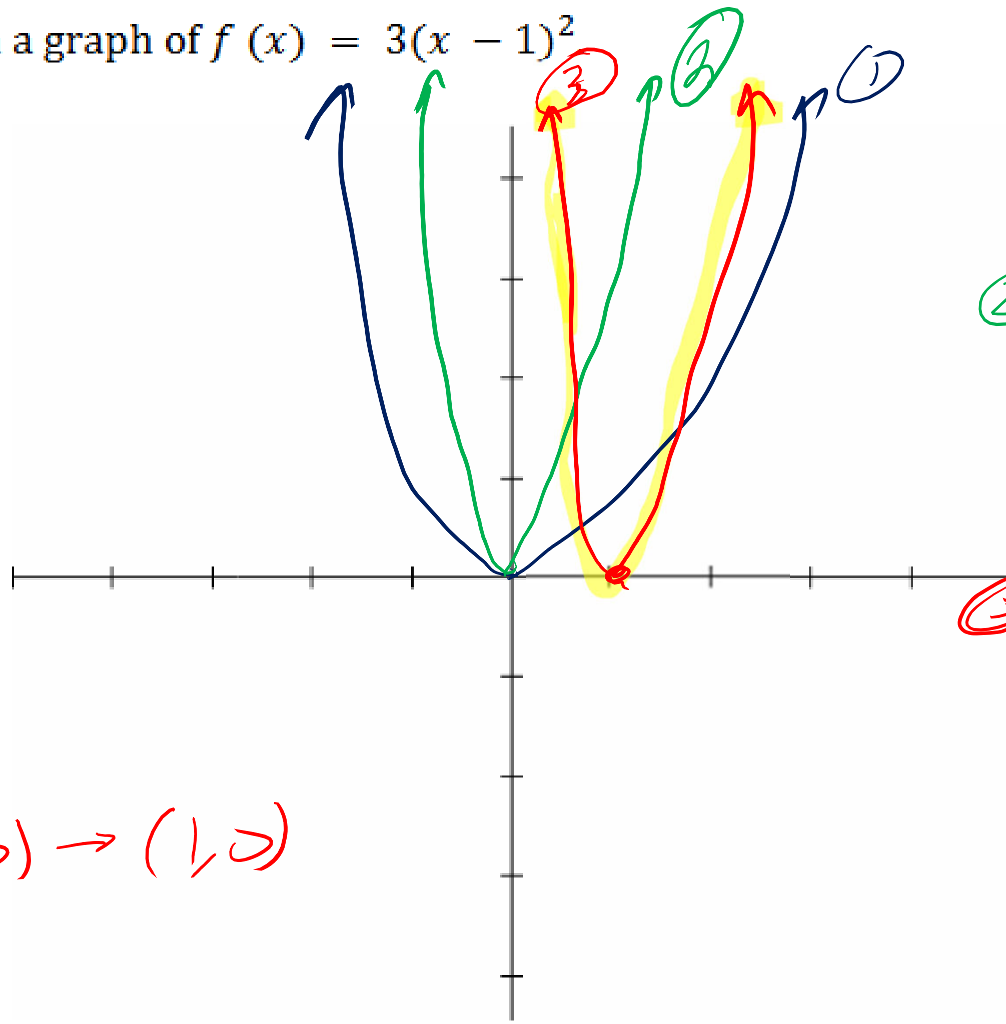


reciprical

Multiplier is larger than 1 . stretch

Multiplior is between 0 and 1, shrink

Example 6: Sketch a graph of $f(x) = 3(x-1)^2$



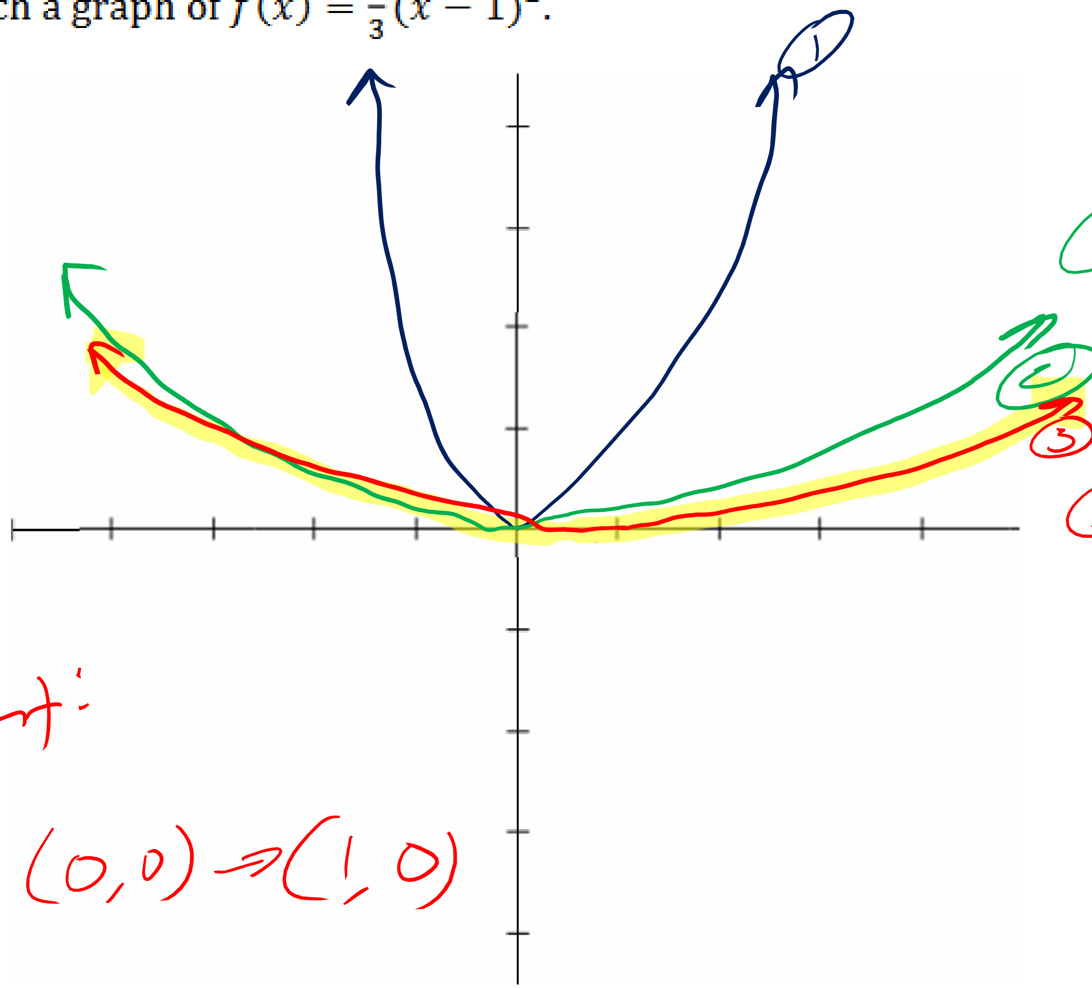
① Parent Function
$y = x^2$

② Vertical Stretch of Factor 3
$y = 3x^2$

③ Horizontal Shift Right 1.

Key Point:
$(0,0) \rightarrow (0,0) \rightarrow (1,0)$

Example 7: Sketch a graph of $f(x) = \frac{1}{3}(x-1)^2$.



① Parent Function
$y = x^2$

② Vertical Shrink of Factor 3
$y = \frac{1}{3}x^2$

③ Right 1
$f(x) = \frac{1}{3}(x-1)^2$

Key Point:

$(0,0) \to (0,0) \to (1,0)$

Recommended order for transforming functions:

1. Vertically stretch or shrink the function.
2. Reflect the function about the *x* axis.
3. Translate the function vertically and/or horizontally.
4. Reflect the function about the *y* axis.

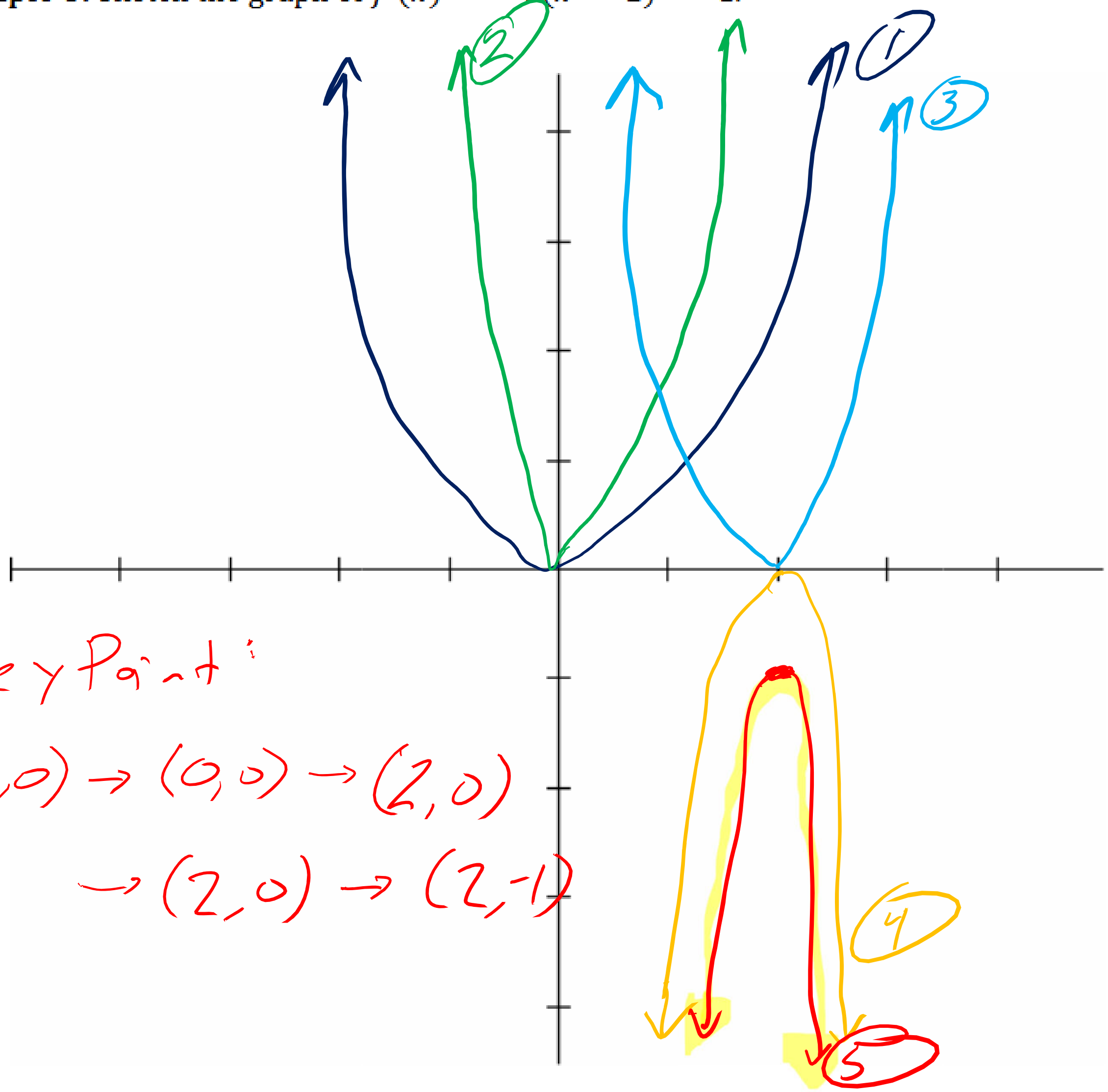
**Note, not all of these transformations will be presented in each problem. This is not the only order that works, but this order will get the job done with the fewest mistakes. Memorize this order!!!

Alternate Order:

1. Vertically Stretch/Shrink
2. Horizontal Shifting
3. Reflections (x or y axis)
4. Vertical Shifting

Heart Rates Vary

Example 8: Sketch the graph of $f(x) = -3(x-2)^2 - 1$.



Key Point:

$(0,0) \to (0,0) \to (2,0)$

$\to (2,0) \to (2,-1)$

① Parent Function

$y = x^2$

② Vertical Stretch of Factor 3

$y = 3x^2$

③ Right 2

$y = 3(x-2)^2$

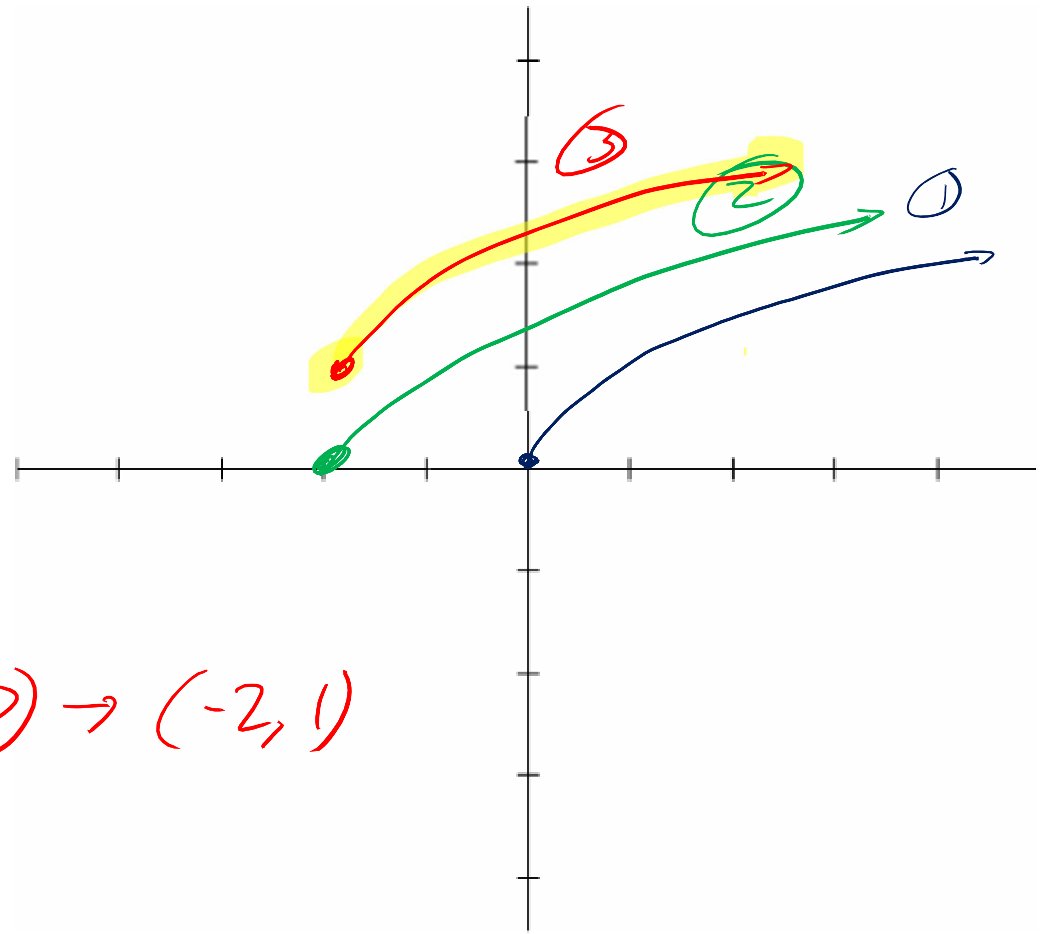
④ x-axis reflection

$y = -3(x-2)^2$

⑤ Down 1

$f(x) = -3(x-2)^2 - 1$

Example 9: Sketch the graph of $f(x) = \sqrt{x+2} + 1$



① Parent Function

$y = \sqrt{x}$

② Left 2

$y = \sqrt{x+2}$

③ up 1

$f(x) = \sqrt{x+2} + 1$

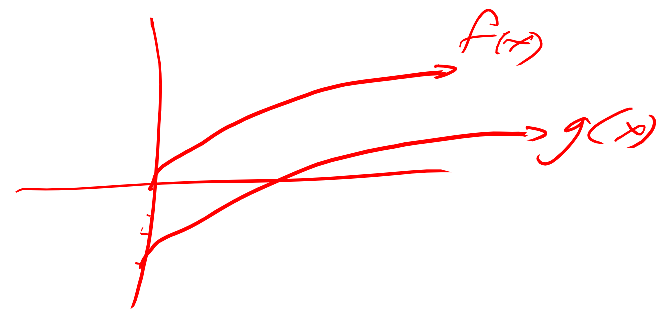
Key Point:

$(0,0) \to (-2,0) \to (-2,1)$

Popper 12

Example 11: Describe how the graph of g is obtained from the graph of f.

5. $f(x) = \sqrt{x}$ $\qquad$ $g(x) = \sqrt{x} - 3$



a. Shift up 3 $\qquad$ b. Shift down 3

c. Shift Left 3 $\qquad$ d. Shift Right 3

Popper 12...continued

$f(x) = x^3$ $\qquad$ $g(x) = -(x + 2)^3 + 1$

① Parent Function
$y = x^3$

② Left 2
$y = (x+2)^3$

③ x-axis refl.
$y = -(x+2)^3$

$(-2, 1)$ ④ up 1

6. Vertical Shift?
a. Up 1 $\qquad$ b. Up 2 $\qquad$ c. Down 1 $\qquad$ d. Down 2

7. Horizontal Shift?
a. Right 1 $\qquad$ b. Right 2 $\qquad$ c. Left 1 $\qquad$ d. Left 2

8. Reflections?
a. Vertical $\qquad$ b. Horizontal $\qquad$ c. Both $\qquad$ d. Neither

$y = (-x+2)^3$

Key Point:
$(0,0) \to (-2,0) \to (-2,0) \to$ $g(x) = -(x+2)^3 + 1$