

MATH 1314

Section 3.4

(We do not cover Section 3.3)

Transforming Functions

In future courses, you will need to be able to sketch the graph of a function quickly and accurately. You can use transformations to do this. There are two types of transformations:

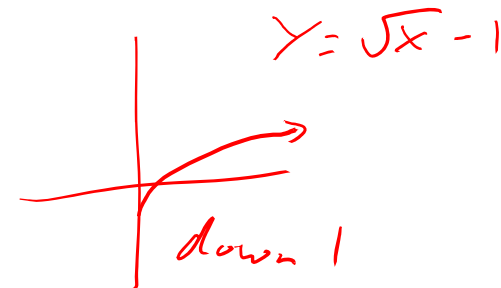
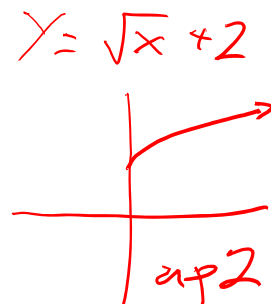
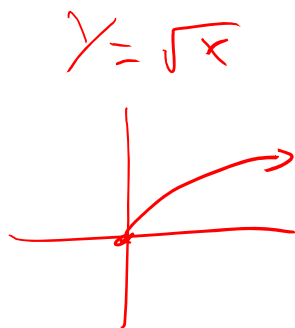
- Translation
- Reflections

We'll start with **translations**. To **translate** a graph means to shift it horizontally, vertically or both.

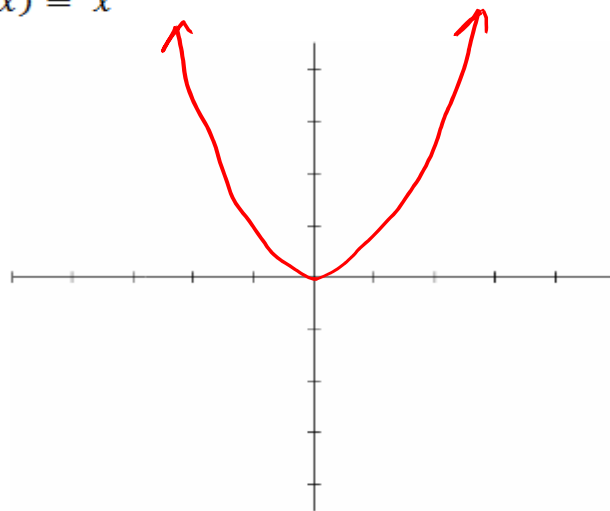
Vertical shifting:

To graph $y = f(x) + c, c > 0$, start with the graph of $f(x)$ and shift it upward c units.

To graph $y = f(x) - c, c > 0$, start with the graph of $f(x)$ and shift it downward c units.

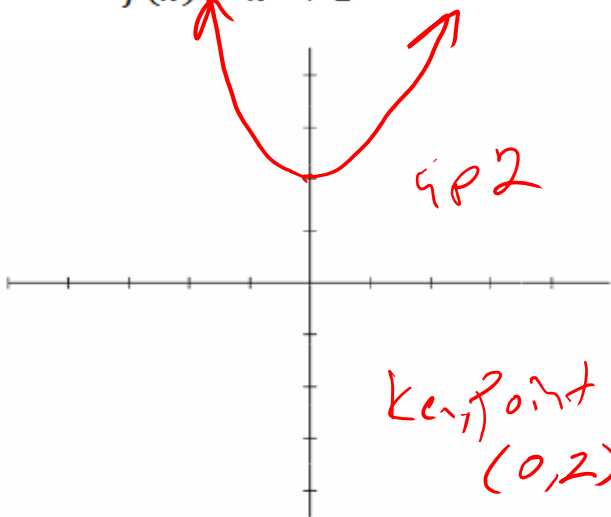


Example 1: Sketch $f(x) = x^2$

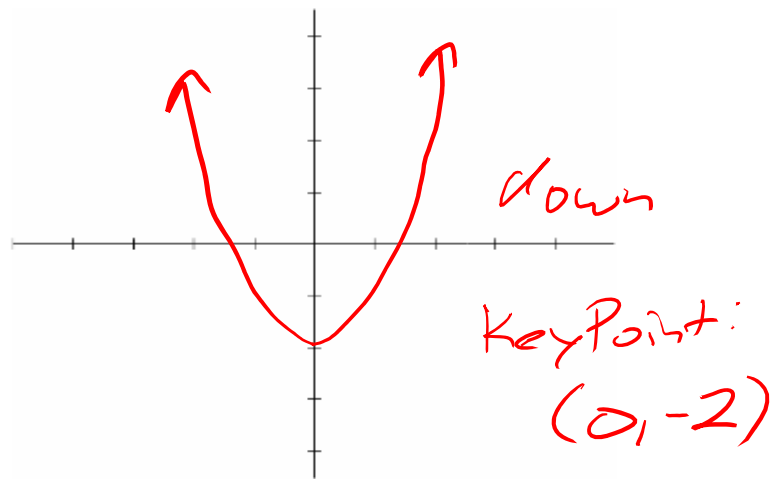


Key Point:
 $(0,0)$

$f(x) = x^2 + 2$



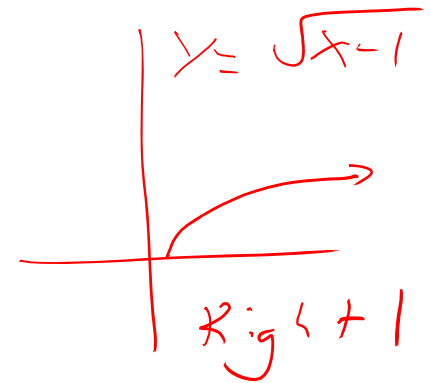
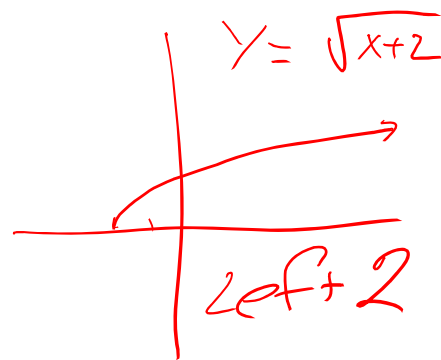
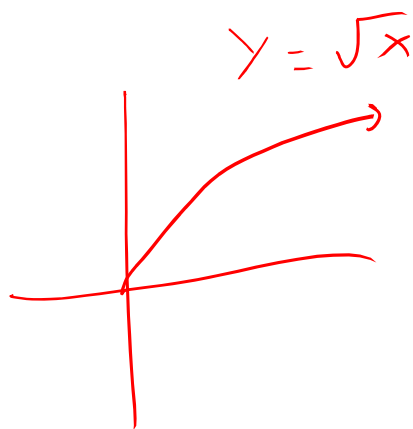
$f(x) = x^2 - 2$



Horizontal shifting:

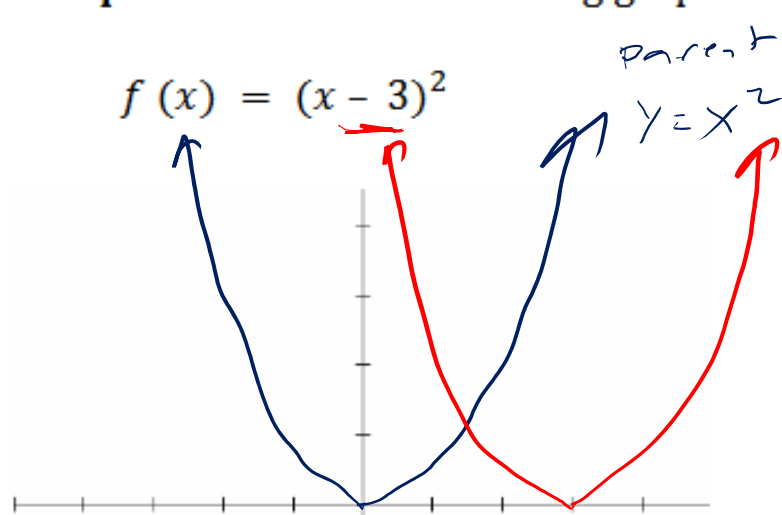
To graph $y = f(x + c)$, $c > 0$, start with the graph of $f(x)$ and shift it left c units.

To graph $y = f(x - c)$, $c > 0$, start with the graph of $f(x)$ and shift it right c units.



* Note: Horizontal shifting is the opposite of the direction it appears to be.
(+ \rightarrow left; - \rightarrow right)

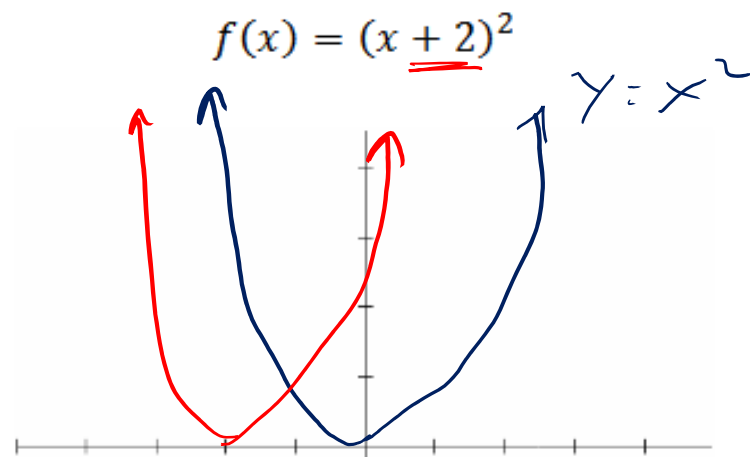
Example 2: Sketch the following graphs



Right 3

keypoint:

$(0,0) \rightarrow (3,0)$

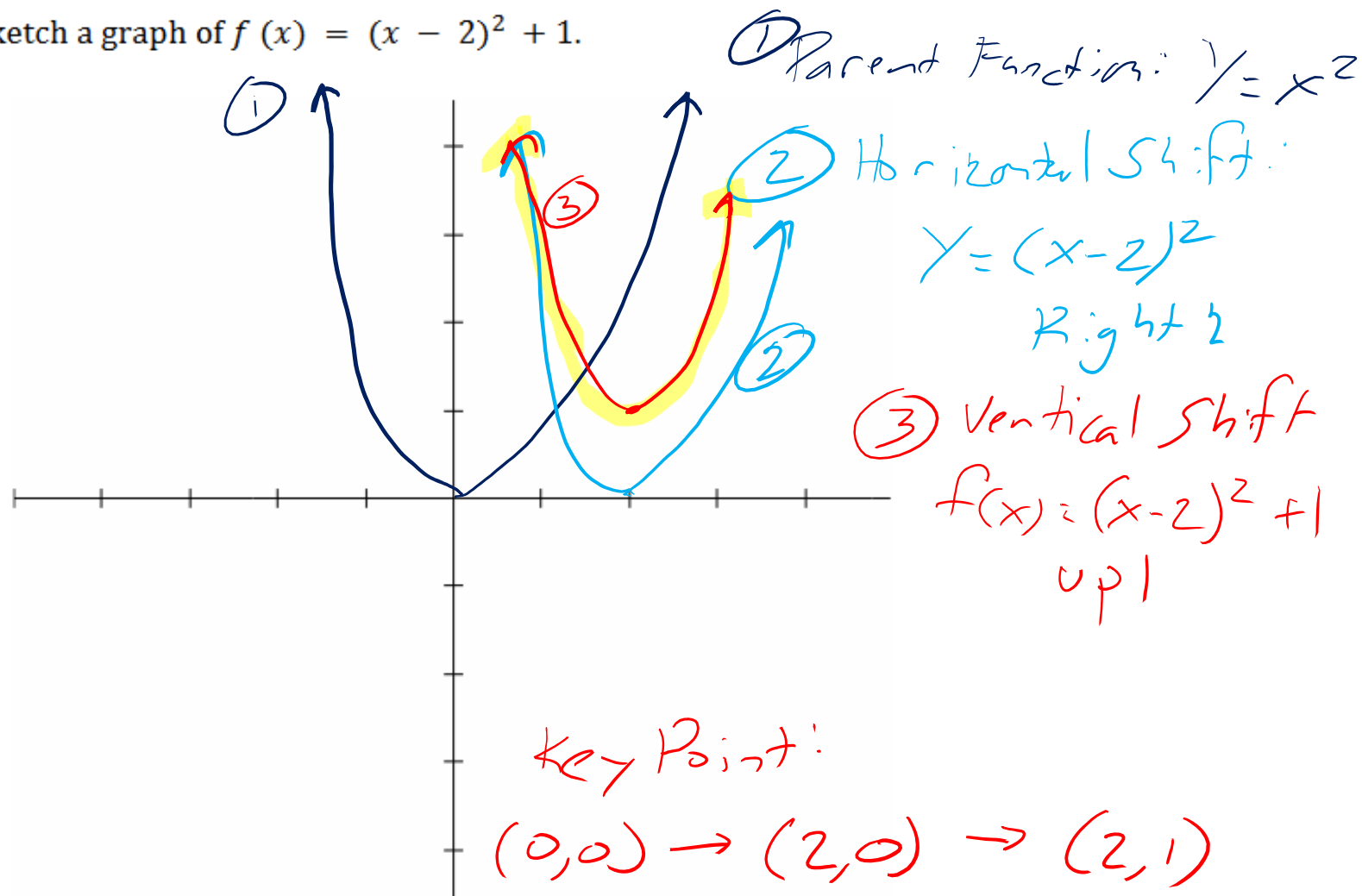


Left 2:

keypoint:

$(0,0) \rightarrow (-2,0)$

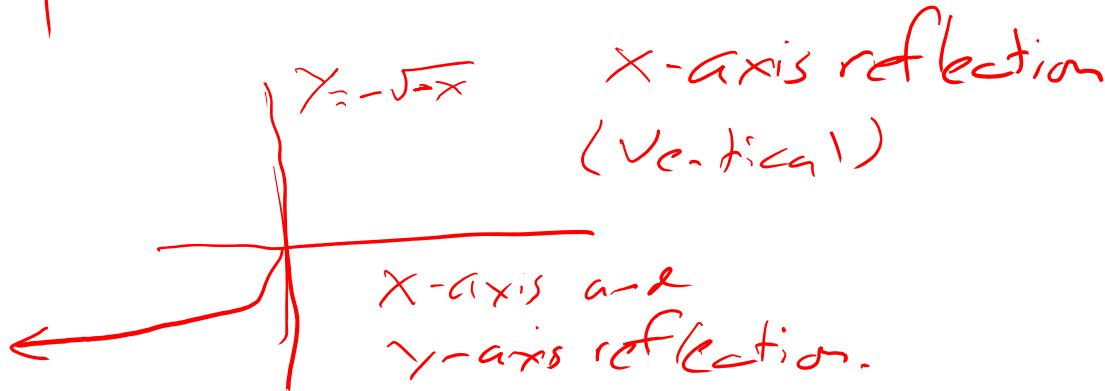
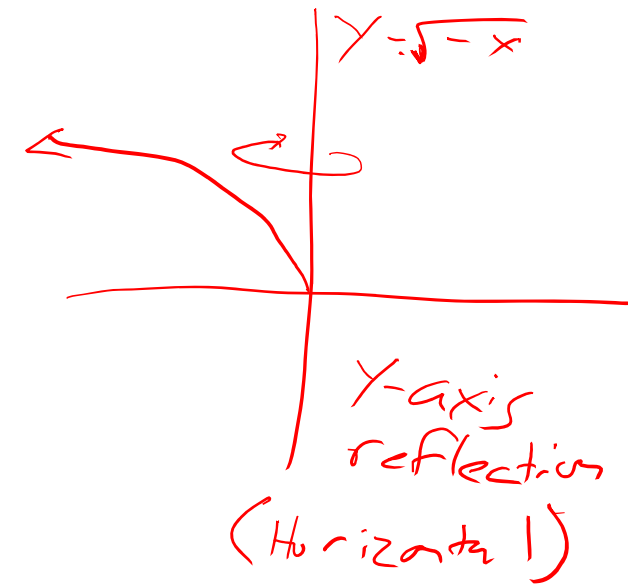
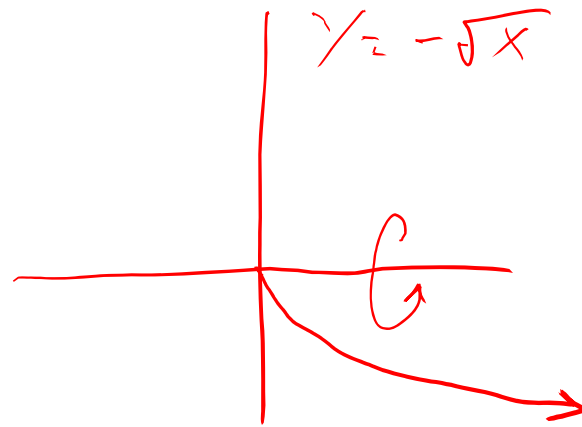
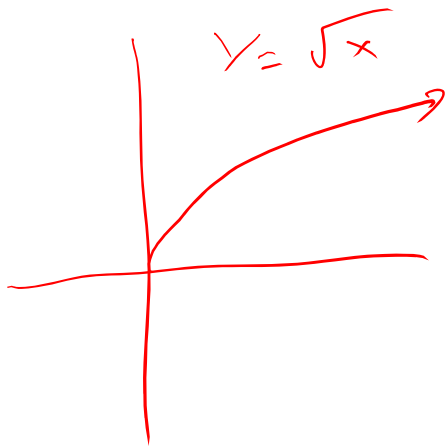
Example 3: Sketch a graph of $f(x) = (x - 2)^2 + 1$.



We can also reflect a function. A reflection of a function is its mirror image about the x axis or the y axis.

To graph $-f(x)$, reflect the graph of $f(x)$ about the x axis.

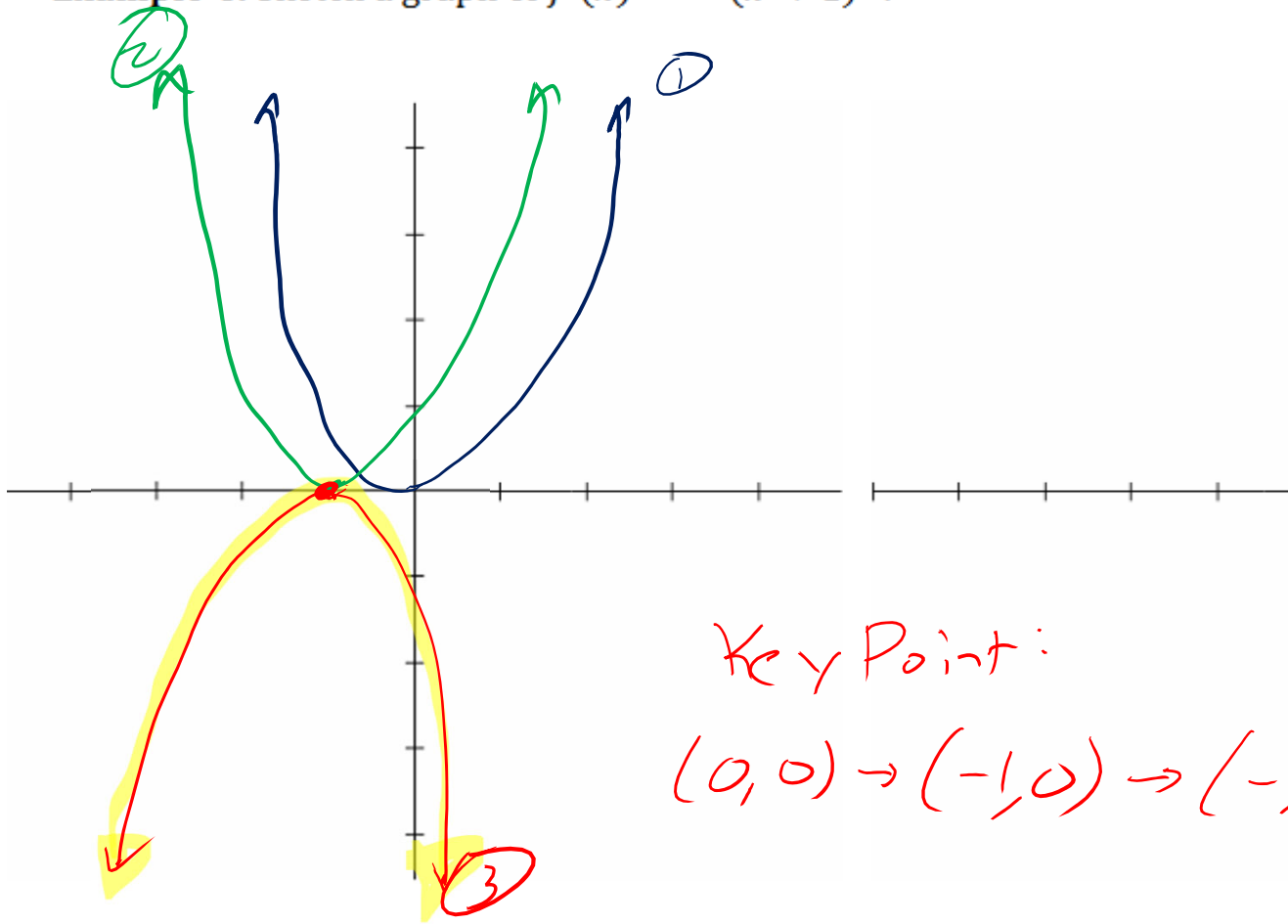
To graph $f(-x)$, reflect the graph of $f(x)$ about the y axis.



x-axis reflection
(Vertical)

x-axis and
y-axis reflection.

Example 4: Sketch a graph of $f(x) = -(x + 1)^2$.



① Parent Function:

$$y = x^2$$

② Horizontal Shift

$$y = (x+1)^2$$

Left + 1

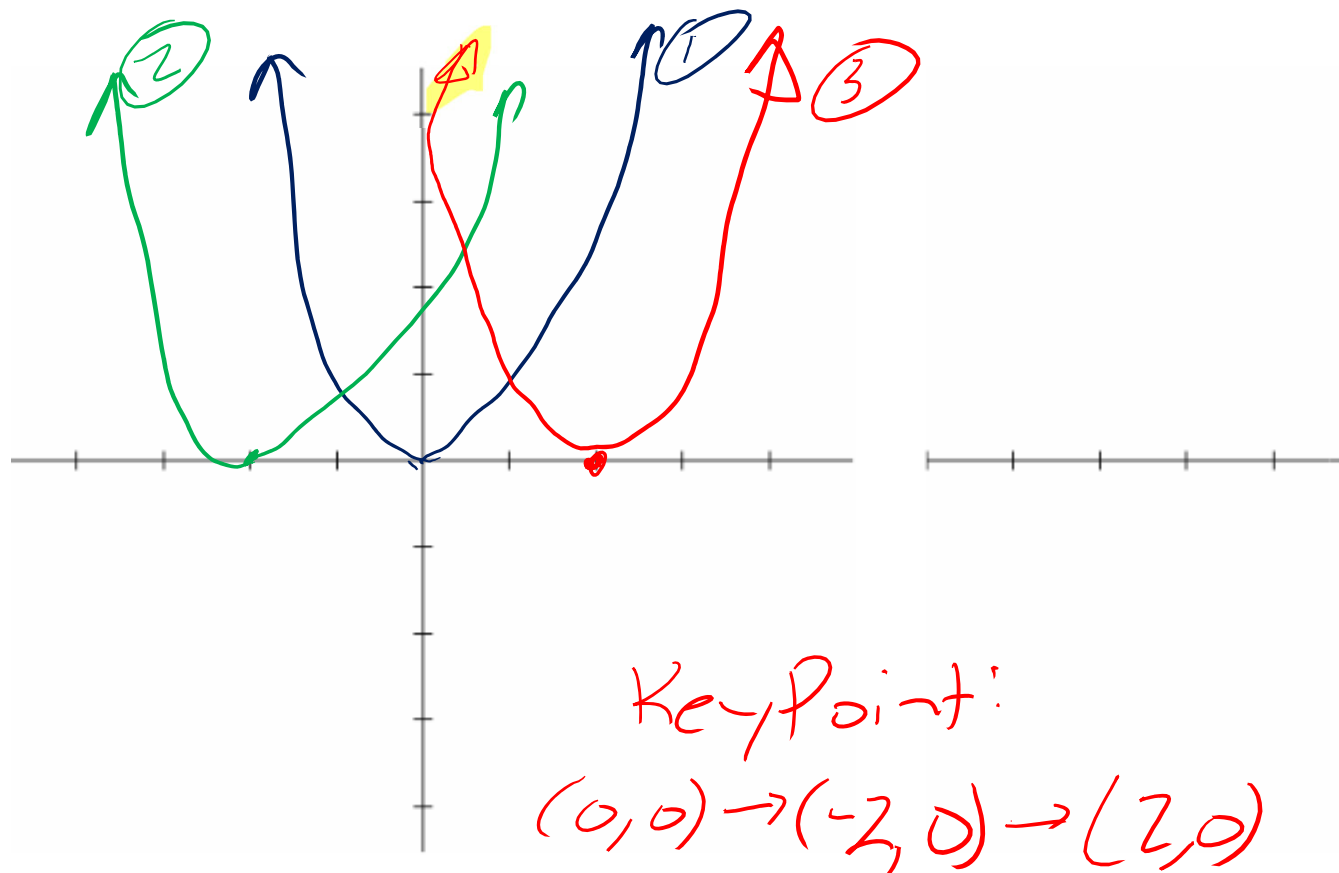
③ x-axis reflection

$$f(x) = -(x+1)^2$$

Key Point:

$$(0, 0) \rightarrow (-1, 0) \rightarrow (-1, 0)$$

Example 5: Sketch a graph of $f(x) = (-x + 2)^2$.



① Parent Function
 $y = x^2$

② Horizontal Shift
Left 2
 $y = (x + 2)^2$

③ X-axis Reflection
 $f(x) = (-x + 2)^2$

Popper 12: $f(x) = -\sqrt{x+2} - 5$

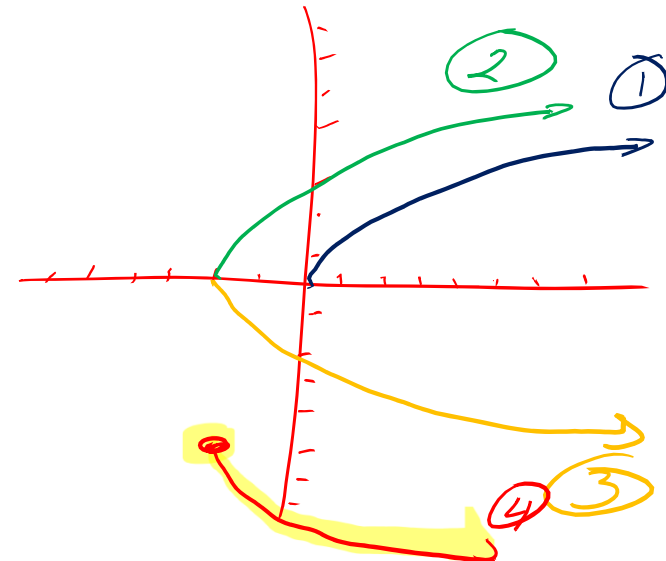
↳ vertical

1. Identify the Parent Function: $y = \sqrt{x}$
- a. Linear $y = mx + b$
 - b. Quadratic $y = (x+k)^2 + l$
 - c. Rational $y = \frac{1}{x+2} + 1$
 - d. Radical

2. Is there a vertical shift? $y = \sqrt{x} - 5$
- a. up 5
 - b. up 2
 - c. down 5
 - d. down 2

3. Is there a horizontal (y-axis) reflection? $y = \sqrt{-x}$
- a. Yes
 - b. No

4. Is there a vertical (x-axis) reflection? $y = -\sqrt{x}$
- a. Yes
 - b. No

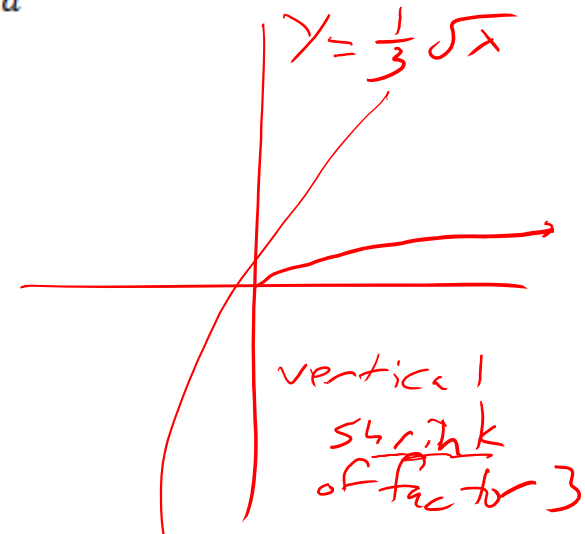
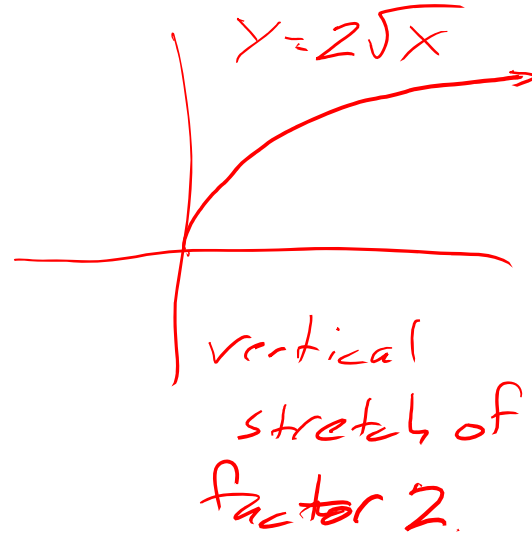
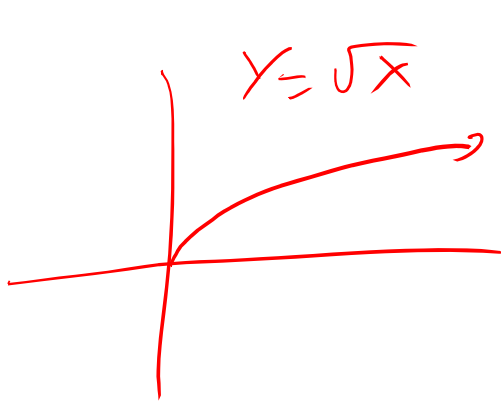


Finally, you can stretch or shrink your graph vertically. A vertical stretch will move your graph closer to the y axis, while a vertical shrink will move it closer to the x axis. It may be helpful to graph one or two points when your problem has a stretch or shrink.

To graph $y = af(x)$, $a > 1$, stretch the graph of $f(x)$ by a factor of a .

To graph $y = af(x)$, $0 < a < 1$, shrink the graph of $f(x)$ by a factor of $\frac{1}{a}$

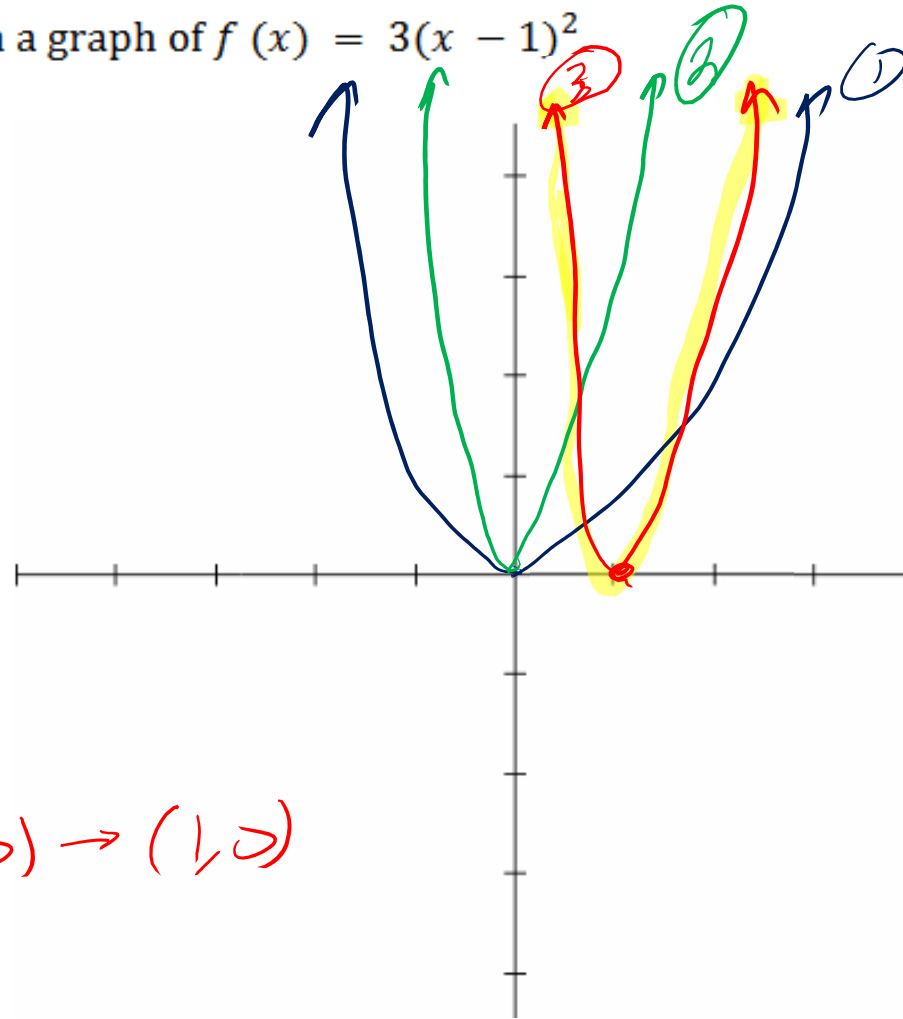
Non-Rigid Transformations



multiplier is larger than 1. stretch
multiplier is between 0 and 1, shrink

reciprocal

Example 6: Sketch a graph of $f(x) = 3(x - 1)^2$



① Parent Function
 $y = x^2$

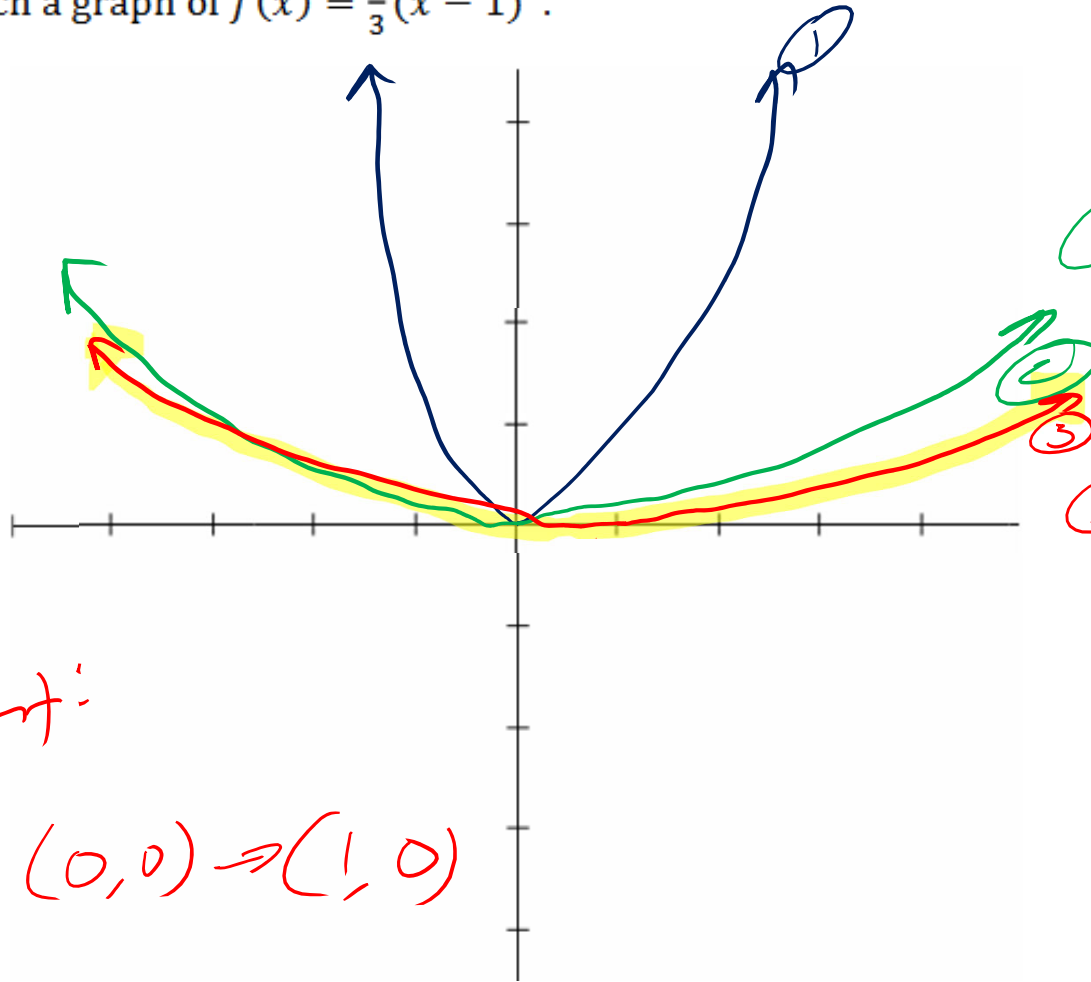
② Vertical Stretch
of Factor 3
 $y = 3x^2$

③ Horizontal Shift
Right 1.

Key Point:

$(0,0) \rightarrow (0,0) \rightarrow (1,0)$

Example 7: Sketch a graph of $f(x) = \frac{1}{3}(x-1)^2$.



① Parent Function
 $y = x^2$

② Vertical Shrink
of Factor 3
 $y = \frac{1}{3}x^2$

③ Right 1

$$f(x) = \frac{1}{3}(x-1)^2$$

Key Point:

$$(0,0) \rightarrow (0,0) \rightarrow (1,0)$$

Recommended order for transforming functions:

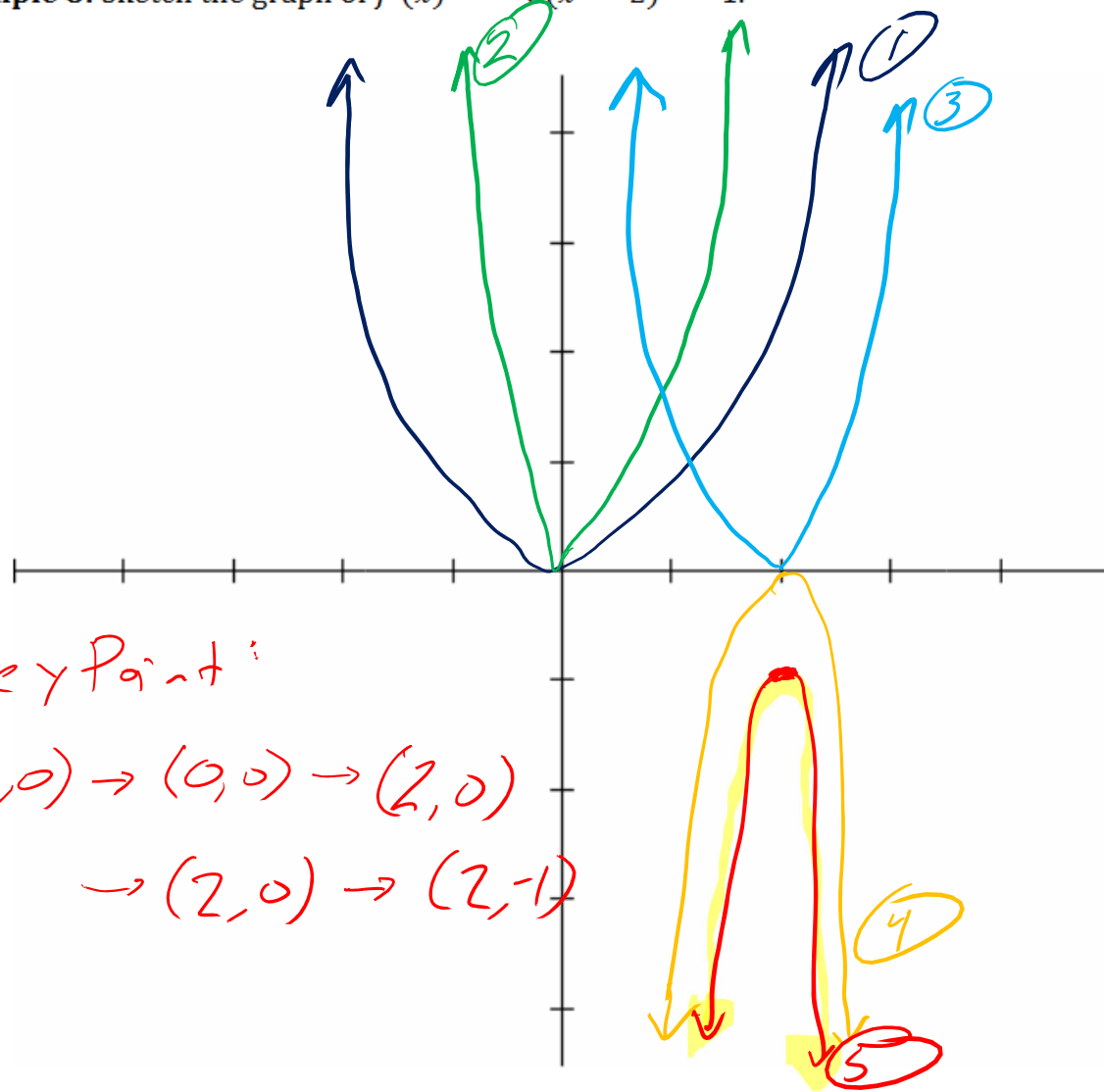
1. Vertically stretch or shrink the function.
2. Reflect the function about the x axis.
3. Translate the function vertically and/or horizontally.
4. Reflect the function about the y axis.

**Note, not all of these transformations will be presented in each problem. This is not the only order that works, but this order will get the job done with the fewest mistakes. Memorize this order!!!

Alternate Order:

- | | |
|-------------------------------------|--------------------|
| ① Vertically stretch/shrink |] Heart Rates Vary |
| ② Horizontal shifting | |
| ③ Reflections (x - or y -axis) | |
| ④ vertical shifting | |

Example 8: Sketch the graph of $f(x) = -3(x - 2)^2 - 1$.



Key Point:

$(0, 0) \rightarrow (0, 0) \rightarrow (2, 0)$
 $\rightarrow (2, 0) \rightarrow (2, -1)$

① Parent Function
 $y = x^2$

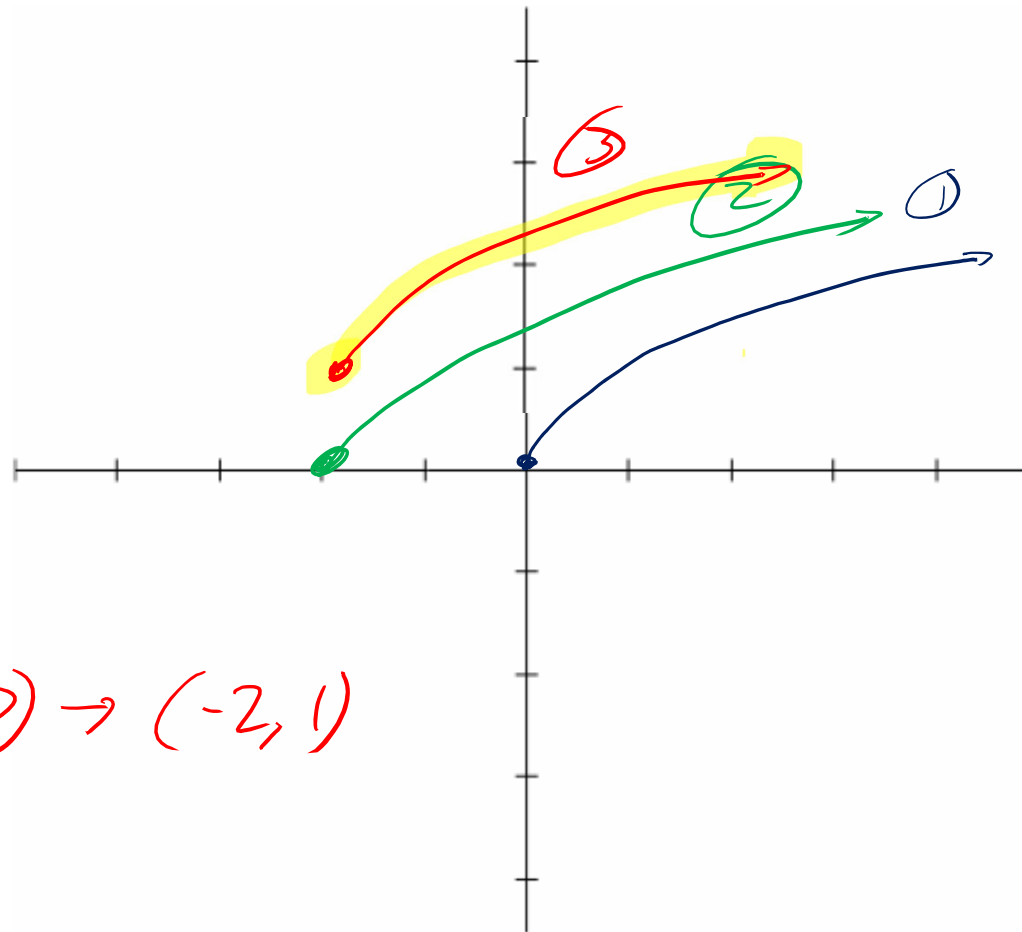
② Vertical Stretch of
Factor 3
 $y = 3x^2$

③ Right 2
 $y = 3(x - 2)^2$

④ x-axis reflection
 $y = -3(x - 2)^2$

⑤ Down 1
 $f(x) = -3(x - 2)^2 - 1$

Example 9: Sketch the graph of $f(x) = \sqrt{x+2} + 1$



① Parent Function

$$y = \sqrt{x}$$

② Left + 2

$$y = \sqrt{x+2}$$

③ up!

$$f(x) = \sqrt{x+2} + 1$$

Key Point:

$$(0, 0) \rightarrow (-2, 0) \rightarrow (-2, 1)$$

Popper 13

Example 11: Describe how the graph of g is obtained from the graph of f .

1. $f(x) = \sqrt{x}$

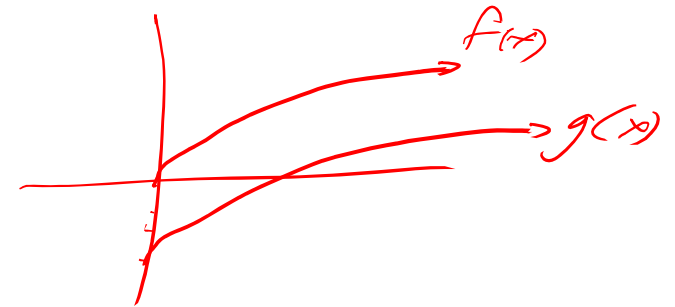
$g(x) = \sqrt{x} - 3$

a. Shift up 3

b. Shift down 3

c. Shift Left 3

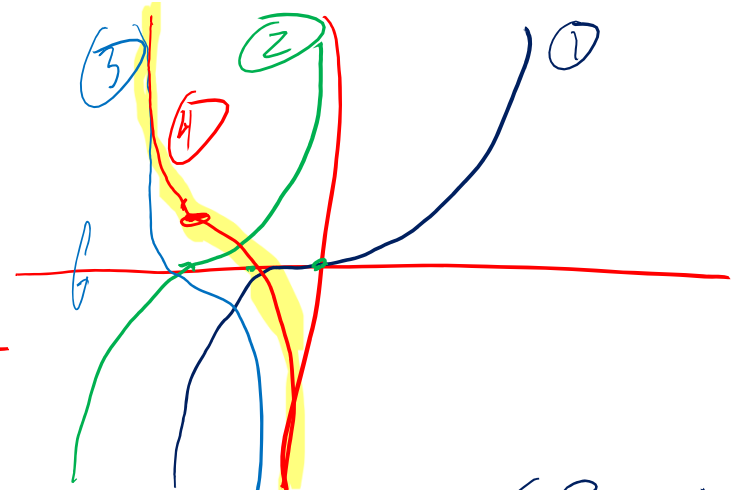
d. Shift Right 3



Popper 13...continued

$$f(x) = x^3$$

$$g(x) = -(x+2)^3 + 1$$



2. Vertical Shift?

a. Up 1

b. Up 2

c. Down 1

d. Down 2

3. Horizontal Shift?

a. Right 1

b. Right 2

c. Left 1

d. Left 2

4. Reflections?

a. Vertical

b. Horizontal

c. Both

d. Neither

① Parent Function

$$y = x^3$$

② Left + 2

$$y = (x+2)^3$$

③ x-axis refl.

$$y = -(x+2)^3$$

(-2, 0) ④ up 1

$$y = (-x+2)^3$$

Key Point:
 $(0,0) \rightarrow (-2,0) \rightarrow (-2,0)^3$
 $g(x) = -(x+2)^3 + 1$