

MATH 1314

Section 3.4

(We do not cover Section 3.3)

Transforming Functions

In future courses, you will need to be able to sketch the graph of a function quickly and accurately. You can use transformations to do this. There are two types of transformations:

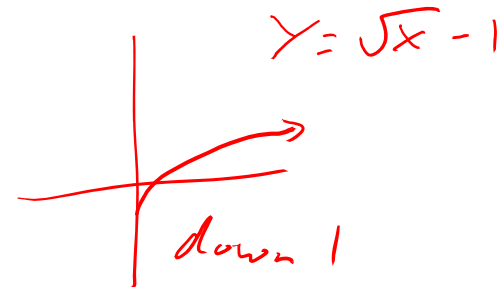
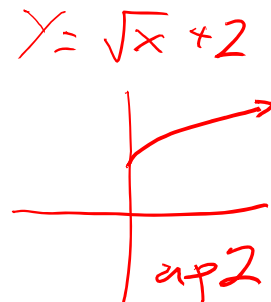
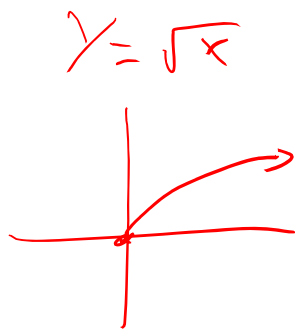
- Translation
- Reflections

We'll start with **translations**. To **translate** a graph means to shift it horizontally, vertically or both.

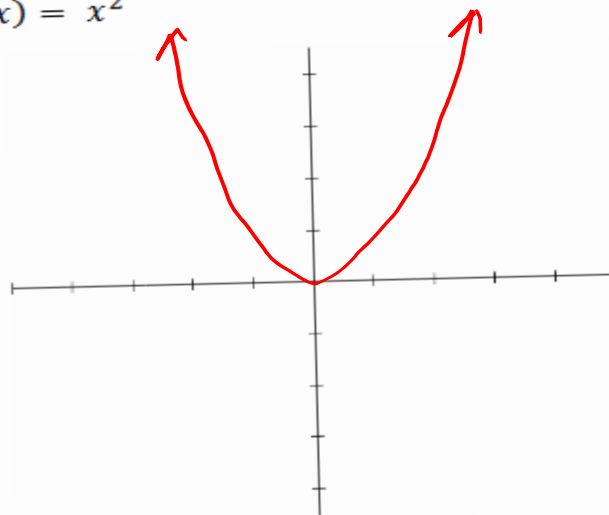
Vertical shifting:

To graph $y = f(x) + c$, $c > 0$, start with the graph of $f(x)$ and shift it upward c units.

To graph $y = f(x) - c$, $c > 0$, start with the graph of $f(x)$ and shift it downward c units.

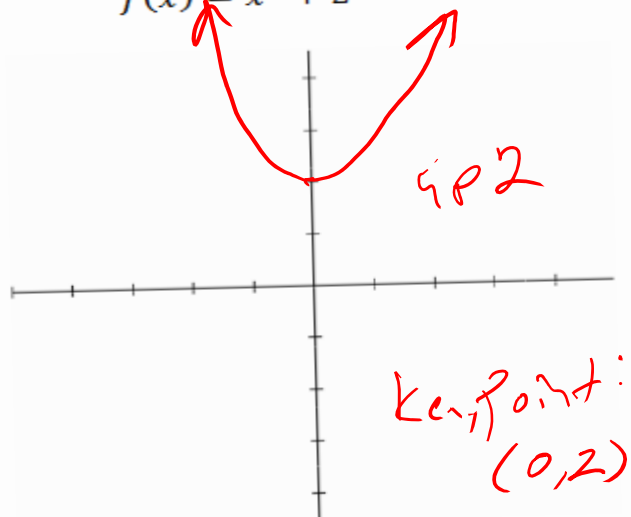


Example 1: Sketch $f(x) = x^2$



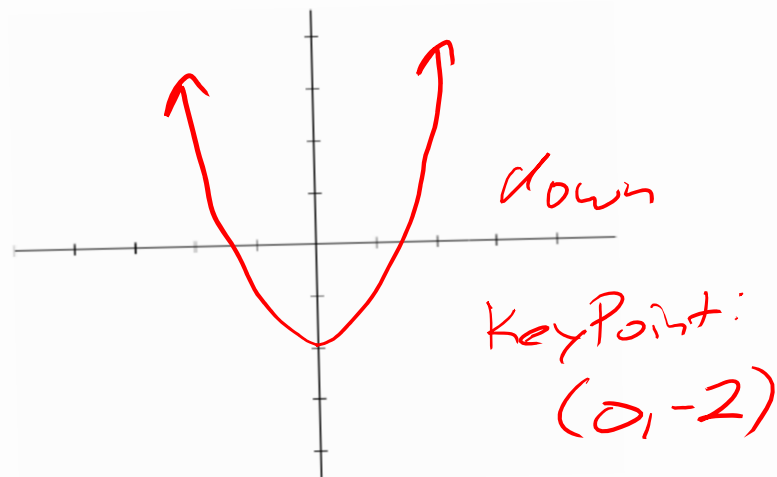
Key Point:
 $(0,0)$

$f(x) = x^2 + 2$



Key Point:
 $(0,2)$

$f(x) = x^2 - 2$

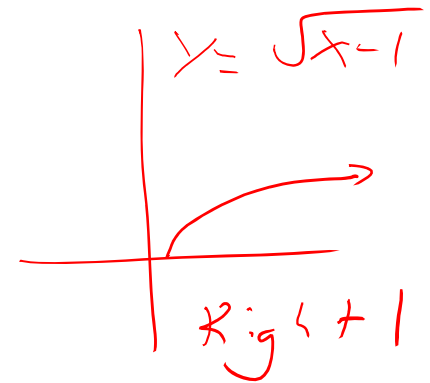
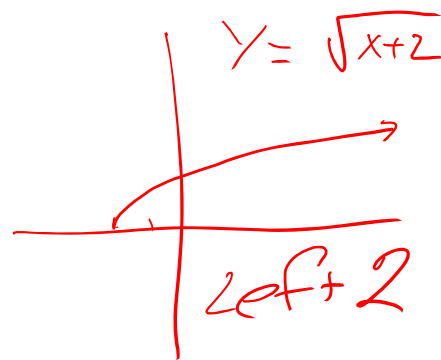
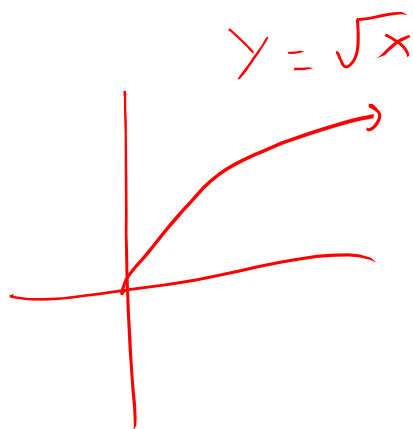


Key Point:
 $(0,-2)$

Horizontal shifting:

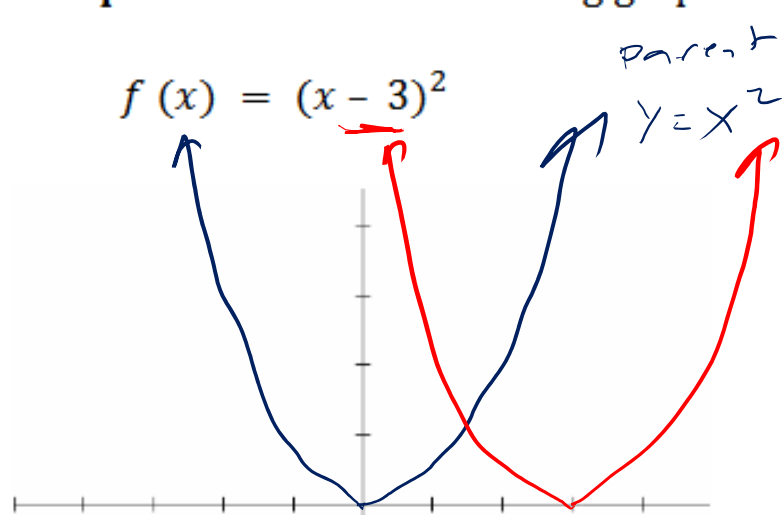
To graph $y = f(x + c)$, $c > 0$, start with the graph of $f(x)$ and shift it left c units.

To graph $y = f(x - c)$, $c > 0$, start with the graph of $f(x)$ and shift it right c units.



* Note: Horizontal shifting is the opposite of the direction it appears to be.
(+ \rightarrow left; - \rightarrow right)

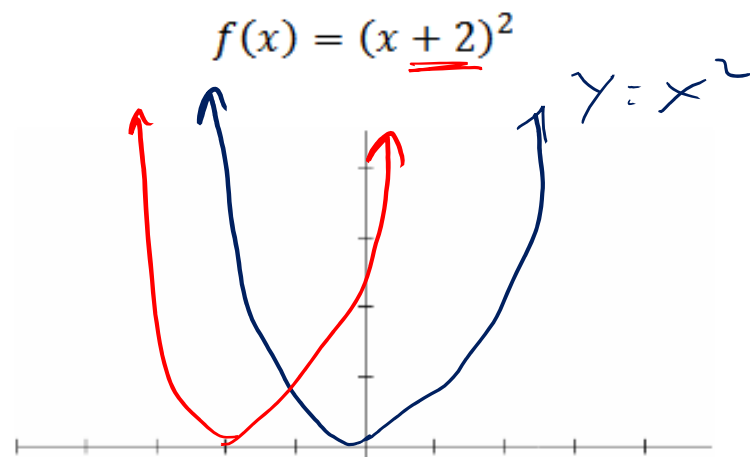
Example 2: Sketch the following graphs



Right 3

keypoint:

$(0,0) \rightarrow (3,0)$

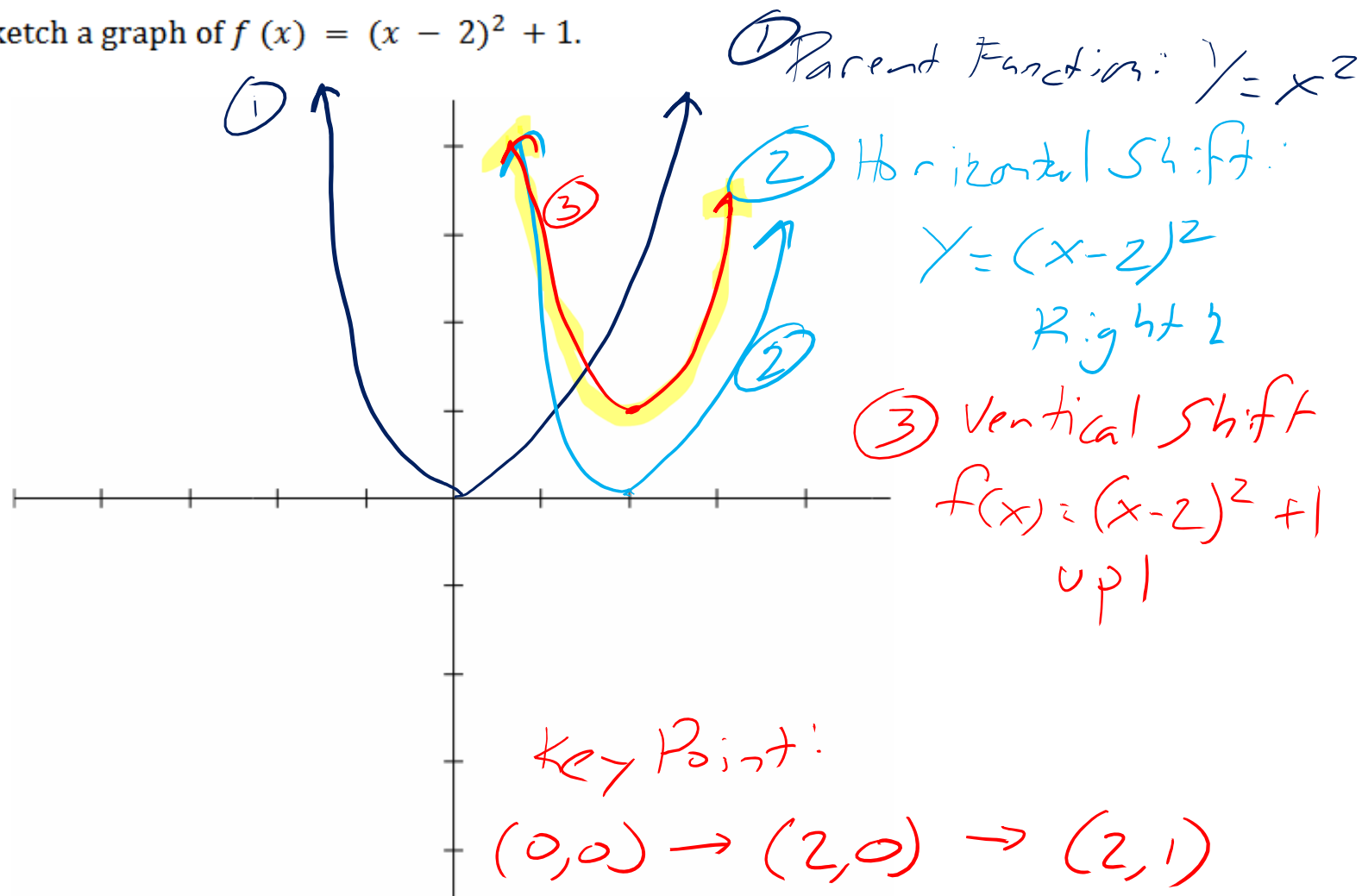


Left 2:

keypoint:

$(0,0) \rightarrow (-2,0)$

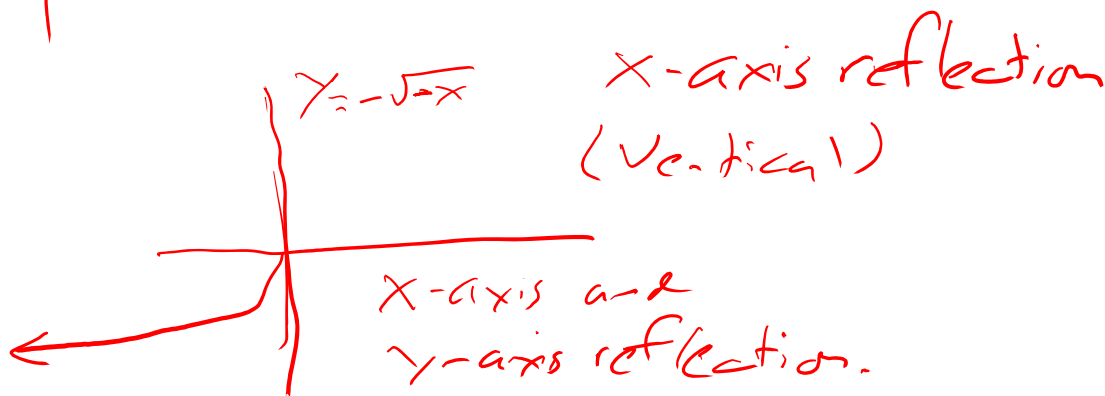
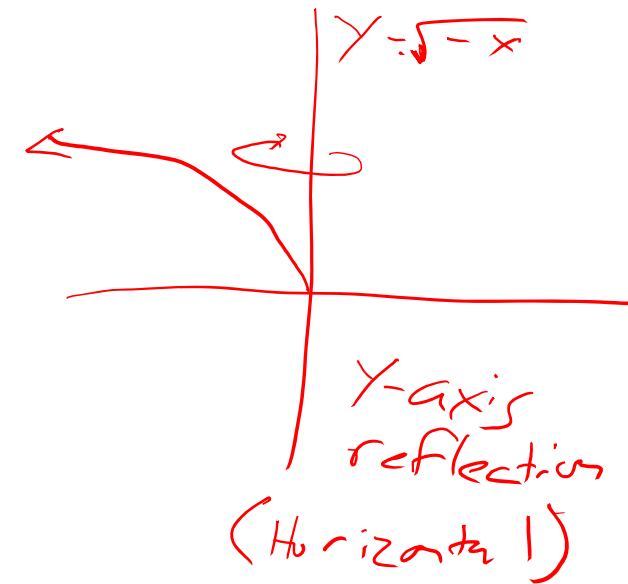
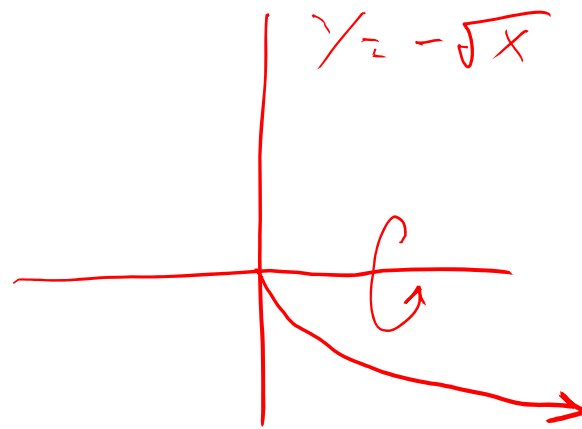
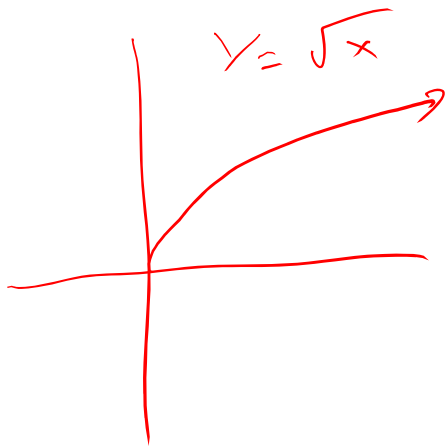
Example 3: Sketch a graph of $f(x) = (x - 2)^2 + 1$.



We can also reflect a function. A reflection of a function is its mirror image about the x axis or the y axis.

To graph $-f(x)$, reflect the graph of $f(x)$ about the x axis.

To graph $f(-x)$, reflect the graph of $f(x)$ about the y axis.

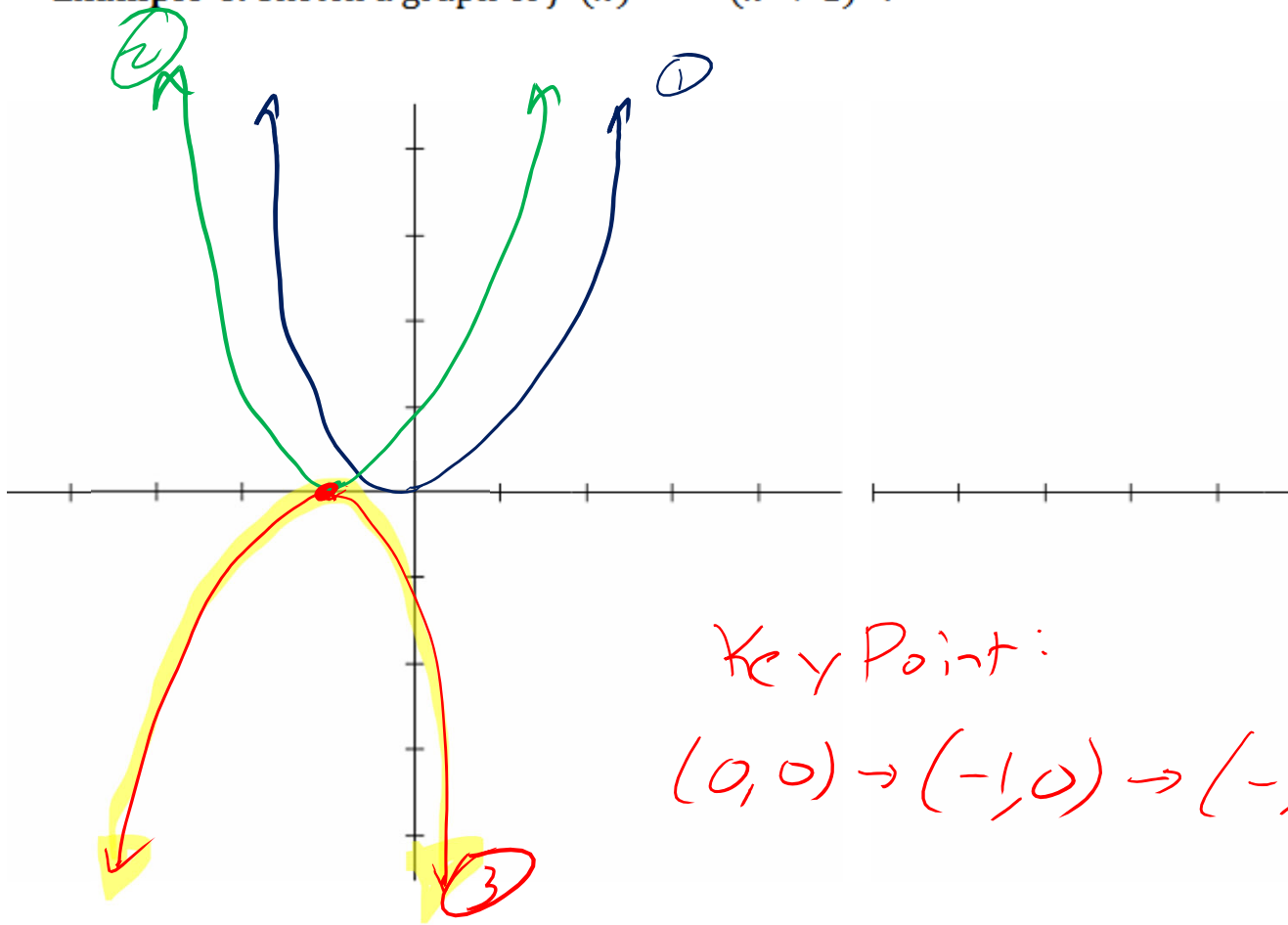


X-axis reflection
(Vertical)

Y-axis
reflection
(Horizontal)

X-axis and
y-axis reflection.

Example 4: Sketch a graph of $f(x) = -(x + 1)^2$.



① Parent Function:

$$y = x^2$$

② Horizontal Shift

$$y = (x + 1)^2$$

Left 1

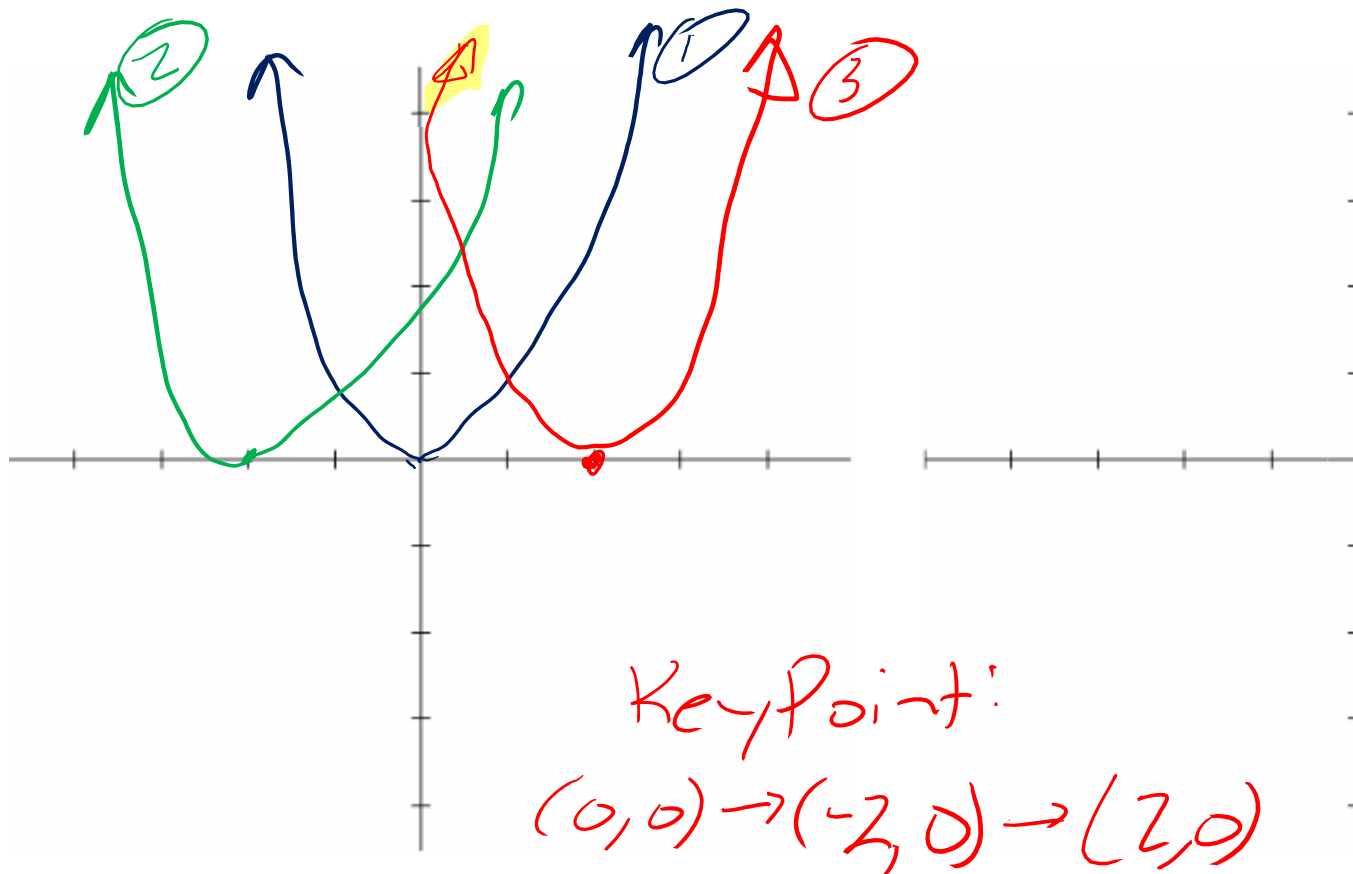
③ x-axis reflection

$$f(x) = -(x + 1)^2$$

Key Point:

$$(0, 0) \rightarrow (-1, 0) \rightarrow (-1, 0)$$

Example 5: Sketch a graph of $f(x) = (-x + 2)^2$.



① Parent Function
 $y = x^2$

② Horizontal Shift
Left 2
 $y = (x + 2)^2$

③ X-axis Reflection
 $f(x) = (-x + 2)^2$

Popper 12: $f(x) = -\sqrt{x+2} - 5$

\hookrightarrow vertical

1. Identify the Parent Function: $y = \sqrt{x}$
- a. Linear $y = mx + b$
 - b. Quadratic $y = (x+k)^2 + l$
 - c. Rational $y = \frac{1}{x+2} + 1$
 - d. Radical

2. Is there a vertical shift? $y = \sqrt{x} - 5$
- a. up 5
 - b. up 2
 - c. down 5
 - d. down 2

3. Is there a horizontal (y-axis) reflection? $y = \sqrt{-x}$
- a. Yes
 - b. No

4. Is there a vertical (x-axis) reflection? $y = -\sqrt{x}$
- a. Yes
 - b. No

