

MATH 1314

Section 3.4

Transforming Functions

In future courses, you will need to be able to sketch the graph of a function quickly and accurately. You can use transformations to do this. There are two types of transformations:

- Translation
- Reflections

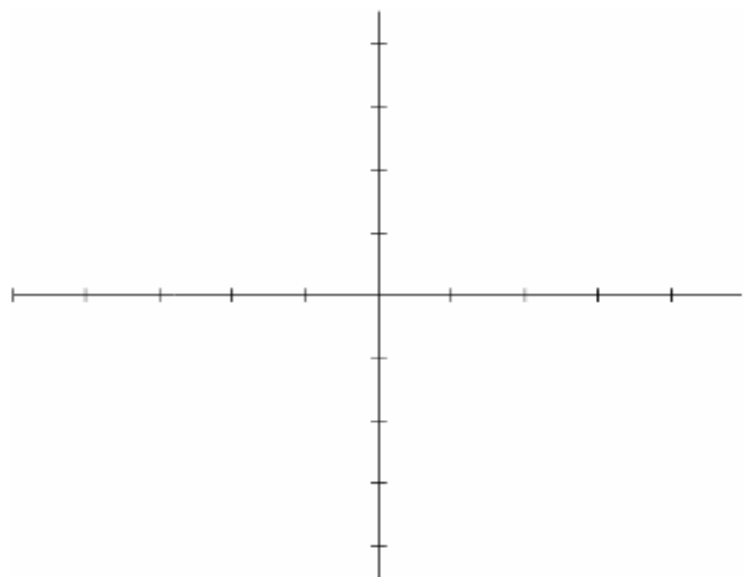
We'll start with **translations**. To **translate** a graph means to shift it horizontally, vertically or both.

Vertical shifting:

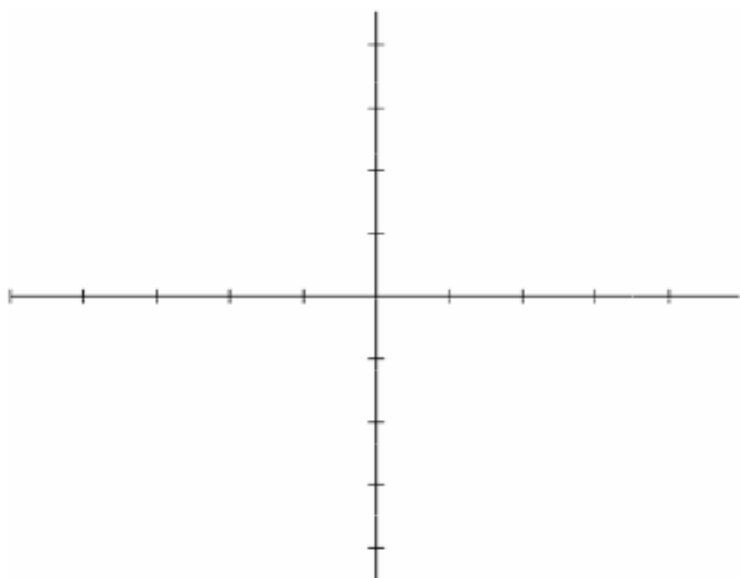
To graph $y = f(x) + c$, $c > 0$, start with the graph of $f(x)$ and shift it upward c units.

To graph $y = f(x) - c$, $c > 0$, start with the graph of $f(x)$ and shift it downward c units.

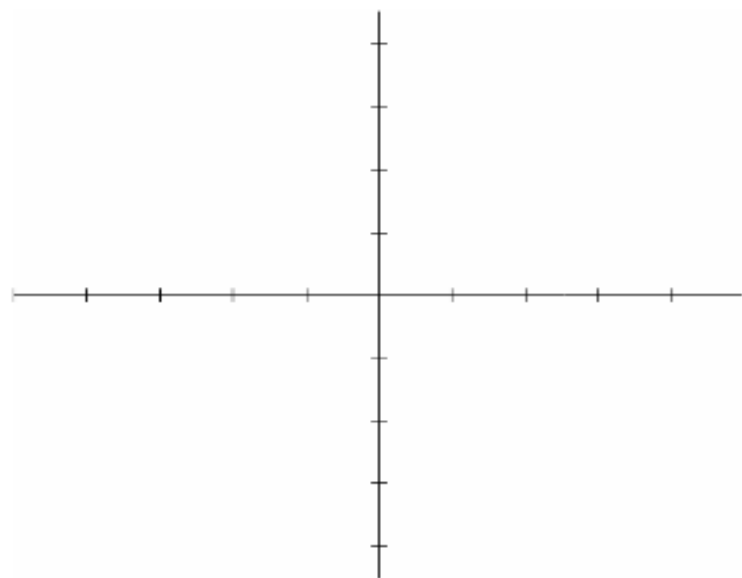
Example 1: Sketch $f(x) = x^2$



$$f(x) = x^2 + 2$$



$$f(x) = x^2 - 2$$



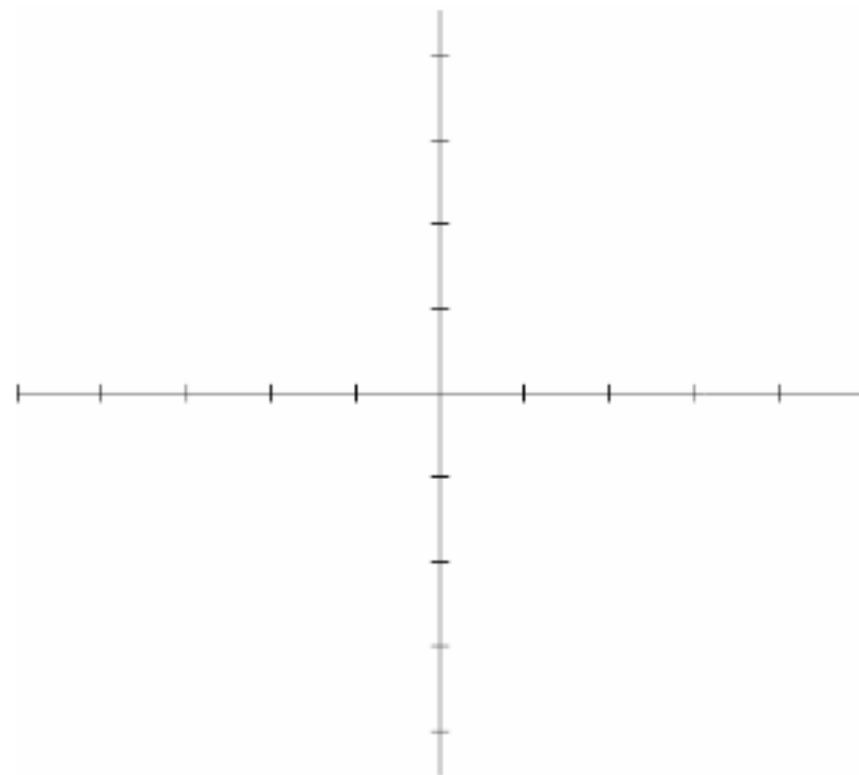
Horizontal shifting:

To graph $y = f(x + c)$, $c > 0$, start with the graph of $f(x)$ and shift it left c units.

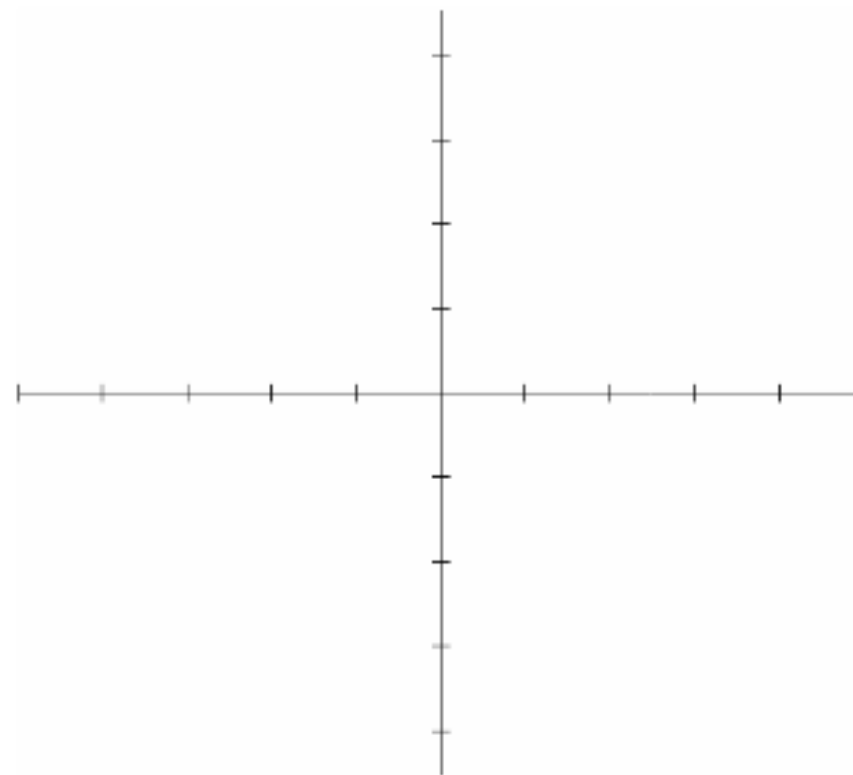
To graph $y = f(x - c)$, $c > 0$, start with the graph of $f(x)$ and shift it right c units.

Example 2: Sketch the following graphs

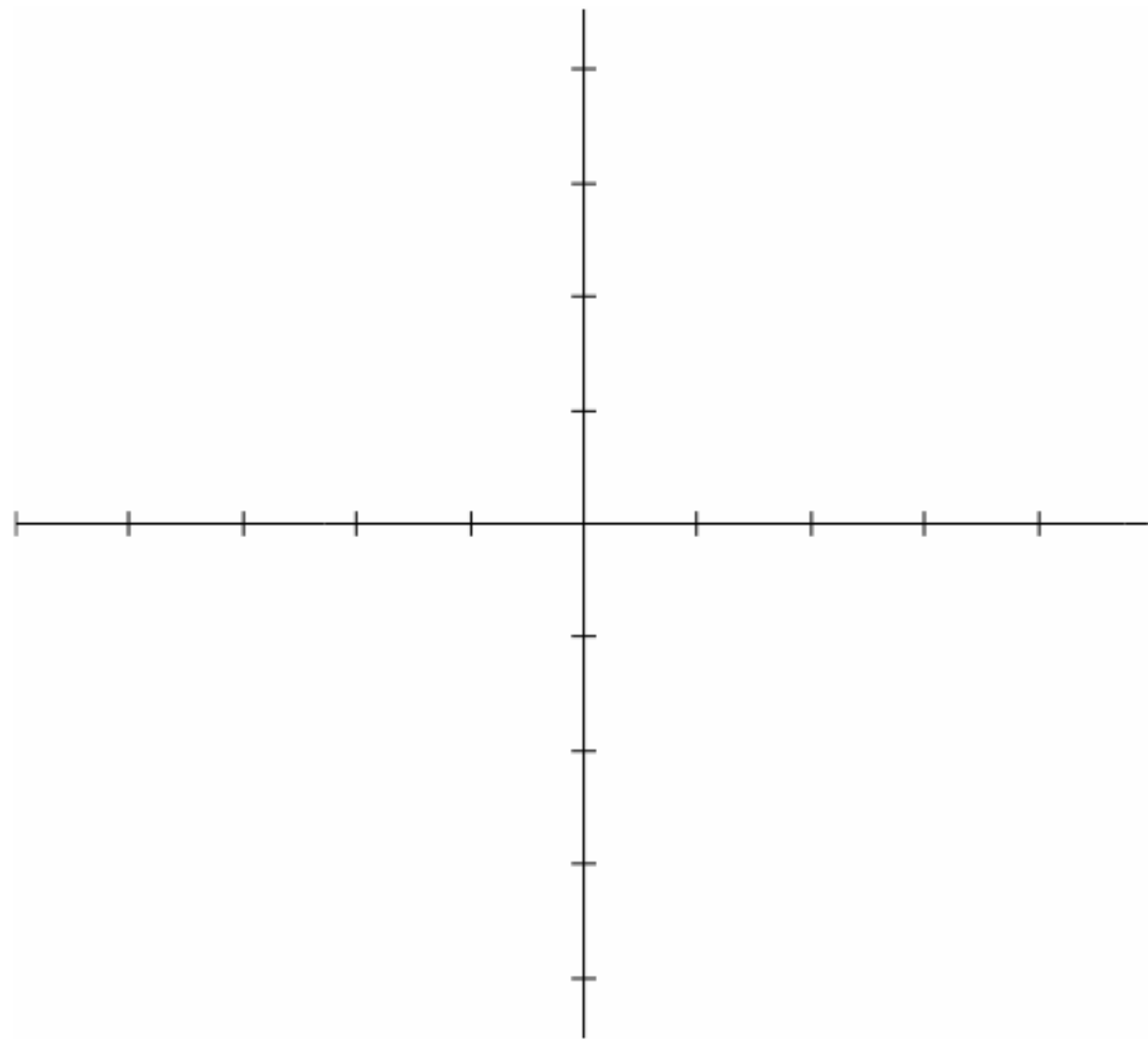
$$f(x) = (x - 3)^2$$



$$f(x) = (x + 2)^2$$



Example 3: Sketch a graph of $f(x) = (x - 2)^2 + 1$.

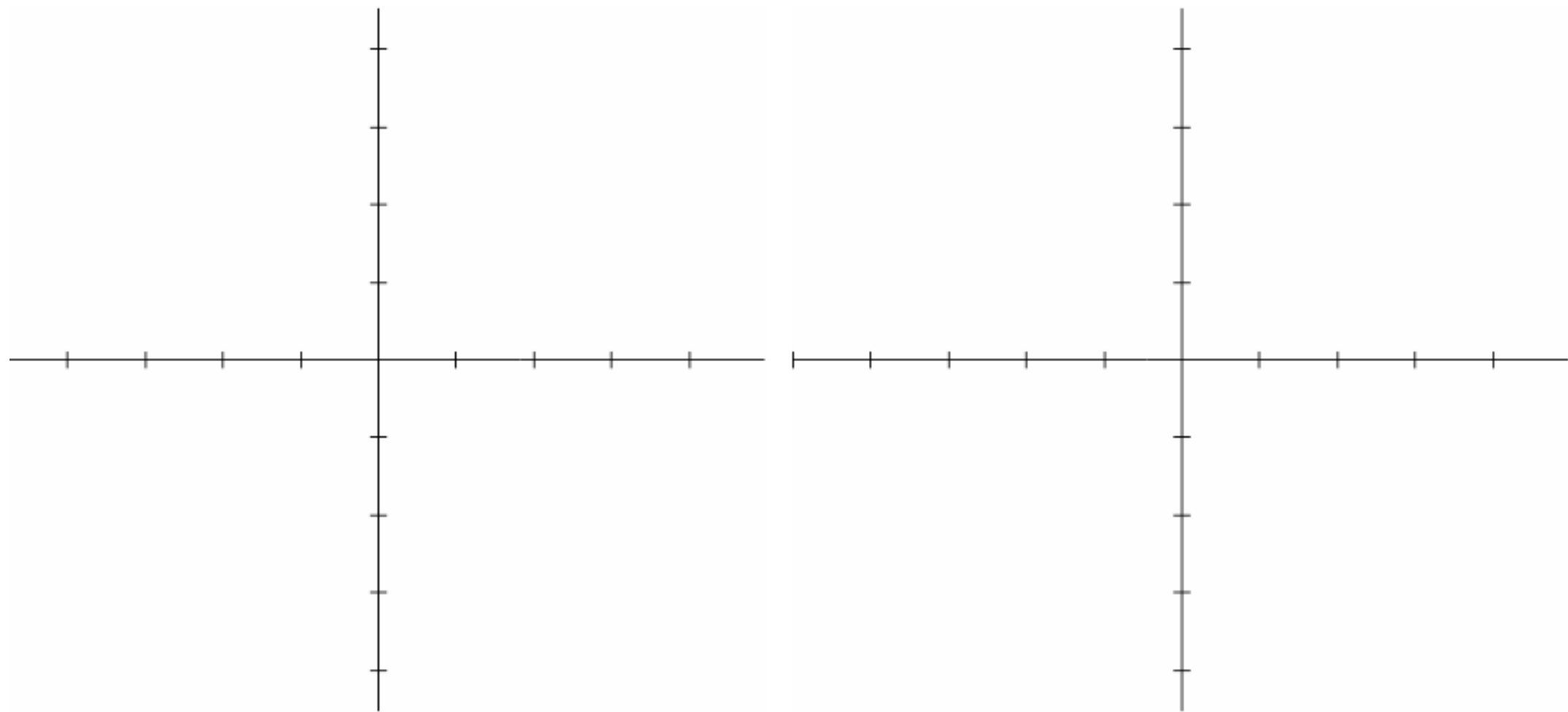


We can also reflect a function. A reflection of a function is its mirror image about the x axis or the y axis.

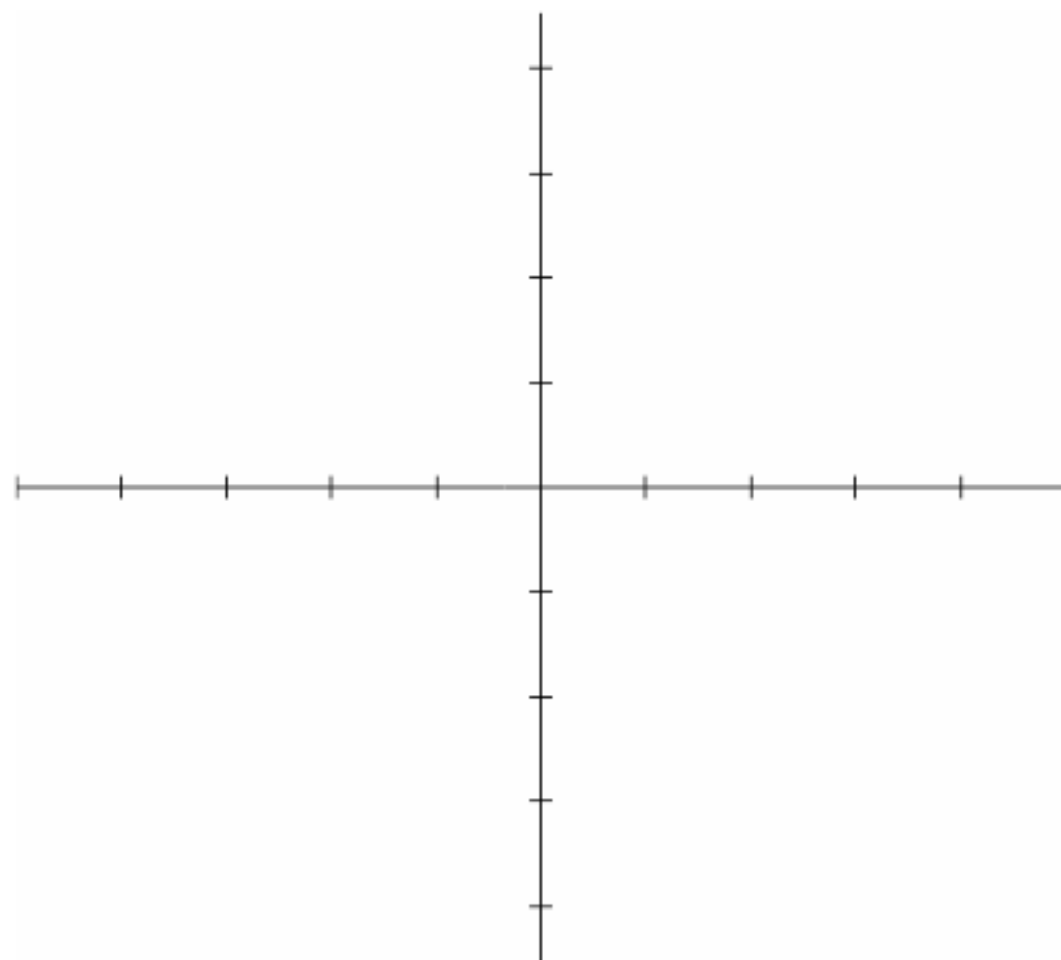
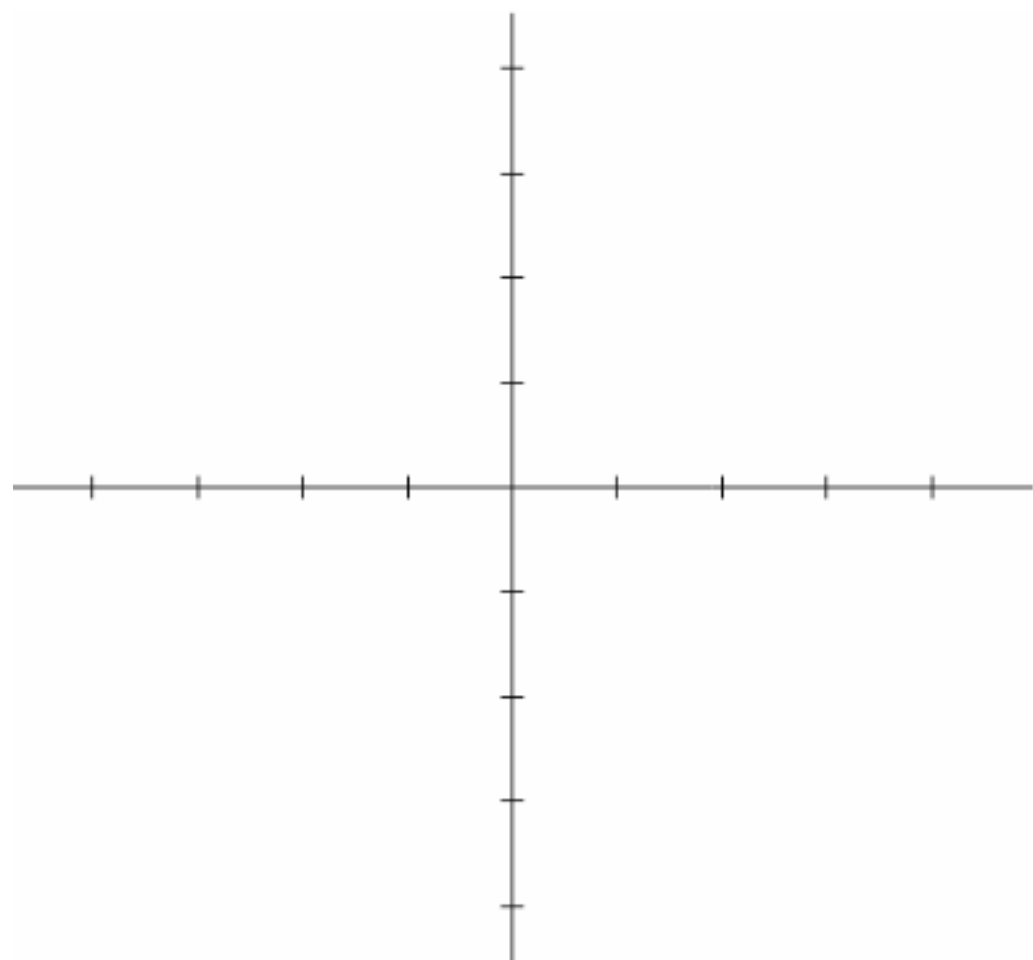
To graph $-f(x)$, reflect the graph of $f(x)$ about the x axis.

To graph $f(-x)$, reflect the graph of $f(x)$ about the y axis.

Example 4: Sketch a graph of $f(x) = -(x + 1)^2$.



Example 5: Sketch a graph of $f(x) = (-x + 2)^2$.



$$f(x) = -\sqrt{x + 2} - 5$$

Identify the Parent Function:

Is there a vertical shift?

Is there a horizontal (y-axis) reflection?

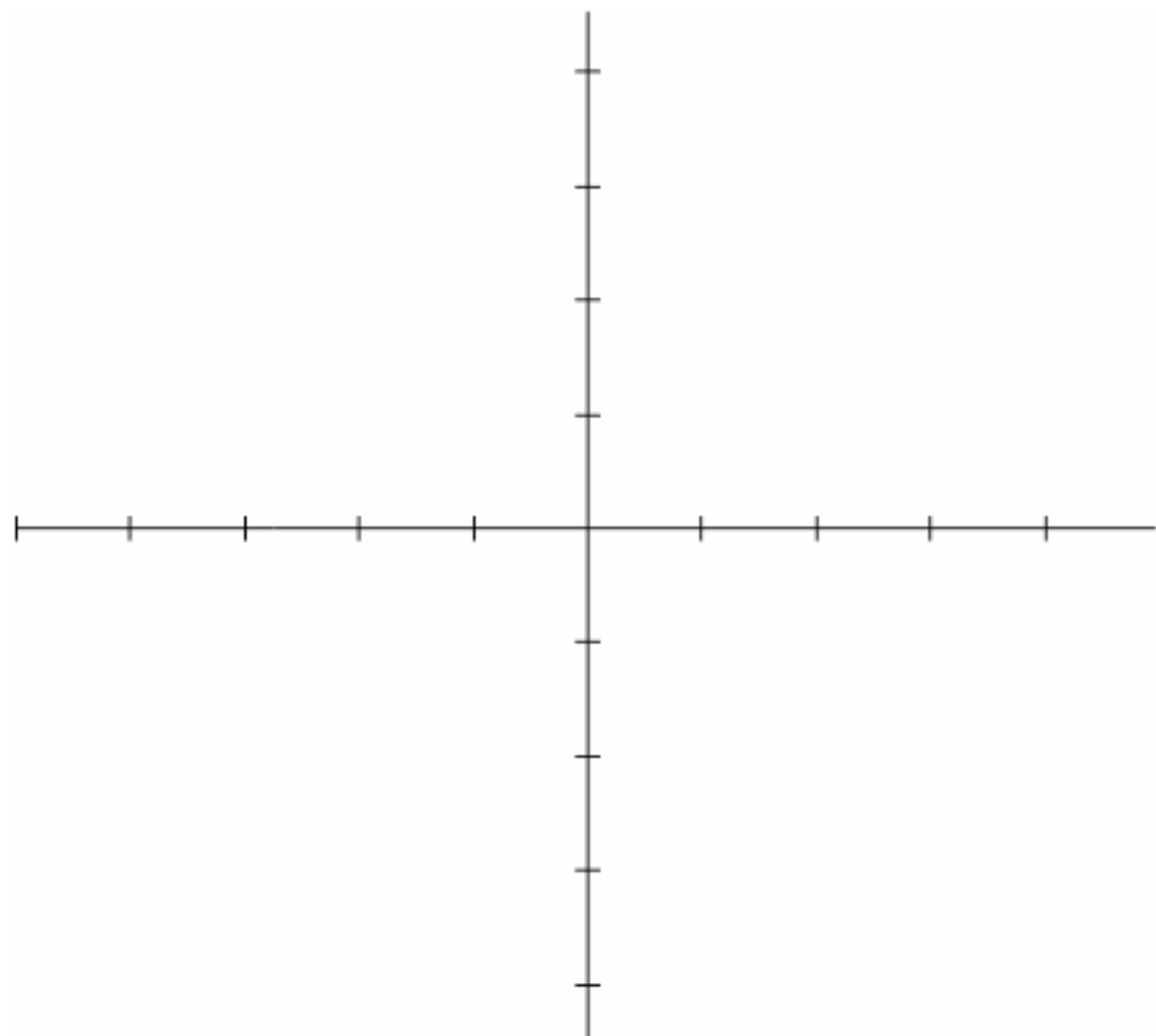
Is there a vertical (x-axis) reflection?

Finally, you can stretch or shrink your graph vertically. A vertical stretch will move your graph closer to the y axis, while a vertical shrink will move it closer to the x axis. It may be helpful to graph one or two points when your problem has a stretch or shrink.

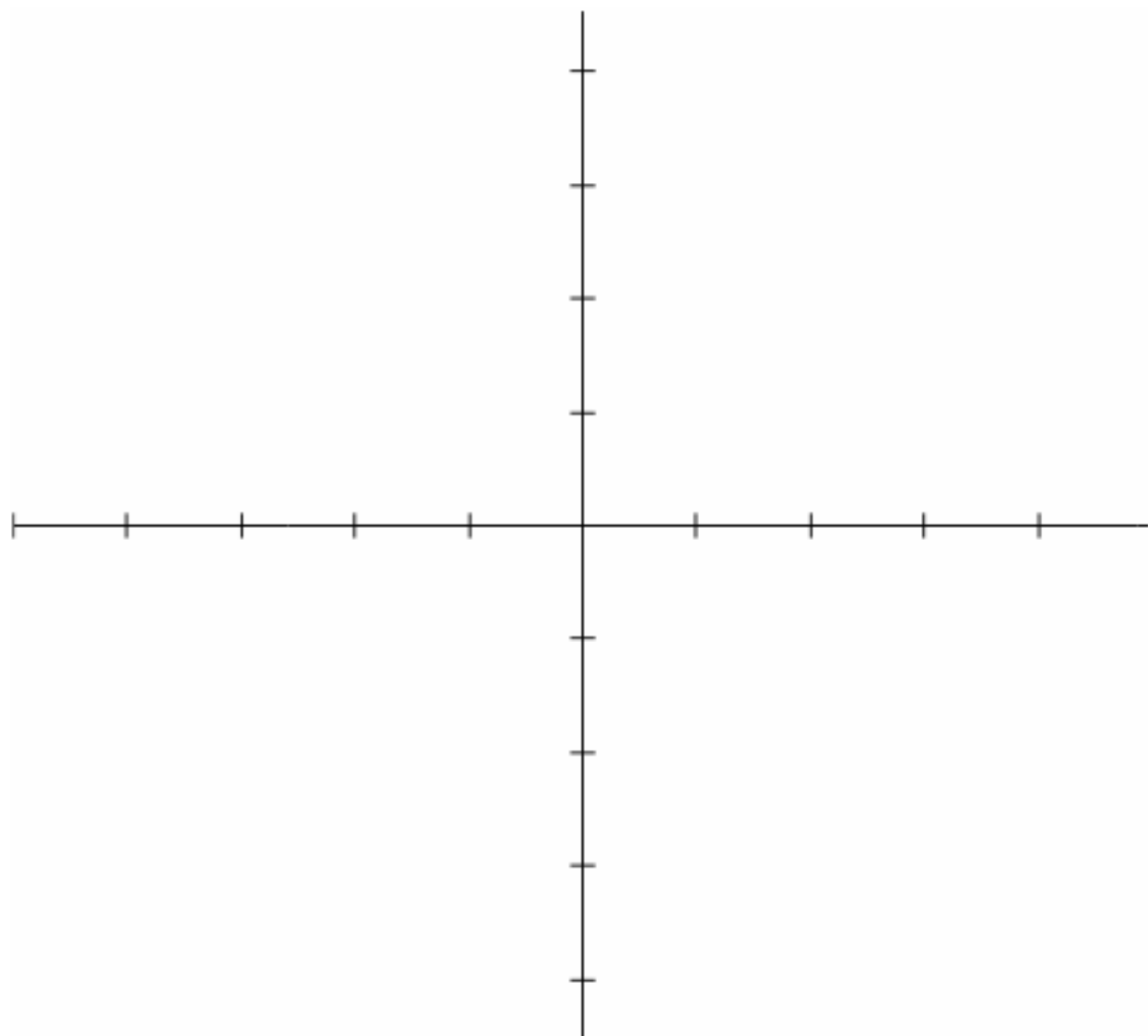
To graph $y = af(x)$, $a > 1$, stretch the graph of $f(x)$ by a factor of a .

To graph $y = af(x)$, $0 < a < 1$, shrink the graph of $f(x)$ by a factor of $\frac{1}{a}$

Example 6: Sketch a graph of $f(x) = 3(x - 1)^2$.



Example 7: Sketch a graph of $f(x) = \frac{1}{3}(x - 1)^2$.

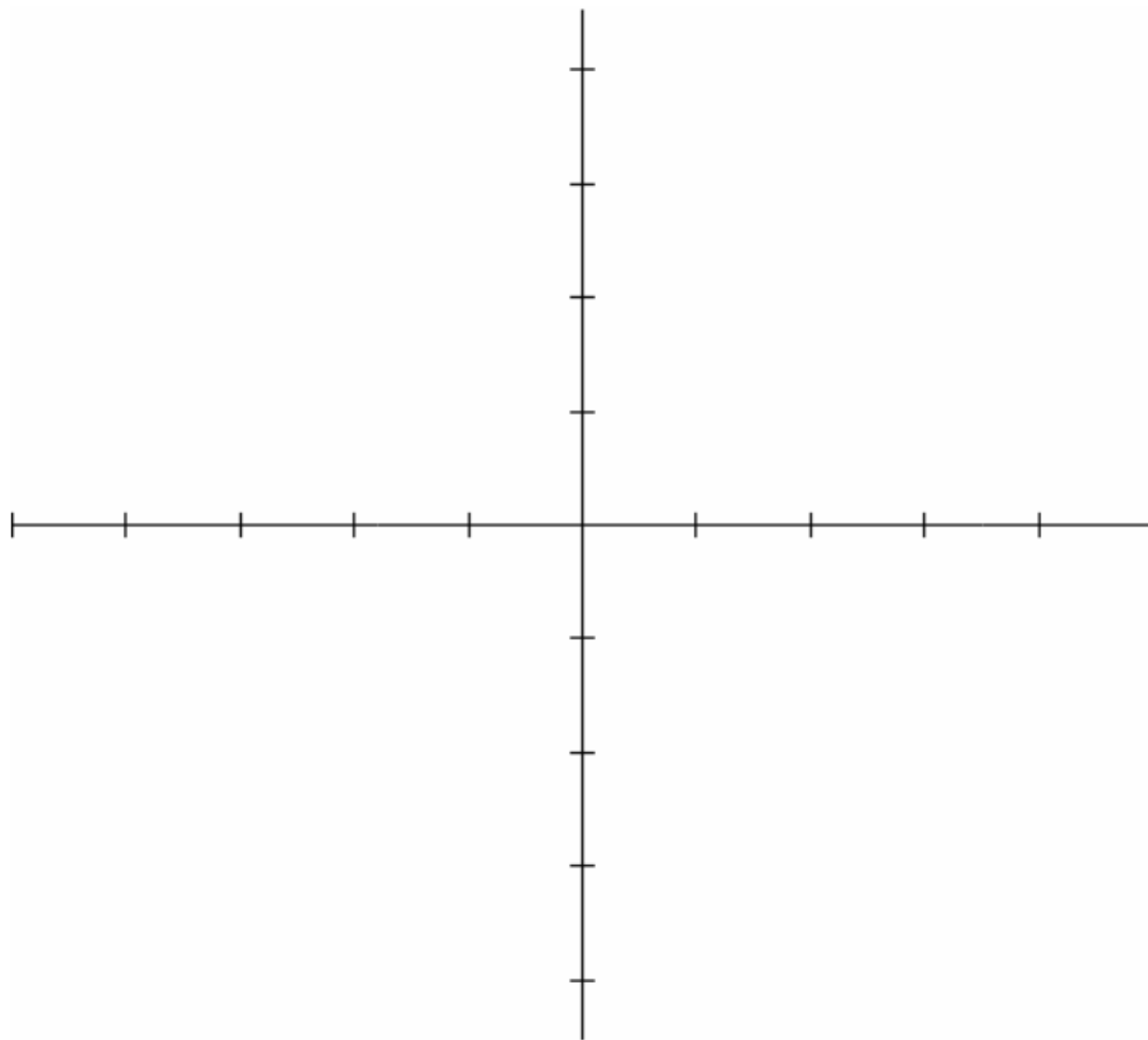


Recommended order for transforming functions:

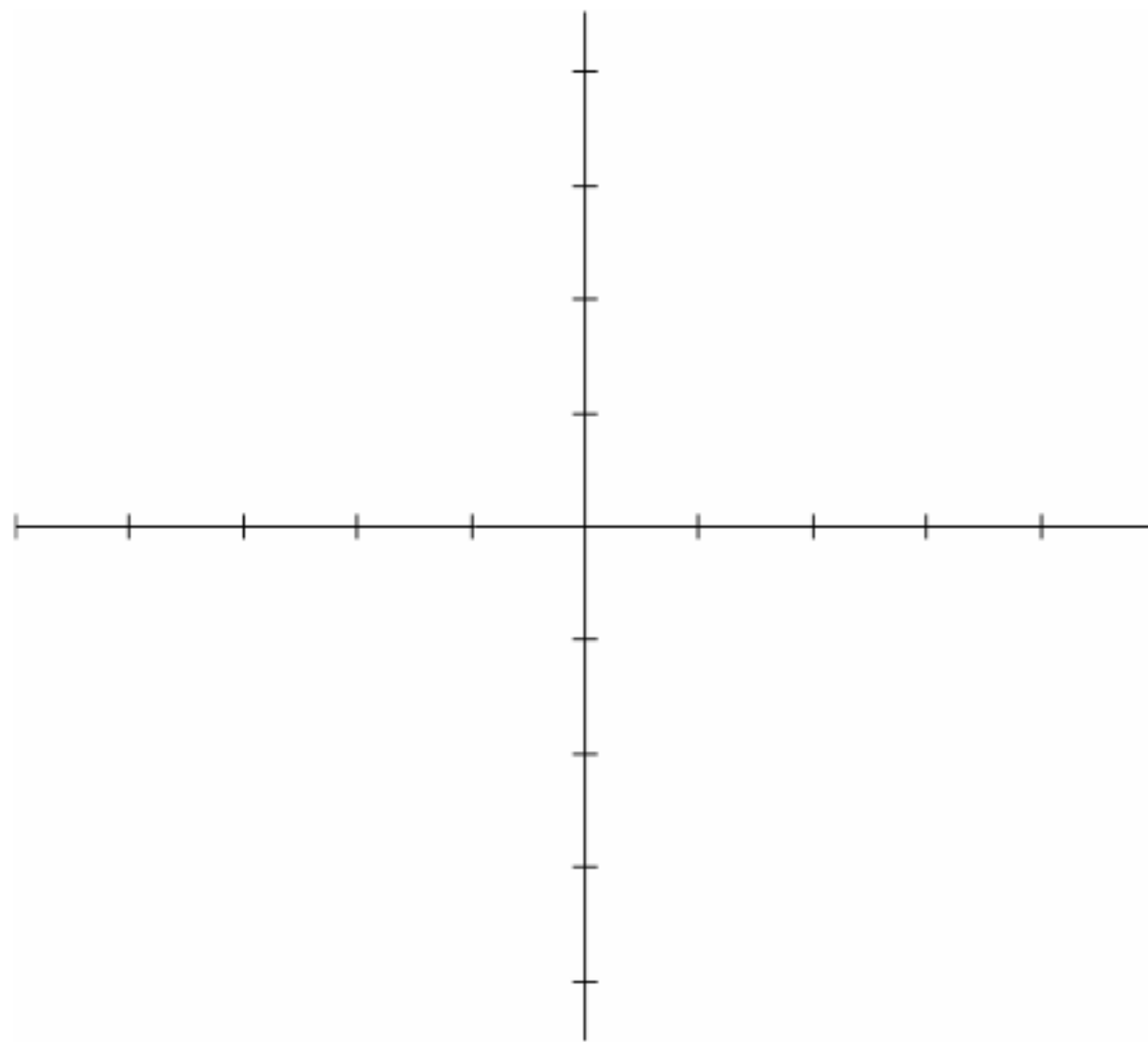
1. Vertically stretch or shrink the function.
2. Reflect the function about the x axis.
3. Translate the function vertically and/or horizontally.
4. Reflect the function about the y axis.

****Note, not all of these transformations will be presented in each problem. This is not the only order that works, but this order will get the job done with the fewest mistakes. Memorize this order!!!**

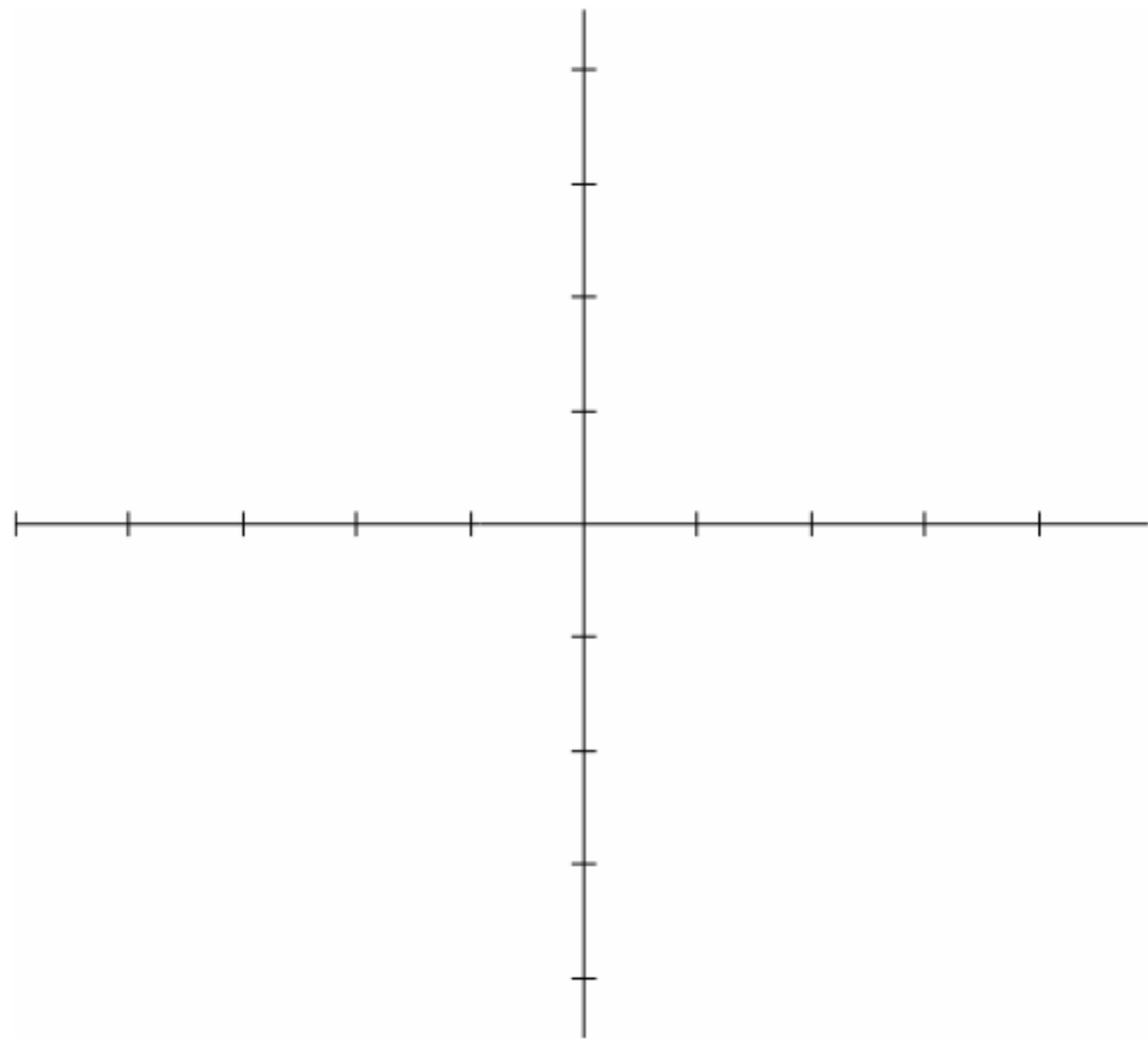
Example 8: Sketch the graph of $f(x) = -3(x - 2)^2 - 1$.



Example 9: Sketch the graph of $f(x) = \sqrt{x + 2} + 1$



Example 10: Sketch the graph of $f(x) = -\sqrt[3]{-x + 1} + 2$



Example 11: Describe how the graph of g is obtained from the graph of f .

a. $f(x) = \sqrt{x}$ $g(x) = \sqrt{x} - 3$

$f(x) = x^3$ $g(x) = -(x + 2)^3 + 1$