MATH 1314

Section 3.5

Maximum and Minimum Values

A quadratic equation is of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real and $a \neq 0$

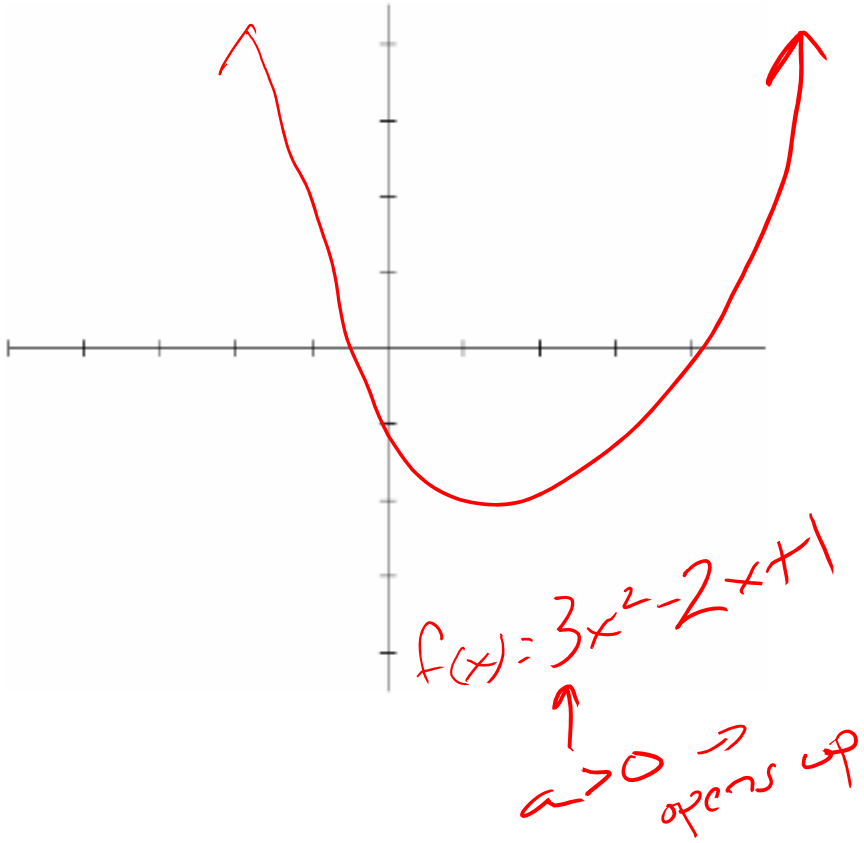
We have seen the graphs of **parabolas**.

$f(x) = ax^2 + bx + c$ Generic Form

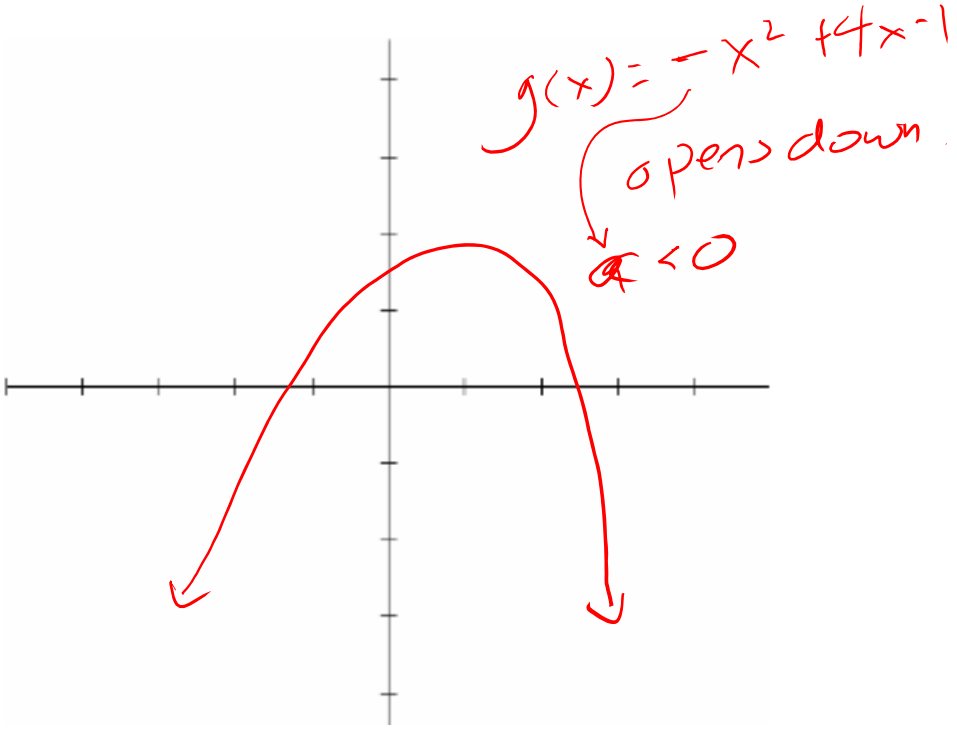
$f(x) = -2x^2 - 3x + 1$ Example

Opening Up or Opening Down

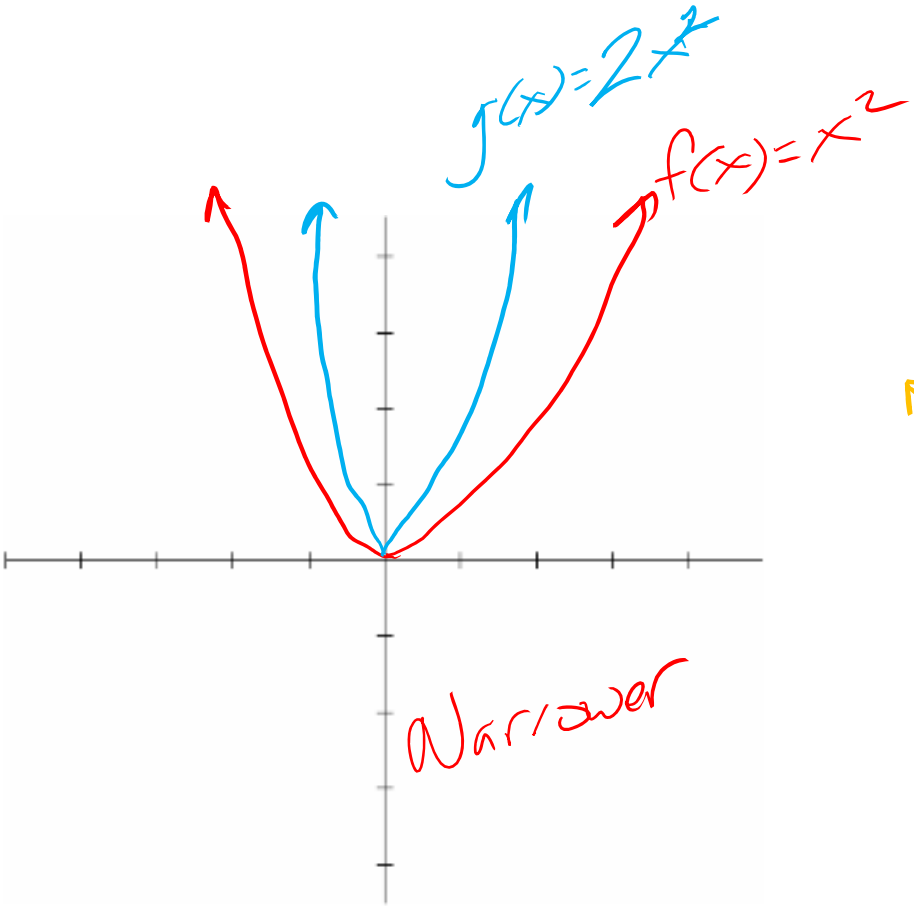
If $a > 0$ then the parabola will open upwards.

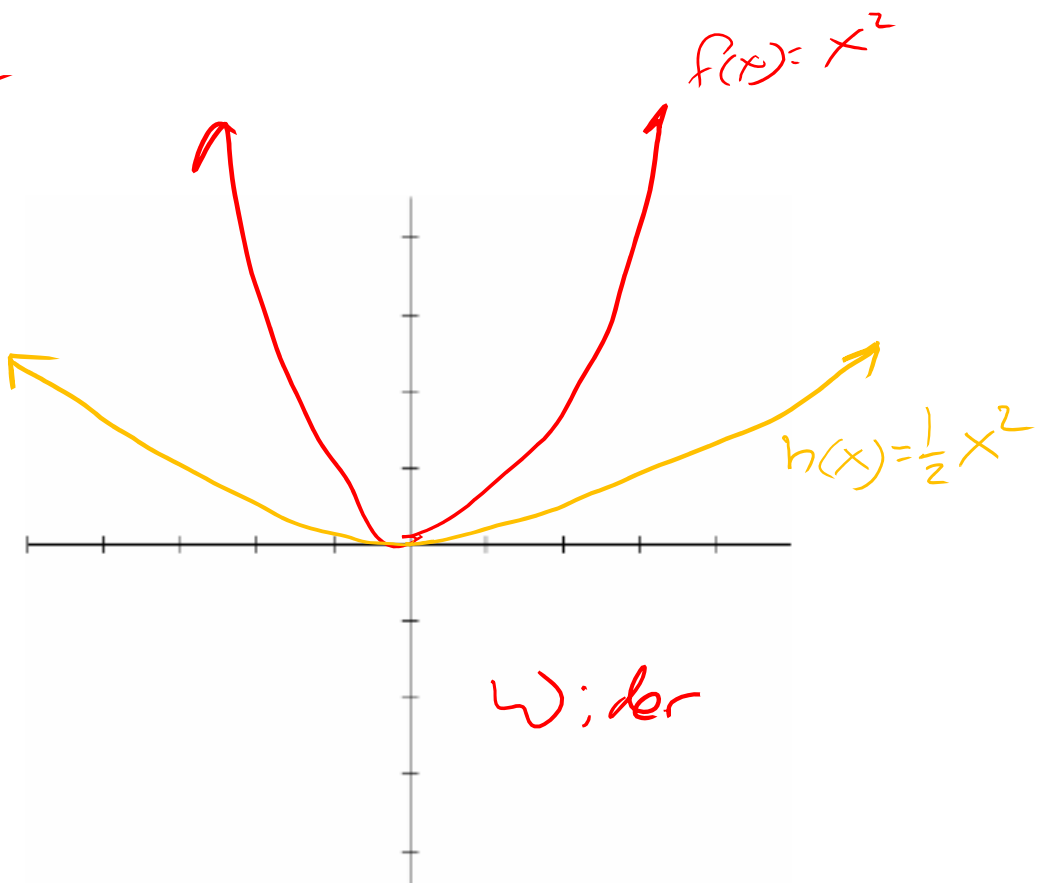


If $a < 0$ then the parabola will open downwards.



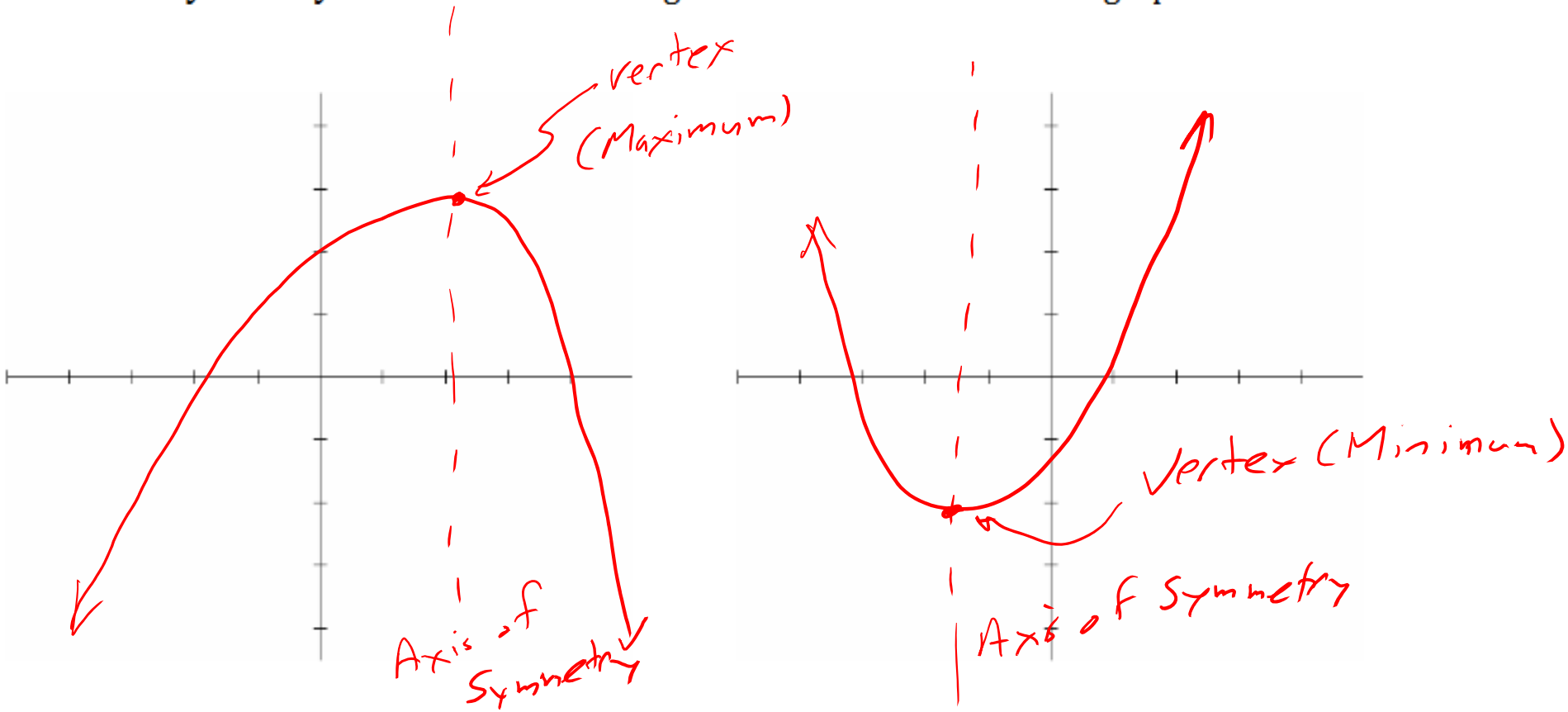
Note: The larger $|a|$, the narrower the parabola





The **vertex** is the turning point of the parabola and is the **minimum point** on the graph when it opens upward and the **maximum point** on the graph when it opens downward. Every parabola has a maximum or minimum, but **NOT** both.

The **axis of symmetry** is a vertical line through the vertex that divides the graph in half.



The Standard form of a Quadratic Function

The quadratic function $f(x) = a(x-h)^2 + k$ is in **standard form**

The vertex is the point $(h,\ k)$ and the axis of symmetry is $x = h$

The domain is $(-\infty, \infty)$.

The range is $[k,\ \infty)$ if $a > 0$ or $(-\infty,\ k]$ if $a < 0$

x-int: set $y=0$ and solve.

y-int: set $x=0$ and solve.

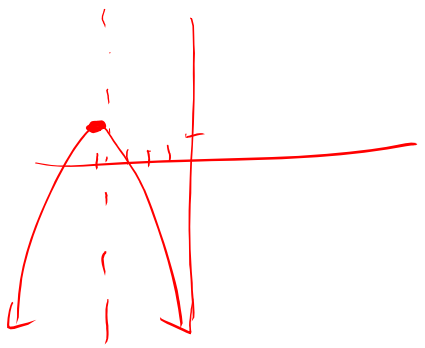
$f(x) = -2(x+4)^2 + 1$

$f(x) = a(x-h)^2 + k$

$a = -2$

$h = -4$

$k = 1$



vertex: $(-4, 1)$

Axis of Symmetry: $x = -4$

Domain: $(-\infty, \infty)$

Direction: $a<0$ open down

Range: $(-\infty, 1]$

Our first task will be to change a given quadratic function from the form $f(x) = ax^2 + bx + c$ to standard form. We'll complete the square to do this. Once the function is in standard form, we can sketch a graph using transformations and then read off the maximum or minimum value

General Form into Standard Form: Complete the Square

Standard Form into General: Simplify the expression

$f(x) = -2(x+4)^2 + 1$

$f(x) = -2(x+4)(x+4) + 1$

$f(x) = -2(x^2+4x+4x+16) + 1$

$f(x) = -2(x^2+8x+16) + 1$

$f(x) = -2x^2 - 16x - 32 + 1 \rightarrow$

$\boxed{f(x) = -2x^2 - 16x - 31}$

Example 1: Write the following quadratic in standard form. Then find the vertex and the axis of symmetry.

a. $f(x) = 3x^2 - 12x - 1$

$f(x) = 3(x^2 - 4x) - 1 = 3(x^2 - 4x + 4) - (3)(4) - 1$

$b = -4$

$\frac{b}{2} = -2$

$\left(\frac{b}{2}\right)^2 = 4$

$f(x) = 3(x-2)^2 - 12 - 1$

$f(x) = 3(x-2)^2 - 13$

$f(x) = a(x-h)^2 + k$

$a = 3$

$h = 2$

$k = -13$

vertex: $(h, k) = (2, -13)$

Axis of Symmetry: $x = h$

$x = 2$

b. $f(x) = -x^2 + 2x + 3$

$f(x) = -1(x^2 - 2x) + 3 = -1(x^2 - 2x + 1) - (-1)(1) + 3$

a ↓ -1 ; $(b/2)^2$ ↓ 1 ; $-(a)(b/2)^2$

$b = -2$

$\frac{b}{2} = -1$ (b/2 →)

$(b/2)^2 = 1$

$f(x) = -(x-1)^2 + 1 + 3$

$f(x) = -(x-1)^2 + 4$

$a = -1$
$h = 1$
$k = 4$

vertex: $(h, k) = (1, 4)$

axis of symmetry: $x = 4 \rightarrow x = 1$

Domain: $(-\infty, \infty)$

Range: $(-\infty, k] \rightarrow (-\infty, 4]$

direction: opens down

c. $f(x) = -10x^2 + 60x$

$f(x) = -10(x^2 - 6x) = -10(x^2 - 6x + 9) - (-10)(9)$

$b = -6$

$\frac{b}{2} = -3$

$\left(\frac{b}{2}\right)^2 = 9$

$f(x) = -10(x-3)^2 + 90$

vertex: $(h, k) \rightarrow (3, 90)$

Axis of Symmetry: $x = h \rightarrow x = 3$

What is the minimum/maximum value? Maximum of 90 opening down ($a = -10$)

what is: y-value

where is: x-value

Popper 14: $f(x) = -2x^2 - 16x - 27$

1. Complete the square and rewrite in standard form:

   a. $f(x) = -2(x + 9)^2$

   b. $f(x) = 2(x + 4)^2 - 5$

   c. $f(x) = -2(x + 4)^2 + 5$

   d. $f(x) = 2(x - 4)^2 - 5$

2. Determine the direction of the parabola:

   a. Opening Up

   b. Opening Down

3. Determine the equation of the axis of symmetry:

   a. $x = 4$

   b. $x = -4$

   c. $x = 5$

   d. $x = -5$

4. Determine the coordinates of the vertex:

   a. (-4, 5)

   b. (-5, 10)

   c. (4, 5)

   d. (-4, -5)

5. Is the vertex of this parabola a minimum or a maximum?

   a. Minimum

   b. Maximum

   c. ~~Neither~~

$-2(x^2+8x)-27$

$b = 8$

$\frac{b}{2} = 4$

$\left(\frac{b}{2}\right)^2 = 16$

$a = -2$

$f(x) = -2(x^2+8x+16)-(-2)(16)-27$

$f(x) = -2(x+4)^2+5$

$a = -2$

$h = -4$

$k = 5$

$x = 4$

(h,k)