MATH 1314

Section 3.6

Combining Functions:

Suppose we have two functions $f(x)$ and $g(x)$. The domain of $f(x)$ is the set A. The domain of $g(x)$ is the set B. We can combine these two functions together in five different ways:

Sum of Functions
Difference of Functions
Product of Functions
Quotient of Functions
Composition of Functions

Sum of Functions: $(f+g)(x) = f(x) + g(x)$ with domain A ∩ B

Difference of Functions: $(f-g)(x) = f(x) - g(x)$with domain A ∩ B

Product of Functions: $(fg)(x) = f(x)g(x)$ with domain A ∩ B

Quotient of Functions: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ with domain $\{x \in A \cap B \mid g(x) \neq 0\}$

Example 1: Suppose $f(x) = 2x - 5$ and $g(x) = x^2 - 3x + 5$. Find each of the following and state the domain:

D: $(-\infty, \infty)$ D: $(-\infty, \infty)$

a. $(f+g)(x) = f(x) + g(x) = 2x - \cancel{5} + x^2 - 3x + \cancel{5} =$

$(f+g)(x) = x^2 - x$ D: $(-\infty, \infty)$

b. $(f-g)(x) = f(x) - g(x) = 2x - 5 - (x^2 - 3x + 5) =$

$2x - 5 - x^2 + 3x - 5 = -x^2 + 5x - 10$

$(f-g)(x) = -x^2 + 5x - 10$ D: $(-\infty, \infty)$

Example 1: Suppose $f(x) = 2x - 5$ and $g(x) = x^2 - 3x + 5$. Find each of the following and state the domain:

c. $(fg)(x) = f(x) \cdot g(x) = (2x-5)(x^2-3x+5)$

$-5x^2 + 15x - 25 + 2x^3 - 6x^2 + 10x$

$D: (-\infty, \infty)$ $\quad (f \cdot g)(x) = 2x^3 - 11x^2 + 25x - 25$

d. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x-5}{x^2-3x+5}$ $\quad D: (-\infty, \infty)$

where $g(x) = 0$

$D = b^2 - 4ac$

$(-3)^2 - 4(1)(5)$

$9 - 20 = -11$

$D < 0$ meaning $g(x) = 0$ has No Real Solutions.

Popper 14: $D:(-\infty,\infty)$ $D:(-\infty,\infty)$

Example 2: Given $f(x) = 3x^4 + 2x^3 - 8$ and, $g(x) = -x^2$, find each of the following functions and its domain.

$(g-f)(x) = g(x) - f(x) = -x^2 - (3x^4 + 2x^3 - 8)$

$= -x^2 - 3x^4 - 2x^3 + 8$

Question 1: Function Answer:

a. $3x^4 + 2x^3 - x^2 - 8$ b. $-3x^4 - 2x^3 - x^2 + 8$ c. $-3x^4 - 3x^3 + 8$ d. $-3x^4 - 2x^3 + x^2 + 8$

Question 2: Domain:

a. $(-\infty, \infty)$ b. $(0, \infty)$ c. $[0, \infty)$ d. $(-\infty, 8]$

$\frac{f(x)}{g(x)} = \frac{3x^4 + 2x^3 - 8}{-x^2} = \frac{3x^4}{-x^2} + \frac{2x^3}{-x^2} - \frac{8}{-x^2} = -3x^2 - 2x + \frac{8}{x^2}$

Question 3: Function Answer:

a. $3x^2 - (8/x^2)$ b. $3x^2 + 2x - (8/x^2)$ c. $-3x^2 - 2x + (8/x^2)$ d. $3x^2 + (8/x^2)$

Question 4: Domain:

a. $(-\infty, \infty)$ b. $[0, \infty)$ c. $(-\infty, 0)$ d. $(-\infty, 0) \cup (0, \infty)$

$g(x) \neq 0$

$-x^2 \neq 0$

$x \neq 0$

Popper 14…continued:

Example 3: Let $f(x) = x^2 - 3x - 1$ and $g(x) = -3x - 10$. Find

$(f + g)(1) = f(1) + g(1) = (-3) + (-13)$
$= -16$

$f(1) = 1^2 - 3(1) - 1$
$= 1 - 3 - 1 = -3$

$g(1) = -3(1) - 10$
$= -3 - 10 = -13$

Question 5:

a. -16 b. 39 c. 10 d. No Solution

$g(-1) = -3(-1) - 10$
$= 3 - 10 = -7$

$(gg)(-1) = g(-1) \cdot g(-1) = (-7)(-7) = 49$

Question 6:

a. -7 b. 49 c. 0 d. No Solution

Composition of Functions

Domain example: g(x) undefined at x = 5: ① Remove x = 5 ② Remove where f(x) = 5

The composition of the function *f* with *g* is denoted $f \circ g$ by and is defined by the

$$(f \circ g)(x) = \underline{f\big(g(x)\big)}$$

Height in inches:

$$h(x) = 5\boxed{f(x)} + 10$$

$$\boxed{f(x) = 12}$$

$$h(x) = 5(12) + 10 = 70$$

The domain of the composition $f \circ g$ is the set of all *x* such that

1. *x* is in the domain of *g* (the "inside" function)
2. *g(x)* is in the domain of *f* (the "outside" function)

$(f \circ g)(x)$: "f follows g(x)"

(g(x) is handled first)

$$(f \circ g)(x) = f(g(x))$$

Remove all x values that make g(x) undefined and all x-values that make f(x) equal where g(x) is undefined.

Example 4: Let $f(x) = x^2 + 1$ and $g(x) = -2x + 5$, find $(f \circ g)(x)$.

$(f \circ g)(x) = f(g(x)) = f(-2x+5) = (-2x+5)^2 + 1$

$= (-2x+5)(-2x+5) + 1$

$= 4x^2 - 10x - 10x + 25 + 1$

$(f \circ g)(x) = 4x^2 - 20x + 26$

Example 5: Let $f(x) = \frac{1}{x}$ and $g(x) = \frac{5}{x+4}$, find $(g \circ f)(x)$

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x}\right) = \frac{5}{\frac{1}{x}+4}$$

$$= \frac{5 \cdot x}{x \cdot \frac{1}{x} + 4 \cdot x} = \frac{5x}{1+4x}$$

$$(g \circ f)(x) = \frac{5x}{4x+1}$$

Example 6: Let $f(x) = \sqrt{4 - x^2}$ and $g(x) = \sqrt{3 - x}$, find $(f \circ g)(x)$.

$(f \circ g)(x) = f(g(x)) = f(\sqrt{3-x}) = \sqrt{4 - (\sqrt{3-x})^2}$

$= \sqrt{4 - (3-x)} = \sqrt{4 - 3 + x}$

$\sqrt{1 + x}$

$(f \circ g)(x) = \sqrt{x+1}$

Example 7: Suppose $f(x) = 3x - 5$ and $g(x) = x^2 + 4x + 3$. Find each of the following.

a. $(f \circ g)(2)$

$f(g(2)) = f(15) = 3(15) - 5 = 45 - 5 = \boxed{40}$

$g(2) = (2)^2 + 4(2) + 3 = 4 + 8 + 3 = 15$

c. $(g \circ f)(x)$ → Next slide

b. $(g \circ f)(-1)$

$g(f(-1)) = g(-8) = (-8)^2 + 4(-8) + 3 = 64 - 32 + 3 = \boxed{35}$

$f(-1) = 3(-1) - 5 = -3 - 5 = -8$

d. $(g \circ g)(0) = g(g(0)) = g(3)$

$g(0) = 0^2 + 4(0) + 3 = 3$

$g(3) = 3^2 + 4(3) + 3 = 9 + 12 + 3 = \boxed{24}$

Example 7: Suppose $f(x) = 3x - 5$ and $g(x) = x^2 + 4x + 3$. Find each of the following.

c. $(g \circ f)(x)$

$$g(f(x)) = g(3x-5) = (3x-5)^2 + 4(3x-5) + 3$$

$$= (3x-5)(3x-5) + 4(3x-5) + 3$$

$$= 9x^2 - 15x - 15x + 25 + 12x - 20 + 3$$

$$(g \circ f)(x) = 9x^2 - 18x + 8$$

Popper 14 continued:

Consider the functions: $f(x) = x^2 + 2x - 8 \; and \; g(x) = \sqrt{x+3}$.

7. Determine $(f \circ g)(x)$ $= f(g(x)) = f(\sqrt{x+3}) = \sqrt{x+3}^2 + 2\sqrt{x+3} - 8$

a. $\sqrt{x+3} - 8$ b. $x^2 + 8x + 7$

c. $x + 2\sqrt{x+3} - 5$ d. $\sqrt{x^2 + 2x - 5}$

$= x + 3 + 2\sqrt{x+3} - 8$

$= x + 2\sqrt{x+3} - 5$

8. Determine $(f \circ g)(6)$ $= 6 + 2\sqrt{6+3} - 5 = 6 + 2\sqrt{9} - 5 = 6 + 2(3) - 5$

$= 6 + 6 - 5 = 7$

a. 2.36 b. 7 c. $\sqrt{43}$ d. 3

9. Determine: $(f \circ g)(-3)$ $= -3 + 2\sqrt{-3+3} - 5 = -3 + \cancel{2\sqrt{0}} - 5 = -8$

a. -8 b. $\sqrt{10}$ c. -5 d. No Answer

10. Determine $(f \circ g)(-5)$ $= -3 + 2\sqrt{-5+3} - 5 = -3 + 2\sqrt{-2} - 5$

$\hookrightarrow$ imaginary

a. -2 b. $\sqrt{10}$ c. 0 d. No Answer

Given the following table of values, determine: $(f \circ g)(4)$. $= f(g(4)) = f(8) = \boxed{2}$

Step 1: $g(4) = 8$

Step 2: $f(8) = 2$

x	f(x)	g(x)
0	4	0
2	-1	6
step 1 4	3	8
6	8	2
step 2 8	2	8