

MATH 1314

Section 3.6

Combining Functions:

Suppose we have two functions $f(x)$ and $g(x)$. The domain of $f(x)$ is the set A. The domain of $g(x)$ is the set B. We can combine these two functions together in five different ways:

Sum of Functions

Difference of Functions

Product of Functions

Quotient of Functions

Composition of Functions

Sum of Functions: $(f + g)(x) = f(x) + g(x)$ with domain $A \cap B$

Difference of Functions: $(f - g)(x) = f(x) - g(x)$ with domain $A \cap B$

Product of Functions: $(fg)(x) = f(x)g(x)$ with domain $A \cap B$

Quotient of Functions: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ with domain $\{x \in A \cap B \mid g(x) \neq 0\}$

Example 1: Suppose $f(x) = 2x - 5$ and $g(x) = x^2 - 3x + 5$. Find each of the following and state the domain:

$$D: (-\infty, \infty) \quad D: (-\infty, \infty)$$

a. $(f + g)(x) = f(x) + g(x) = \underline{2x - 5} + \underline{x^2 - 3x + 5} =$
 $(f + g)(x) = x^2 - x \quad D: (-\infty, \infty)$

b. $(f - g)(x) = f(x) - g(x) = 2x - 5 - (x^2 - 3x + 5) =$
 $\underline{2x - 5} - \underline{x^2 + 3x - 5} = -x^2 + 5x - 10$
 $(f - g)(x) = -x^2 + 5x - 10 \quad D: (-\infty, \infty)$

Example 1: Suppose $f(x) = 2x - 5$ and $g(x) = x^2 - 3x + 5$. Find each of the following and state the domain:

c. $(fg)(x) = f(x) \cdot g(x) = (2x-5)(x^2-3x+5)$

$$= \underbrace{-5x^2 + 15x - 25}_{\text{blue}} + \underbrace{2x^3 - 6x^2 + 10x}_{\text{green}}$$

$D: (-\infty, \infty)$ $(fg)(x) = 2x^3 - 11x^2 + 25x - 25$

d. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x-5}{x^2-3x+5}$ $D: (-\infty, \infty)$

where $g(x) = 0$

$$D = b^2 - 4ac$$

$$(-3)^2 - 4(1)(5)$$

$$9 - 20 = -11$$

$D < 0$ meaning $g(x) = 0$ has no Real solutions.

Popper 16: $D: (-\infty, \infty)$ $D: (-\infty, \infty)$

Example 2: Given $f(x) = 3x^4 + 2x^3 - 8$ and, $g(x) = -x^2$, find each of the following functions and its domain.

$$(g - f)(x) = g(x) - f(x) = -x^2 - (3x^4 + 2x^3 - 8)$$

$$= -x^2 - 3x^4 - 2x^3 + 8$$

Question 1: Function Answer:

- a. $3x^4 + 2x^3 - x^2 - 8$ b. $-3x^4 - 2x^3 - x^2 + 8$ c. $-3x^4 - 3x^3 + 8$ d. $-3x^4 - 2x^3 + x^2 + 8$

Question 2: Domain:

- a. $(-\infty, \infty)$ b. $(0, \infty)$ c. $[0, \infty)$ d. $(-\infty, 8]$

$$\frac{f(x)}{g(x)} = \frac{3x^4 + 2x^3 - 8}{-x^2} = \frac{3x^4}{-x^2} + \frac{2x^3}{-x^2} - \frac{8}{-x^2} = -3x^2 - 2x + \frac{8}{x^2}$$

Question 3: Function Answer:

- a. $3x^2 - (8/x^2)$ b. $3x^2 + 2x - (8/x^2)$ c. $-3x^2 - 2x + (8/x^2)$ d. $3x^2 + (8/x^2)$

Question 4: Domain:

- a. $(-\infty, \infty)$ b. $[0, \infty)$ c. $(-\infty, 0)$ d. $(-\infty, 0) \cup (0, \infty)$

$$g(x) \neq 0$$

$$-x^2 \neq 0$$

$$x \neq 0$$

Popper 16...continued:

Example 3: Let $f(x) = x^2 - 3x - 1$ and $g(x) = -3x - 10$. Find

$$(f + g)(1) = f(1) + g(1) = (-3) + (-13) = -16$$

Question 5:

a. -16

b. 39

c. 10

d. No Solution

$$f(1) = 1^2 - 3(1) - 1 = 1 - 3 - 1 = -3$$

$$g(1) = -3(1) - 10 = -3 - 10 = -13$$

$$(gg)(-1) = g(-1) \cdot g(-1) = (-7)(-7) = 49$$

Question 6:

a. -7

b. 49

c. 0

d. No Solution

$$g(-1) = -3(-1) - 10 = 3 - 10 = -7$$

Composition of Functions

Domain example: $g(x)$ is undefined at $x=5$:
① Remove $x=5$
② Remove where $f(x)=5$

The composition of the function f with g is denoted $f \circ g$ by and is defined by the

$$(f \circ g)(x) = \underline{f(g(x))}$$

The domain of the composition $f \circ g$ is the set of all x such that

1. x is in the domain of g (the "inside" function)
2. $g(x)$ is in the domain of f (the "outside" function)

Height in inches:

$$h(x) = 5 \boxed{f(x)} + 10$$
$$\boxed{f(x) = 12}$$

$$h(x) = 5(12) + 10 = 70$$

$(f \circ g)(x)$: "f follows $g(x)$ "
($g(x)$ is handled first)

$$(f \circ g)(x) = f(g(x))$$

Remove all x values that make $g(x)$ undefined and all x -values that make $f(x)$ equal where $g(x)$ is undefined.

Example 4: Let $f(x) = x^2 + 1$ and $g(x) = -2x + 5$, find $(f \circ g)(x)$.

$$(f \circ g)(x) = f(\underline{g(x)}) = f(\underline{-2x+5}) = (-2x+5)^2 + 1$$

$$= (-2x+5)(-2x+5) + 1$$

$$= 4x^2 - 10x - 10x + 25 + 1$$

$$(f \circ g)(x) = 4x^2 - 20x + 26$$

Example 5: Let $f(x) = \frac{1}{x}$ and $g(x) = \frac{5}{\underline{x+4}}$, find $(g \circ f)(x)$

$$(g \circ f)(x) = g(\underline{f(x)}) = g\left(\frac{1}{x}\right) = \frac{5}{\frac{1}{x} + 4}$$

$$= \frac{5 \cdot x}{x \cdot \frac{1}{x} + 4 \cdot x} = \frac{5x}{1 + 4x}$$

$$(g \circ f)(x) = \frac{5x}{4x+1}$$

Example 6: Let $f(x) = \sqrt{4 - x^2}$ and $g(x) = \sqrt{3 - x}$, find $(f \circ g)(x)$.

$$(f \circ g)(x) = f(g(x)) = f(\sqrt{3-x}) = \sqrt{4 - (\sqrt{3-x})^2}$$

$$= \sqrt{4 - (3-x)} = \sqrt{4 - 3 + x}$$

$$\sqrt{1+x}$$

$$(f \circ g)(x) = \sqrt{x+1}$$

Example 7: Suppose $f(x) = 3x - 5$ and $g(x) = x^2 + 4x + 3$. Find each of the following.

a. $(f \circ g)(2)$

$$f(g(2)) = f(15) = 3(15) - 5 = 45 - 5 = \boxed{40}$$

$$g(2) = (2)^2 + 4(2) + 3 = 4 + 8 + 3 = 15 \quad \text{c. } (g \circ f)(x) \rightarrow \text{Next slide}$$

b. $(g \circ f)(-1)$

$$g(f(-1)) = g(-8) = (-8)^2 + 4(-8) + 3 = 64 - 32 + 3 = \boxed{35}$$

$$f(-1) = 3(-1) - 5 = -3 - 5 = -8$$

d. $(g \circ g)(0) = g(g(0)) = g(3)$

$$g(0) = 0^2 + 4(0) + 3 = 3$$
$$g(3) = 3^2 + 4(3) + 3 = 9 + 12 + 3 = \boxed{24}$$

Example 7: Suppose $f(x) = 3x - 5$ and $g(x) = x^2 + 4x + 3$. Find each of the following.

c. $(g \circ f)(x)$

$$\begin{aligned}g(f(x)) &= g(3x-5) = (3x-5)^2 + 4(3x-5) + 3 \\&= (3x-5)(3x-5) + 4(3x-5) + 3 \\&= 9x^2 - 15x - 15x + 25 + 12x - 20 + 3\end{aligned}$$

$$(g \circ f)(x) = 9x^2 - 18x + 8$$

Popper 16 continued:

Consider the functions: $f(x) = x^2 + 2x - 8$ and $g(x) = \sqrt{x+3}$.

7. Determine $(f \circ g)(x) = f(g(x)) = f(\sqrt{x+3}) = \sqrt{x+3}^2 + 2\sqrt{x+3} - 8$

a. $\sqrt{x+3} - 8$

b. $x^2 + 8x + 7$

c. $x + 2\sqrt{x+3} - 5$

d. $\sqrt{x^2 + 2x - 5}$

$= \underbrace{x+3} + 2\sqrt{x+3} - 8$

$= x + 2\sqrt{x+3} - 5$

8. Determine $(f \circ g)(6) = 6 + 2\sqrt{6+3} - 5 = 6 + 2\sqrt{9} - 5 = 6 + 2(3) - 5 = 6 + 6 - 5 = 7$

a. 2.36

b. 7

c. $\sqrt{43}$

d. 3

9. Determine: $(f \circ g)(-3) = -3 + 2\sqrt{-3+3} - 5 = -3 + 2\sqrt{0} - 5 = -3 + 0 - 5 = -8$

a. -8

b. $\sqrt{10}$

c. -5

d. No Answer

10. Determine $(f \circ g)(-5) = -3 + 2\sqrt{-5+3} - 5 = -3 + 2\sqrt{-2} - 5$

a. -2

b. $\sqrt{10}$

c. 0

d. No Answer

\rightarrow imaginary

Given the following table of values,

determine: $(f \circ g)(4) = f(g(4)) = f(8) = \boxed{2}$

x	f(x)	g(x)
0	4	0
2	-1	6
4	3	8
6	8	2
8	2	8

Step 1: $g(4) = 8$

Step 2: $f(8) = 2$

Step 1

Step 2