

MATH 1314

Section 3.6

Combining Functions:

Suppose we have two functions $f(x)$ and $g(x)$. The domain of $f(x)$ is the set A. The domain of $g(x)$ is the set B. We can combine these two functions together in five different ways:

Sum of Functions

Difference of Functions

Product of Functions

Quotient of Functions

Composition of Functions

Sum of Functions: $(f + g)(x) = f(x) + g(x)$ with domain $A \cap B$

Difference of Functions: $(f - g)(x) = f(x) - g(x)$ with domain $A \cap B$

Product of Functions: $(fg)(x) = f(x)g(x)$ with domain $A \cap B$

Quotient of Functions: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ with domain $\{x \in A \cap B \mid g(x) \neq 0\}$

Example 1: Suppose $f(x) = 2x - 5$ and $g(x) = x^2 - 3x + 5$. Find each of the following and state the domain:

a. $(f + g)(x)$

b. $(f - g)(x)$

Example 1: Suppose $f(x) = 2x - 5$ and $g(x) = x^2 - 3x + 5$. Find each of the following and state the domain:

c. $(fg)(x)$

d. $\left(\frac{f}{g}\right)(x)$

Example 2: Given $f(x) = 3x^4 + 2x^3 - 8$ and, $g(x) = -x^2$, find each of the following functions and its domain.

$$(g - f)(x)$$

$$\frac{f(x)}{g(x)}$$

Example 3: Let $f(x) = x^2 - 3x - 1$ and $g(x) = -3x - 10$. Find

$$(f + g)(1)$$

$$(gg)(-1)$$

Composition of Functions

The composition of the function f with g is denoted $f \circ g$ and is defined by the

$$(f \circ g)(x) = f(g(x))$$

The domain of the composition $f \circ g$ is the set of all x such that

1. x is in the domain of g (the “inside” function)
2. $g(x)$ is in the domain of f (the “outside” function)

Example 4: Let $f(x) = x^2 + 1$ and $g(x) = -2x + 5$, find $(f \circ g)(x)$.

Example 5: Let $f(x) = \frac{1}{x}$ and $g(x) = \frac{5}{x+4}$, find $(g \circ f)(x)$

Example 6: Let $f(x) = \sqrt{4 - x^2}$ and $g(x) = \sqrt{3 - x}$, find $(f \circ g)(x)$.

Example 7: Suppose $f(x) = 3x - 5$ and $g(x) = x^2 + 4x + 3$. Find each of the following.

a. $(f \circ g)(2)$

b. $(g \circ f)(-1)$

c. $(g \circ f)(x)$

d. $(g \circ g)(0)$

Consider the functions: $f(x) = x^2 + 2x - 8$ and $g(x) = \sqrt{x + 3}$.

Determine $(f \circ g)(x)$

Determine $(f \circ g)(6)$

Determine: $(f \circ g)(-3)$

Determine $(f \circ g)(-5)$

Given the following table of values,
determine: $(f \circ g)(4)$.

x	f(x)	g(x)
0	4	0
2	-1	6
4	3	8
6	8	2
8	2	8