MATH 1314

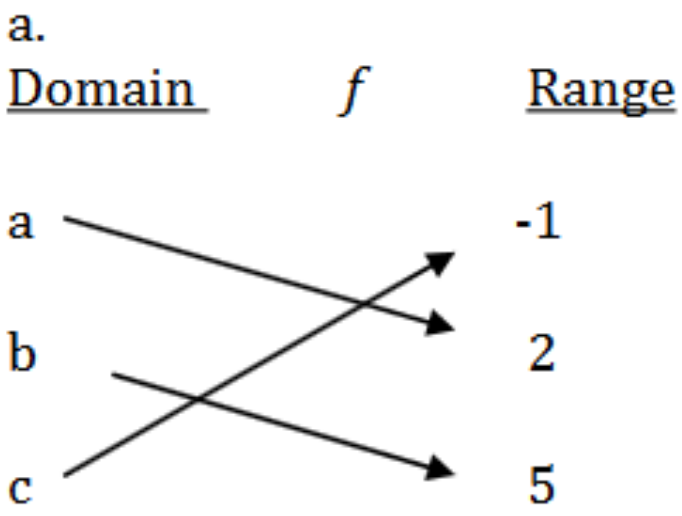
Section 3.7

Inverse Functions

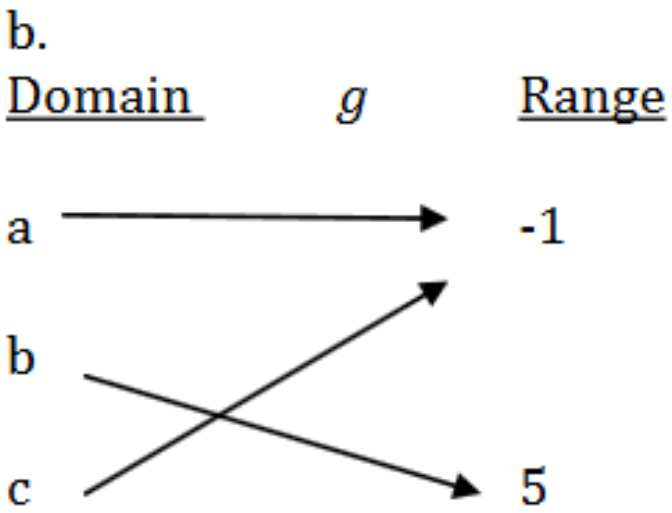
Let f be a function with domain A. f is said to be **one-to-one** if no two elements in A have the same image.

Example 1: Determine if the following function is one-to-one.

a.



b.



A one-to-one function has an inverse. The inverse function reverses whatever the first function did. These two statements mean exactly the same thing:

1. f is one-to-one (1-1)
2. f has an inverse function

The inverse of a function f is denoted by f^{-1} , read "f-inverse".

Note: $f^{-1}(x) \neq \frac{1}{f(x)}$ like $x^{-3} = \frac{1}{x^3}$

Domain and Range

Suppose f is a one-to-one function with domain A and range B. The inverse function has domain B and range A.

Example 1: Suppose f and g are inverse functions. If $f(3) = -1$ and $f(-1) = 4$, then find $g(-1)$.

Property of Inverse Functions

Let f and g be two functions such that $(f \circ g)(x) = x$ for every x in the domain of g and $(g \circ f)(x) = x$ for every x in the domain of f then **f and g are inverses of each other.**

Example 2: Show that the following functions are inverses of each other.

$f(x) = 3x + 7$ and $g(x) = \frac{x}{3} - \frac{7}{3}$

Example 3: Determine whether the following pair of functions are inverses of each other.

$f(x) = 2x - 1$ and $g(x) = \frac{x}{2} + 1$

How to find the equation of the inverse function of a one-to-one function:

1. Replace $f(x)$ by y.
2. Exchange x and y.
3. Solve for y.
4. Replace y by $f^{-1}(x)$
5. Verify.

Example 4: Write the equation of the inverse function for $f\ (x)\ =\ 3x\ -\ 3$

Example 5: Write the equation of the inverse for $f(x) = \frac{6}{4-x}$

Determine the inverse of the following: $f(x) = \frac{2x-3}{x+5}$

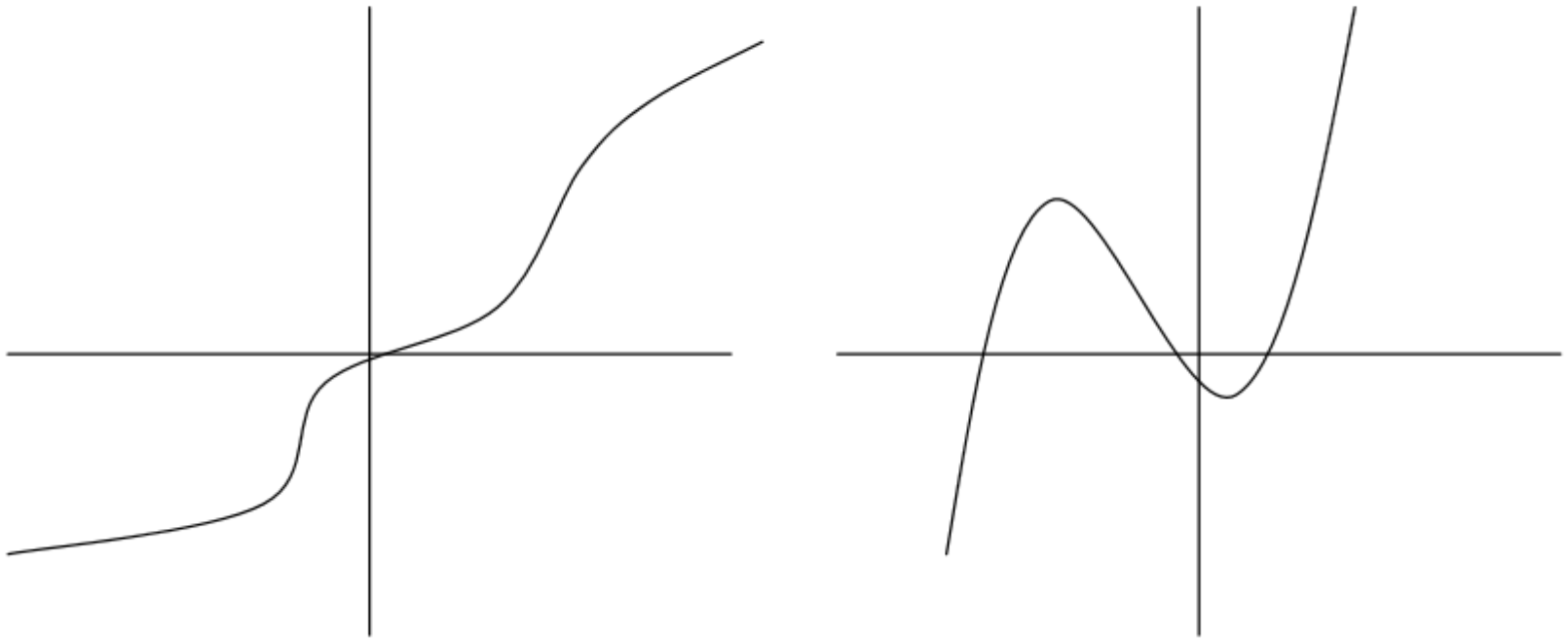
Determine the inverse of $f(x) = x^2 - 3$ for $x \leq 0$

Example 6: Write the equation of the inverse for $f(x) = (x+1)^3 + 1$

Example 7: Write the equation of the inverse for $f(x) = \sqrt[3]{x+4}$

It is easiest to determine if a function is one-to-one by looking at its graph. We can use the Horizontal Line Test to determine if a function is one-to-one.

Horizontal Line Test: A function is one-to-one if no horizontal line intersects its graph in more than one point.

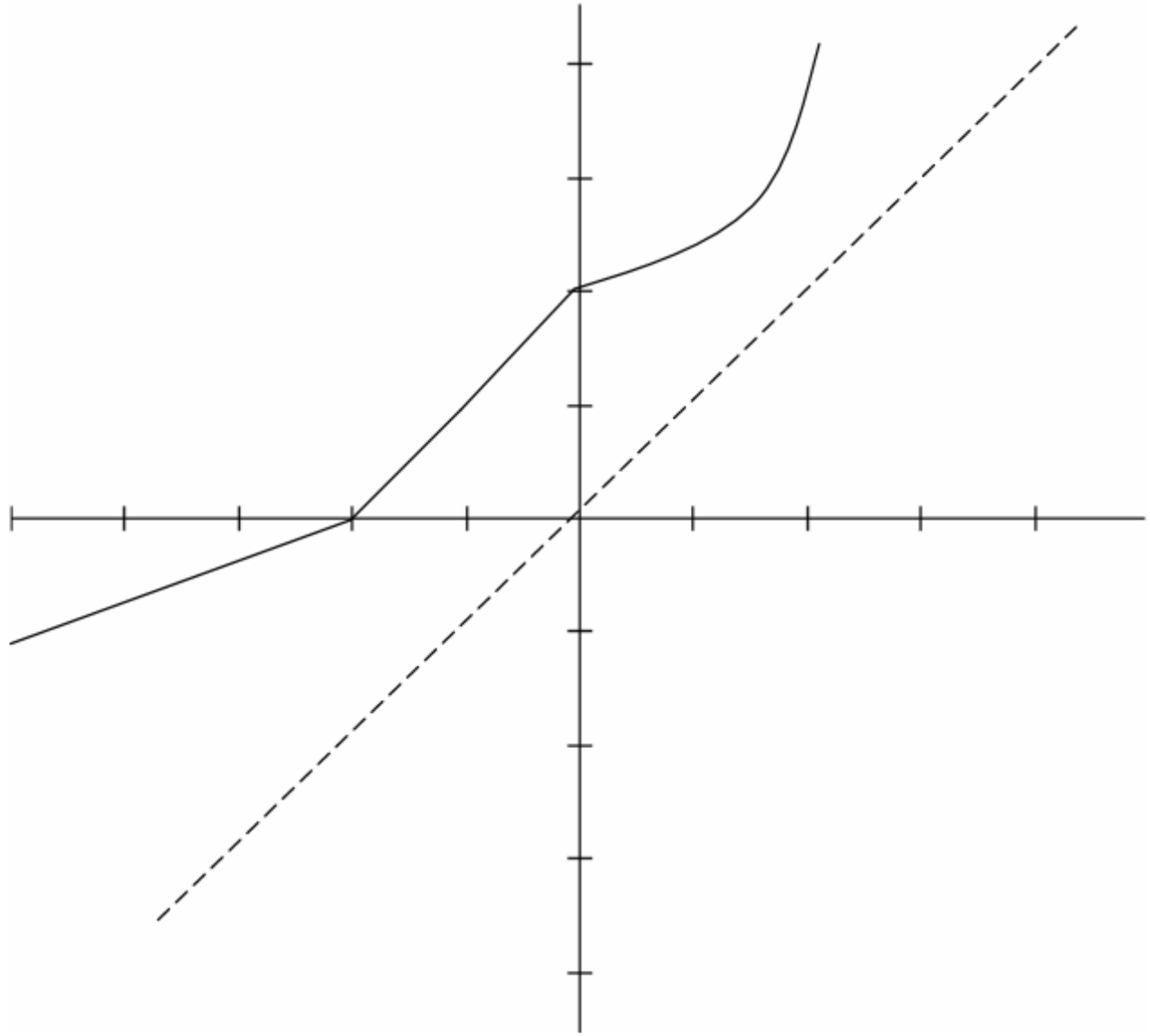


Graphing the Inverse Function

Given that f is 1-1, the graph of f^{-1} is a reflection of the graph of f about the line $y = x$

Remember:

1. The inverse function reverses whatever the first function did.
2. The Domain of f becomes the Range of and the Range of f becomes the Domain of f^{-1} .



Are the following pairs of graphs (on the same axis) inverses to one another?