MATH 1314

Section 4.1

Polynomial Functions:

String of terms that are added or subtracted.

Each term: $2x^3$ ← positive, whole numbers.
(Example)
coefficient

A polynomial function is a function of the form

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^n + \ldots + a_1 x^1 + a_n$$

where $a_n \neq 0$, $a_0, a_1, \ldots, a_n$ are real numbers and n is a whole number.

The degree of the polynomial function is n. We call the term $a_n x^n$ the leading term, and a_n is called the leading coefficient.

Degree: Largest Exponent

$$\boxed{P(0) = a_0}$$

Leading Term: Term with largest exponent

Constant term: Term without the x

Our objectives in working with polynomial functions will be, first, to gather information about the graph of the function and, second, to use that information to generate a reasonably good graph without plotting a lot of points. In later examples, we'll use information given to us about the graph of a function to write its equation.

$$P(x) = \underbrace{-3x^4} + 2x^3 - 5x \underbrace{+2}$$

Leading Term: $-3x^4$
(Degree: 4)

Constant Term: 2

Factored Form:

$p(x) = 2(x - 4)^2(x^2 + 1)^3$

Leading Term: "First" Step of FOIL

$2(x)^2(x^2)^3$ [Each Largest Power of x and the outside exponent]

$2(x^2)(x^6)$

$\underbrace{2x^8}$ Degree: 8

Constant term: → y-intercept

$p(0) = 2(0-4)^2(0^2+1)^3$

$= 2(-4)^2(1)^3$

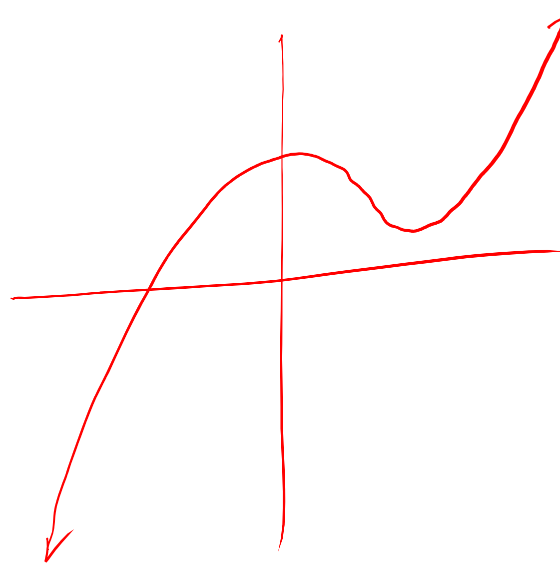
$= 2(16)(1) = \underbrace{32}$ (0, 32)

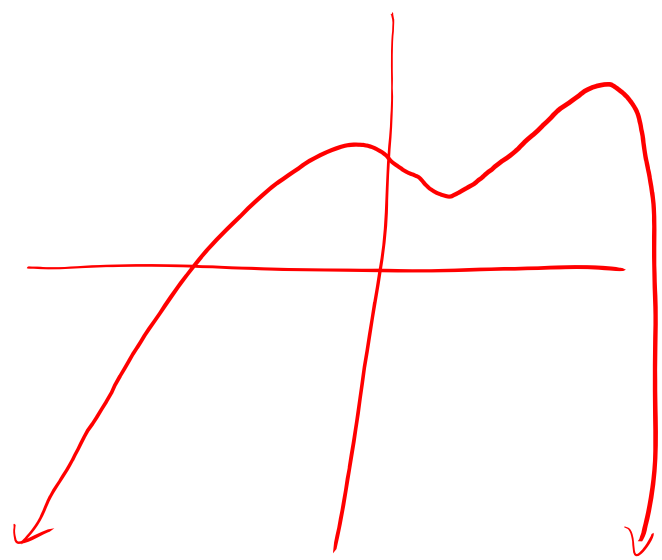
Graph Properties of Polynomial Functions

Let P be any nth degree polynomial function with real coefficients. The graph of P has the following properties.

1. P is continuous for all real numbers, so there are no breaks, holes, jumps in the graph. $D: (-\infty, \infty)$
2. The graph of P is a smooth curve with rounded corners and no sharp corners.
3. The graph of P has at most n x-intercepts.
4. The graph of P has at most $n - 1$ turning points.

degree of n





Example 1: Given the following polynomial functions, state the leading term, the degree of the polynomial and the leading coefficient, constant term

a. $P(x) = 6x^4 - 4x^3 + 7x - 2$

Leading Term: $6x^4$ Constant term: -2

Degree: 4

Leading Coefficient: 6

b. $P(x) = (3x + 4)(x + 1)^2(x - 5)^3$

Leading Term: $(3x)(x)^2(x)^3 = 3x \cdot x^2 \cdot x^3 = 3x^6$

Degree: 6

Leading Coefficient: 3

Constant Term: $P(0) = (3 \cdot 0 + 4)(0+1)^2(0-5)^3 = 4(1)^2(-5)^3 = 4(1)(-125)$

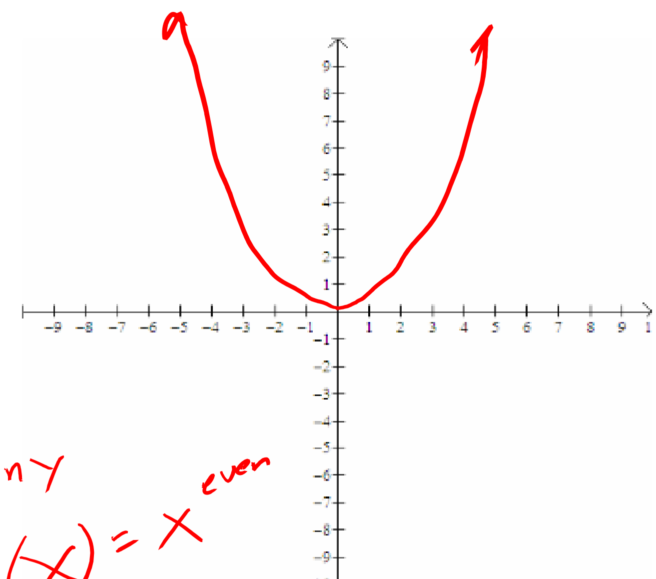
-500

We'll start with the shapes of the graphs of functions of the form $f(x) = x^n, n > 0$.

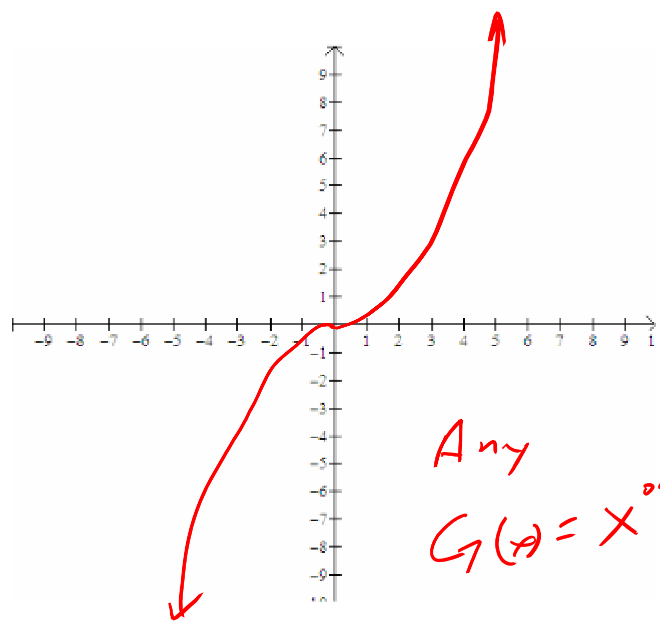
You should be familiar with the graphs of $f(x) = x^2$ and $g(x) = x^3$.

The graph of $f(x) = x^n, n > 0.$, n is even, will resemble the graph of $f(x) = x^2$, and the graph of $f(x) = x^n, n > 0$, n is odd, will resemble the graph of $f(x) = x^3$.

$f(x) = x^2$



$g(x) = x^3$



Any $F(x) = X^{\text{even}}$

Any $G(x) = X^{\text{odd}(>1)}$

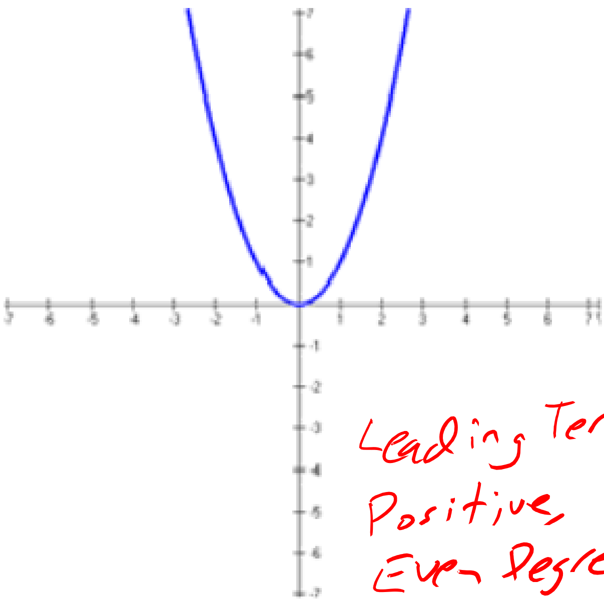
Next, you will need to be able to describe the end behavior of a function.

End Behavior of Polynomial Functions

The behavior of a graph of a function to the far left or far right is called its **end behavior.** The end behavior of a polynomial function is revealed by the leading term of the polynomial function.

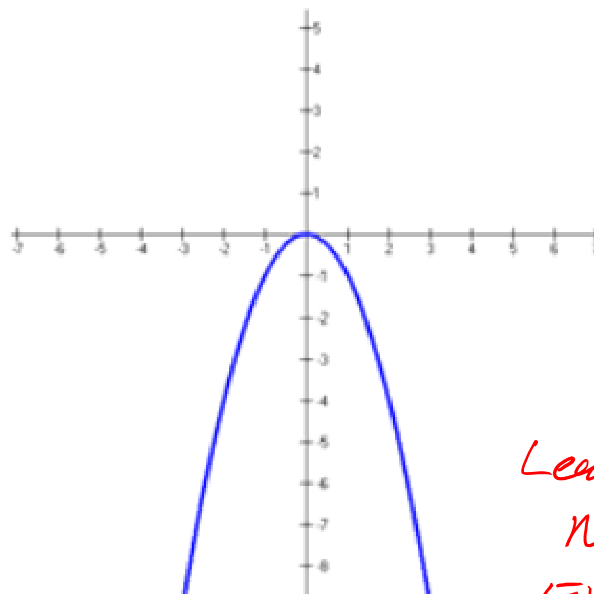
End Behavior: what is happening at the far left and far right of the graph.

1. Even-degree polynomials look like . $y = \pm x^2$



Leading Term: Positive, Even Degree

Left: Rising
Right: Rising

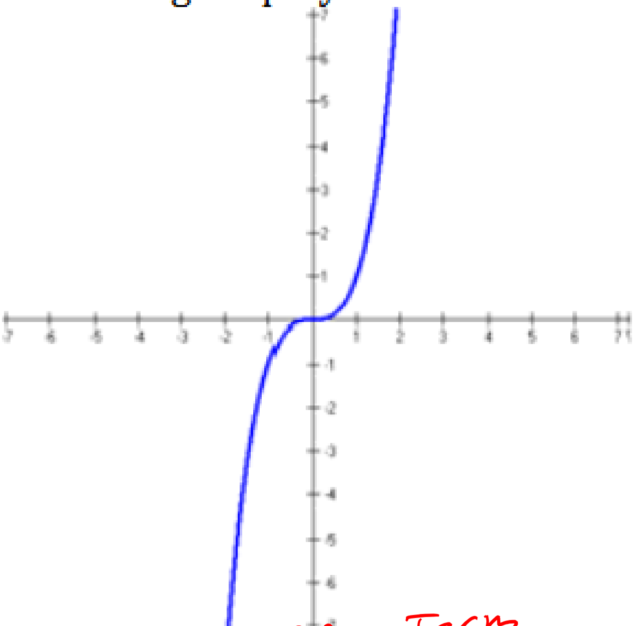


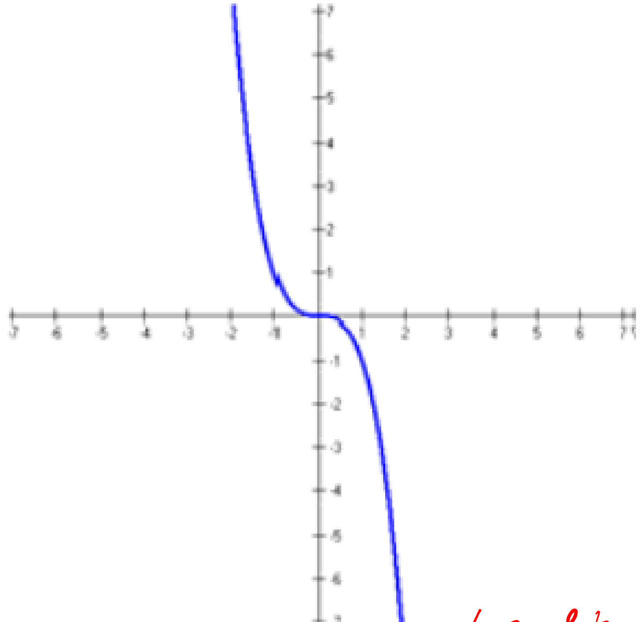
For Polynomials: Rising or Falling

Leading Term: Negative Even Degree

Left: Falling
Right: Falling

2. Odd-degree polynomials look like. $y = \pm x^3$



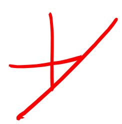


Leading Term
positive, odd exponent

Left: Falling

Right: Rising

Think about a line with positive slope

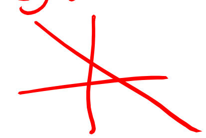


Leading Term:
Negative odd exponent

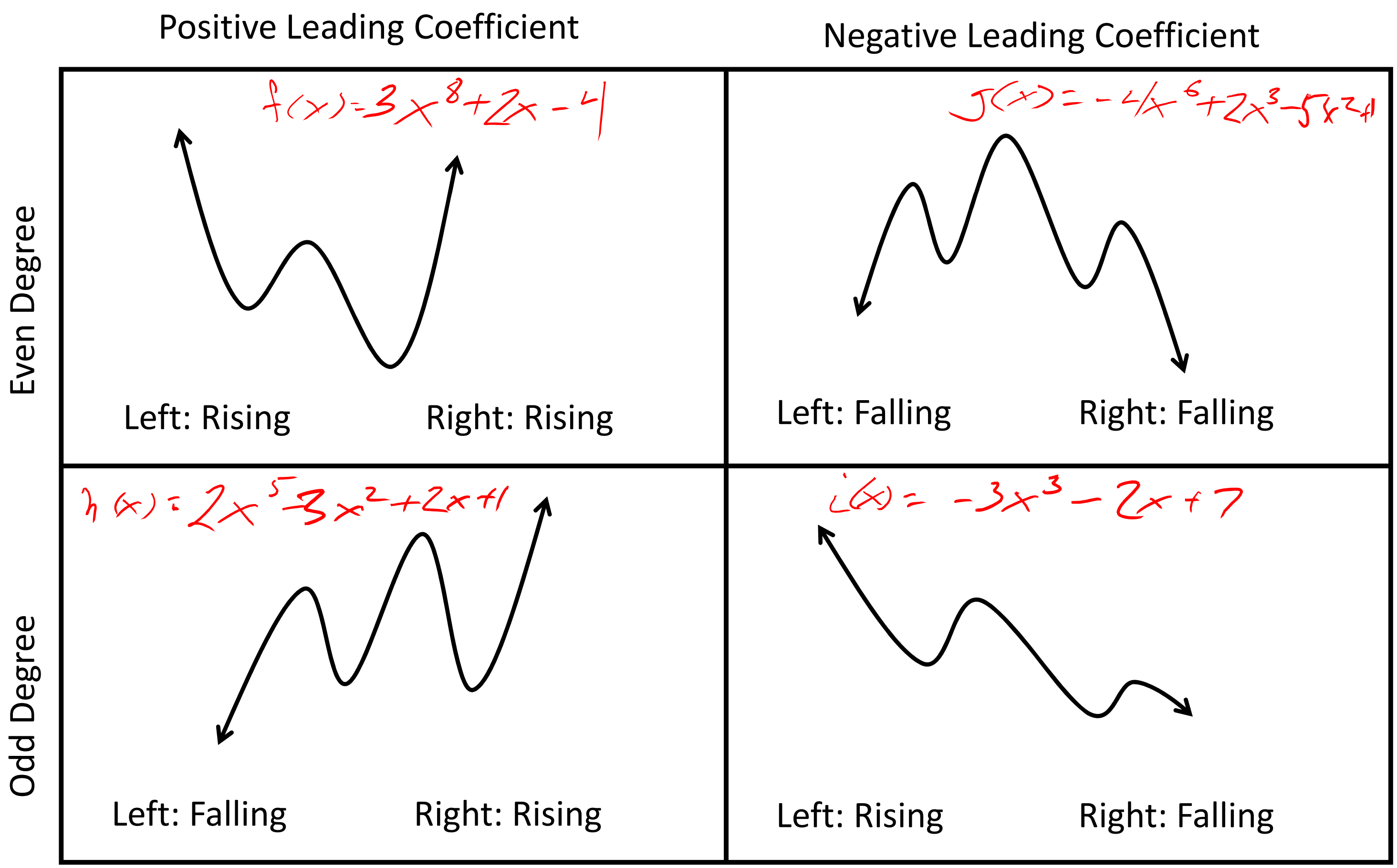
Left: Rising

Right: Falling

Think of a line with a negative slope



End behavior of Polynomials:



Popper 17

For the following, refer to $f(x) = -8x^3 - 2x^2 + 9x - 3$

1. What is the degree of the polynomial?

a. First   b. Sixth   c. Third   d. Eighth

2. What is the leading coefficient?

a. -8   b. -2   c. 9   d. -3

3. What is the end behavior on the left?

a. Rising   b. Falling

4. What is the end behavior on the right?

a. Rising   b. Falling

Leading Term: $-8x^3$

Leading Coefficient: -8

Degree: 3

Left: Rising

Right: Falling

Describe the End Behavior of the following:

$g(x) = (x^2 + 3)(x - 2)^4$

Leading Term: $(x^2)(x)^4 = x^2 \cdot x^4 = x^6$

Leading Coefficient: 1

Degree: 6

positive even

Left: Rising

Right: Rising

Next, you should be able to find the *x* intercept(s) and the *y* intercept of a polynomial function.

Zeros of Polynomial Functions (x- intercepts or Roots)

You will need to set the function equal to zero and then use the Zero Product Property to find the *x*-intercept(s). That means if $ab = 0$, then either $a = 0$ or $b = 0$. To find the *y* intercept of a function, you will find $f(0)$.

Example 2: Find the zeros of:

a. $f(x) = x^4 - x^2$

3 x-intercepts: $-1, 0, 1$

Y-int: $f(0) = 0^4 - 0^2 = 0$

$x^2(x^2-1)$

$x^2(x+1)(x-1)$ [Product of Linear Factors]

$x=0$ $\quad x+1=0$ $\quad x-1=0$ [Set insides equal to zero, ignore any outside exponents]

$x=-1$ $\quad x=+1$

b. $f(x) = -3x\left(x + \frac{1}{2}\right)(x - 4)^3$ [Factored Form]

$-3x=0$ $\quad x+\frac{1}{2}=0$ $\quad x-4=0$

$x=0$ $\quad x=-\frac{1}{2}$ $\quad x=4$

3 x-intercepts: $-\frac{1}{2}, 0, 4$

Y-intercept:

$f(0) = -3(0)(0+\frac{1}{2})(0-4)^3$

$= -3(0)(\frac{1}{2})(-4)^3 = 0$

In some problems, one or more of the factors will appear more than once when the function is factored. The power of a factor is called its multiplicity. So given $\boxed{P(x) = x^3(x-3)^2(x+2)^1}$, then the multiplicity of the first factor is 3, the multiplicity of the second factor is and the multiplicity of the third factor is 1.

$P(0) = 0^3(0-3)^2(0+2)^1 = 0$

$x = 0$ M: 3 | $x - 3 = 0$, $x = 3$ M: 2 | $x + 2 = 0$, $x = -2$ M: 1

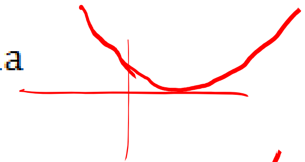
Degree = sum of multiplicities

Leading Term: x^6

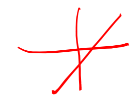
Description of the Behavior at Each *x*-intercept

1. Even Multiplicity: The graph touches the *x*-axis, but does not cross it. It looks like a parabola there.

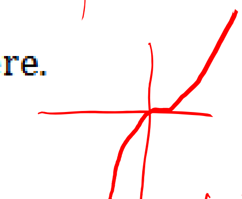
Tangent to the x-axis



2. Multiplicity of 1: The graph crosses the *x*-axis. It looks like a line there.



3. Odd Multiplicity greater than or equal 3: The graph crosses the *x*-axis. It looks like a cubic there.



You can use all of this information to generate the graph of a polynomial function.

- degree of the function
- end behavior of the function
- *x* and *y* intercepts (and multiplicities)
- behavior of the function through each of the *x* intercepts (zeros) of the function

End Behavior: L: ↑ R: ↑
Start End

(the best we can draw is a rough sketch of a polynomial)