MATH 1314

Section 4.2

Dividing Polynomials

In this section, you'll learn two methods for dividing polynomials, long division and synthetic division. You'll also learn two theorems that will allow you to interpret results when you divide.

Suppose $P(x)$ and $D(x)$ are polynomial functions and $D(x) \neq 0$. Then there are unique polynomials $Q(x)$ (called the quotient) and $R(x)$ (called the remainder) such that $P(x) = D(x) * Q(x) + R(x)$.

We call $D(x)$ the divisor. The remainder function, $R(x)$, is either 0 or of degree less than the degree of the divisor.

You can find the quotient and remainder using long division. Recall the steps you learned in elementary school to perform long division:

Example 1: Divide

$$\frac{x^6 + 4x^4 + 4x^2 + 16}{x^2 + 4}$$

Example 2: Divide

$$\frac{12x^3 - x^2 - x}{3x - 1}$$

Example 3: If $D(x) = 2x - 5, Q(x) = 3x^2 + 5$ and $R(x) = 12$, find $P(x)$.

Often it will be more convenient to use synthetic division to divide polynomials. This method is easy to use, as long as your divisor is $x \pm c$, for any real number c.

Dividing Polynomials Using Synthetic Division

Example 4: Divide using synthetic division

$$\frac{6x^4 + x^3 - 10x^2 + 9}{x - 3}$$

Example 5: Divide using synthetic division

$$\frac{x^3 + 5x^2 - 7x + 2}{x - 1}$$

Example 6: Divide using synthetic division

$$\frac{x^3 + 8}{x + 2}$$

Here are two theorems that can be helpful when working with polynomials:

The Remainder Theorem: If $P(x)$ is divided by $x - c$, then the remainder is $P(c)$.

The Factor Theorem: c is a zero of $P(x)$ if and only if $x - c$ is a factor of $P(x)$, that is if the remainder when dividing by $x - c$ is zero.

You can use synthetic division and the remainder theorem to evaluate a function at a given value.

Example 7: Use synthetic division and the remainder theorem to find P(3) for $P(x) = 2x^3 - 5x^2 + 4x + 3$

Example 8: Determine if $x + 2$ is a factor of $P(x) = x^3 + 6x^2 + 3x - 10$.

Use synthetic division to determine p(2) if

$p(x) = 5x^6 - 2x^5 + 7x^4 - 8x^3 + 2x - 3$

And you may also need to work backwards.

Example 9: Find a polynomial with a degree of 4 with zeros at -3, 0, 2, 5.

Example 10: Find a polynomial of degree 3 with zeros at 0, 2 and -3.

If the polynomial $f(x) = x^3 + 3x^2 – 13x – 15$ has one root located at (-5, 0), determine all other roots.

A polynomial has roots of -1, 3, and 5, and a y-intercept at (0,-30). Determine its equation.