MATH 1314

Section 4.3

Roots of Polynomials

Repeated Roots: Multiplicities

Complex Roots: Based on imaginary numbers

You'll need to be able to find all of the zeros of a polynomial. You'll now be expected to find both real and complex zeros of a function.

Degree is the sum of all real and complex roots

A polynomial of degree n has exactly n zeros, counting all multiplicities.

To find all zeros, you'll factor completely. From the factored form of your polynomial, you'll be able to read off all the zeros of the function.

If c is a zero of a polynomial P, then $x = c$ is a **root** of the equation $P(x) = 0$.

If your polynomial has real coefficients, then the polynomial may have complex roots. Complex roots occur in pairs, called complex conjugate pairs. This means that if $a + bi$ is a root of P then so is $a - b$.

$x^2 - 16 = (x+4)(x-4)$

Note: $a^2 + b^2 = (a + bi)(a - bi)$

$x^2 + 16 = (x+4i)(x-4i)$ → complex factors

Complex roots come in conjugate pairs.

So if $x = 2 - 3i$ is a root, we know $x = 2 + 3i$ is also a root

Example 1: Find the zeros of the polynomial write the polynomial in factored form and then state the multiplicity of each zero. (Sometimes it may be easier to factor the polynomial first, then find the zeros.)

a. $f(x) = x^2 - 6x + 9 = 0$

$(x-3)(x-3) = 0$

$(x-3)^2 = 0$

$x - 3 = 0$

$x = 3$ M2

$(x-3)^2$ is a product of linear terms

$\{3\}$

b. $f(x) = x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$x+4=0$ $\quad$ $x-3=0$

$x = -4$ $\quad$ $x = 3$

M:1 $\quad$ M:1

$(x+4)(x-3)$ is a product of linear terms

$\{-4, 3\}$

Popper 23:

Find all the roots of the following:

Question 1:

$f(x) = 9x^2 + 36$

product of linear terms

$9x^2+36=0$

$9x^2=-36$

$\sqrt{x^2}=\pm\sqrt{-4}$

$x=\pm 2i$

$9x^2+36=0$

$(3x+6i)(3x-6i)=0$

$3x+6i=0$ $\quad$ $3x-6i=0$

$3x=-6i$ $\quad$ $3x=6i$

$x=-2i$ $\quad$ $x=2i$

a. 3, 6 b. 3, 6, 3i, 6i c. 2i, -2i d. 2, -2

Question 2:

$f(x) = (x^3 - 4x^2) + (x - 4)$

$f(x)=x^2(x-4)+1(x-4)$

a. 4 b. 4, i, -i c. -1, 1, 4 d. -4i, 1, 4i

$f(x)=(x-4)(x^2+1)$

$x-4=0$ $\quad$ $x^2+1=0$

$x=4$ $\quad$ $x^2=-1$

$x=\pm\sqrt{-1}=\pm i$

You can also work backwards to writing a polynomial with integer coefficients that meets stated conditions.

Example 2: Find a 3rd degree polynomial with integer coefficients given -5, and *i* are zeros

also need $-i$

$x=-5 \quad x=i \quad x=-i$

$P(x) = (x+5)\underbrace{(x-i)(x+i)}$

FOIL conjugates first

$P(x) = (x+5)(x^2+\cancel{ix}-\cancel{ix}-i^2)$

Remember: $i^2=-1$

$-(-1)=+1$

$P(x) = (x+5)(x^2+1)$

$P(x) = x^3+x+5x^2+5$

$$\boxed{P(x) = x^3+5x^2+x+5}$$

Example 3: Find a polynomial with integer coefficients given the zeros at 2 and 2 - 5*i*.

$x=2 \quad x=2-5i \quad x=2+5i \qquad 2+5i$

$P(x)=(x-2)(x-(2-5i))(x-(2+5i))$

$P(x)=(x-2)(x^2-(2+5i)x-(2-5i)x+(2-5i)(2+5i))$

$P(x)=(x-2)(x^2-2x\cancel{-5ix}-2x\cancel{+5ix}+4\cancel{+10i}\cancel{-10i}\underbrace{-25i^2}_{+25})$

$P(x)=(x-2)(x^2-4x+29)$

$P(x)=x^3-4x^2+29x-2x^2+8x-58$

$$\boxed{P(x)=x^3-6x^2+37x-58}$$

Popper 23…Question 3:

Example 4: Write a polynomial with integer coefficients with degree 4 and zeros at -3 (multiplicity 2) and $-3i$.

$+3i$

$x=-3$ $x=-3i$ $x=3i$

$P(x)=(x+3)^2(x+3i)(x-3i)$

$P(x)=(x+3)(x+3)(x+3i)(x-3i)$

$P(x)=(x^2+3x+3x+9)(x^2 \cancel{-3ix} \cancel{+3ix} - 9i^2)$

$+9$

$P(x)=(x^2+6x+9)(x^2+9)$

a. $p(x) = (x^2 + 6x + 9)(x^2 + 9)$

b. $p(x) = (x^2 - 6x + 9)(x^2 + 9)$

c. $p(x) = (x^2 + 6x + 9)(x^2 - 9)$

d. $p(x) = (x + 3)(x^2 + 9)$

Example 5: Write a polynomial with integer coefficients with degree 3 and zeros at 5 and $4+i$ with a constant coefficient of 170.

$4-i$

$p(x)=a(x-5)(x-(4+i))(x-(4-i))$

$p(x)=a(x-5)(x^2-(4-i)x-(4+i)x+(4+i)(4-i))$

$p(x)=a(x-5)(x^2-4x+\cancel{ix}-4x-\cancel{ix}+16-\cancel{4i}+\cancel{4i}\underbrace{-i^2}_{+1})$

$p(x)=a(x-5)(x^2-16x+17)$

$p(x)=a(x^3-16x^2+17x-5x^2+80x-85)$

$p(x)=a(x^3-21x^2+97x-85)$

$-85 \times \square = 170$

$a=-2$

$$\boxed{p(x)=-2x^3+42x^2-194x+170}$$

The polynomial $f(x) = x^3 + 3x^2 - 13x - 15$ has a root at $x = 3$. Determine the value of all its roots.

$x - 3$

opp

3	1	3	−13	−15
	↓	3	18	15
	1	6	5	0

$\rightarrow R(x) = 0$

$Q(x) = x^2 + 6x + 5 = 0$

$(x + 5)(x + 1) = 0$

$x + 5 = 0 \qquad x + 1 = 0$

$x = -5 \qquad x = -1$

All Roots:

$\{-5, -1, 3\}$