MATH 1314

Section 5.1

The Exponential Function

x is in the exponent

$a \neq 0, a \neq 1$

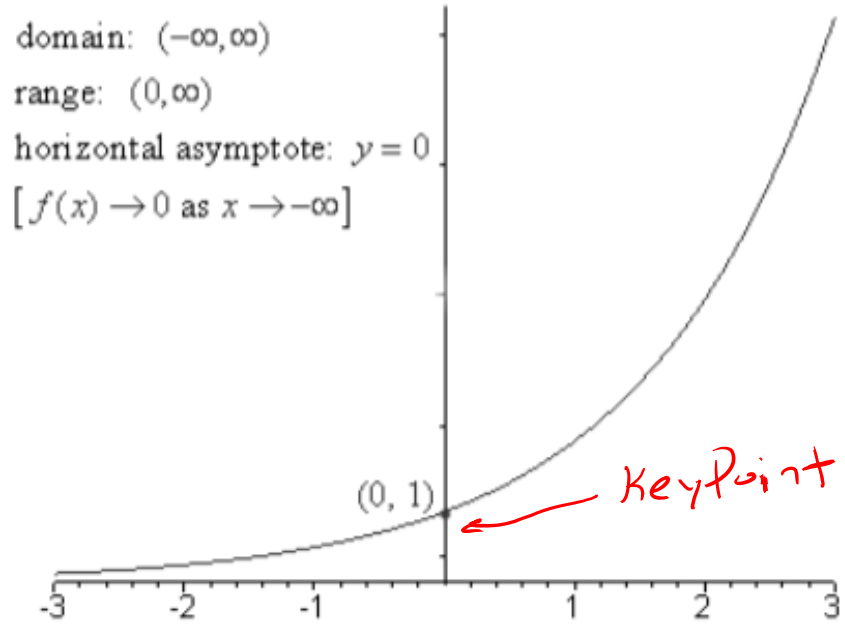
$a > 0$ [positive]

An exponential function is defined as a function of the form:

$f(x) = 2^x$ $\qquad$ $f(x) = a^x$

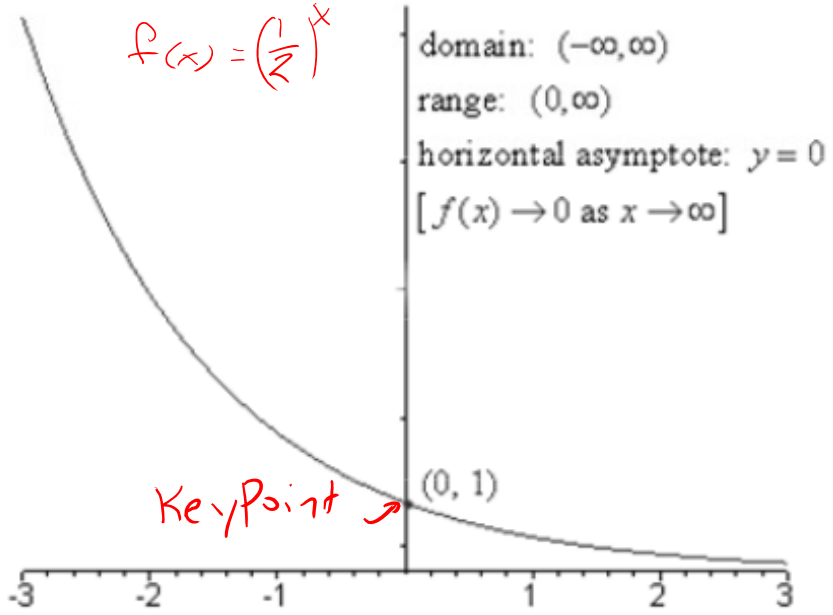
where a > 0. This is considered an exponential function with base a.

The graph of $f(x) = a^x$ for $a > 1$ has the following shape:



The graph of $f(x) = a^x$ for $0 < a < 1$ has the following shape:

$f(x) = \left(\frac{1}{2}\right)^x$



Example Problem 1: $(-2)^x \neq -2^x$

Sketch the graph of the function $f(x) = -2^x$ by starting from the graph of $y = 2^x$.

State the domain, range, and asymptote.

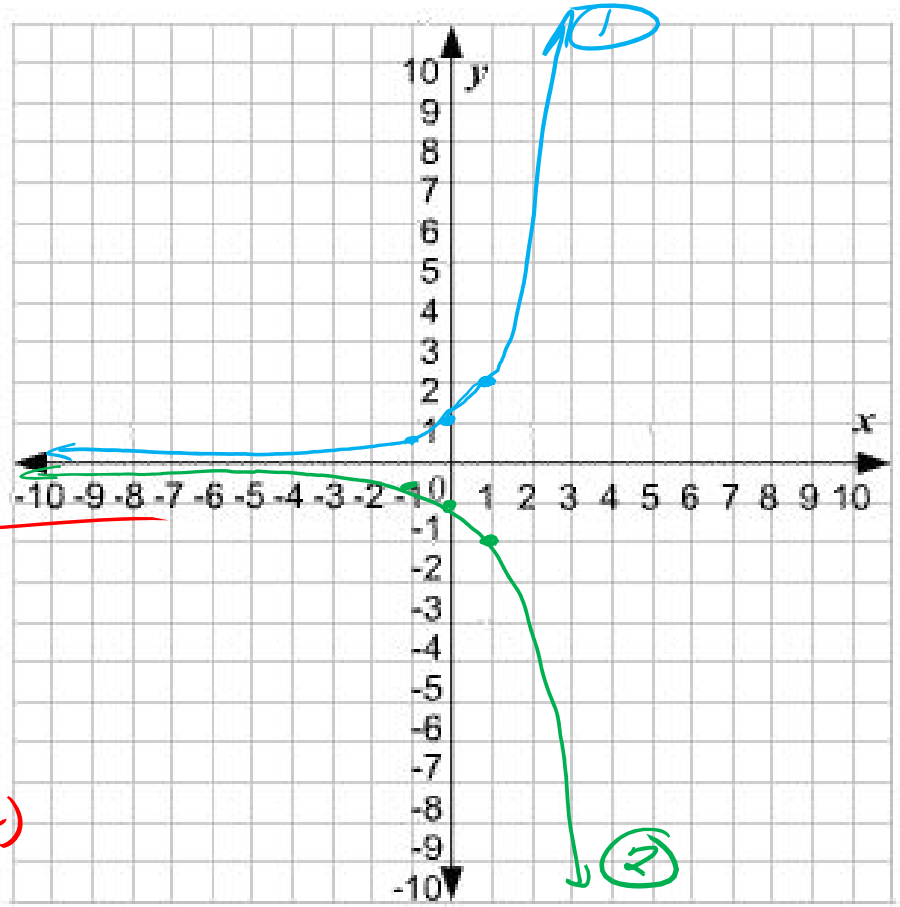
$y = 2^x$ $(0, 1) \rightarrow (0, 1)$

$(1, 2) \rightarrow (1, a)$

$(-1, \frac{1}{2}) \rightarrow (-1, \frac{1}{a})$

① Parent Function

$y = -2^x$ x-axis reflection



Domain: $(-\infty, \infty)$

Range: $(-\infty, 0)$

Asymptote: $y = 0$ (HA)

$f(x) = -2^x + 0$ (HA)

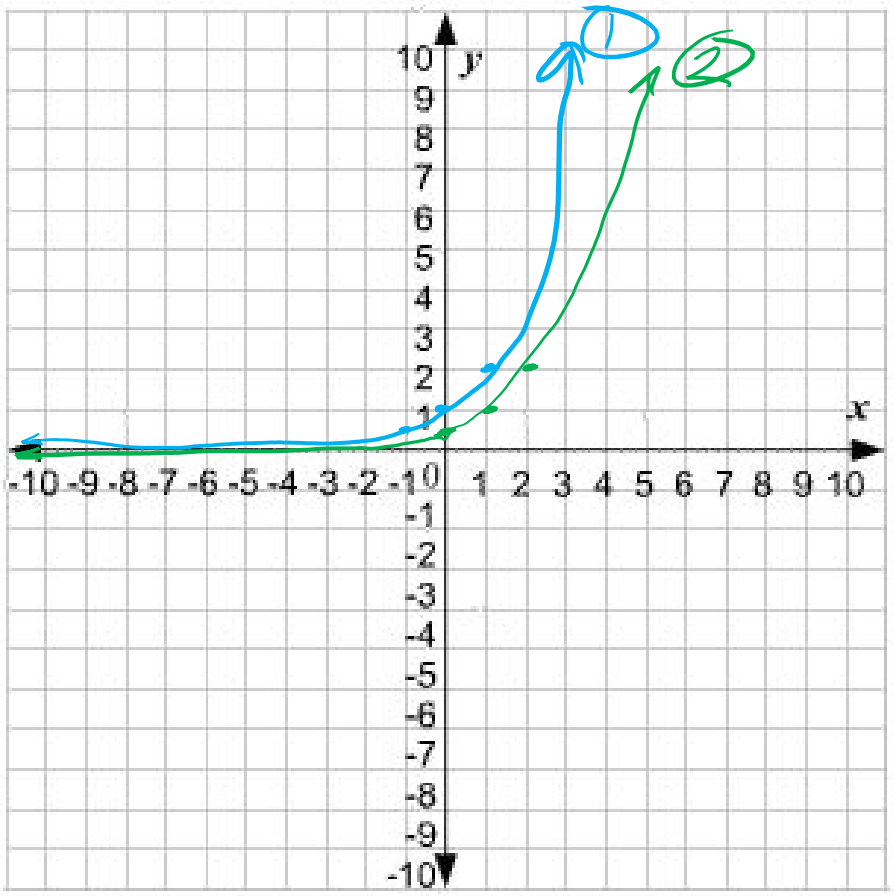
+/−

Range: $(-\infty, HA)$ (neg) (HA, ∞) (pos)

Example Problem 2:

Sketch the graph of the function $f(x) = 2^{x-1}$ by starting from the graph of $y = 2^x$.

State the domain, range, and asymptote.



① Parent Function $y = 2^x$

② $y = 2^{x-1}$ Right 1

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Asymptote: $y = 0$

$f(x) = 2^{x-1} + 0$ (HA)

↑ Positive

(HA, 0)

Example 3:

Sketch the graph of the function $f(x) = -4^x + 10$. Do not plot points, but instead apply transformations to the graph of the function $y = 4^x$. Identify the domain, range, and asymptote.

① Parent Function $Y = 4^x$

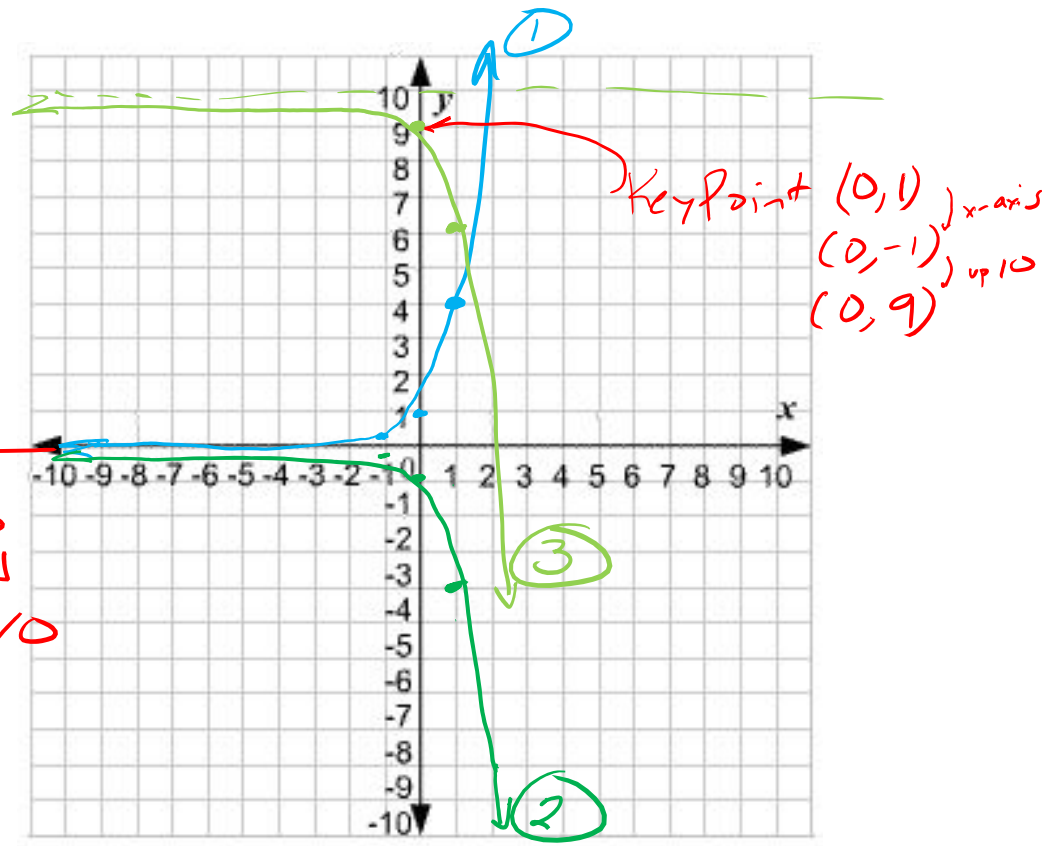
② x-axis reflection $Y = -4^x$

③ up 10 $f(x) = -4^x + 10$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 10)$

Asymptote: $Y = 10$



$f(x) = -4^x + 10$

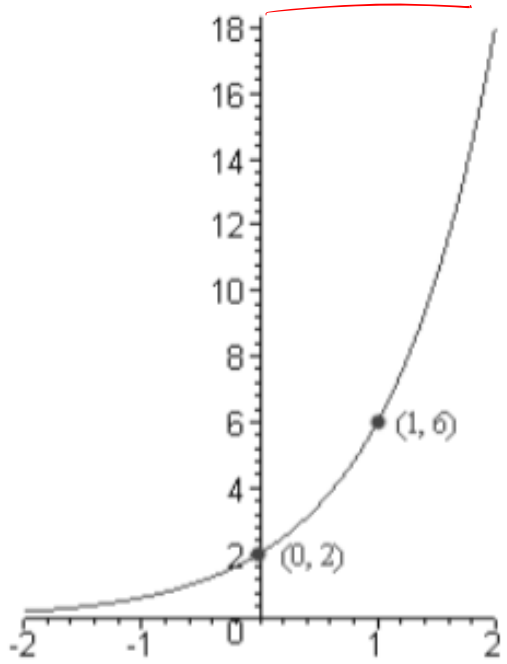
HA: $Y = 10$

Negative: Below

$(-\infty, 10)$

Example 4:

Find the function $f(x) = Ca^x$ whose graph is shown below.



$f(x) = C \cdot a^x$

Plug in (0, 2)

$2 = C \cdot a^0$ (Anything)$^0 = 1$

$2 = C \cdot 1$

$2 = C$

$f(x) = 2 \cdot a^x$

Plug in (1, 6)

$6 = 2 \cdot a^1$

$6 = 2 \cdot a$

$3 = a$

$f(x) = 2 \cdot 3^x$

Note: Base must be positive not 0, not 1

$f(x) = c\,a^x$ (1,6) and (2, 12)

$f(x) = c \cdot a^x$

plug in (1, 6)

$6 = c \cdot a^1$

$6 = c \cdot a$

$\frac{6}{c} = a$

$f(x) = c \cdot \left(\frac{6}{c}\right)^x$

$12 = c \cdot \left(\frac{6}{c}\right)^2$

$12 = c \cdot \frac{36}{c^2}$

$\frac{12}{1} = \frac{36}{c}$

$12c = 36$

$c = 3$

$a = \frac{6}{c} = \frac{6}{3} = 2$

$a = 2$

$f(x) = 3 \cdot 2^x$

Keep in Mind: $3 \cdot 2^x \neq 6^x$

$f(x) = 3(2^x)$

Popper 21:

Consider the function: $f(x) = 4\left(\frac{2}{3}\right)^{x-1} + 5$

HA

1. What is this function called?

a. Quadratic b. Rational c. Exponential d. Polynomial

2. Is this function increasing or decreasing? Base: $a = \frac{2}{3} \rightarrow 0 < a < 1$

a. Increasing b. Decreasing c. Cannot be determined

3. What is the horizontal asymptote of the function?

a. y = 5 b. y = 4 c. y = 2/3 d. None

Popper 21…continued:

$f(0) = 4(2/3)^{0-1} + 5$

$4(2/3)^{-1} + 5$

$4(3/2) + 5$

$\frac{12}{2} + 5$

$6 + 5 = 11$

$(0, 11)$

Consider the function: $f(x) = 4\,(^2/_3)_{x-1} + 5$

4. What is the domain of the function?

a. $(-\infty, \infty)$ b. $(5, \infty)$ c. $[5, \infty)$ d. $(4, \infty)$

5. What is the range of the function? HA: 5 positive exp term: Above 5

a. $(-\infty, \infty)$ b. $(5, \infty)$ c. $[5, \infty)$ d. $(4, \infty)$

6. Which of the following points does the function pass through?

a. (0, 1) b. (0, 5) c. (0, 4) d. (0, 11)

7. Which transformations are present?

a. Horizontal Shift b. Vertical Shift c. Vertical Stretch d. All of above

Right 1 up 5 4×

Sketch the graph: $f(x) = 4\left(\frac{2}{3}\right)^{x-1} + 5$