MATH 1314

Section 5.2

The number "e."

More on transformations of the exponential function $f(x) = a^x$, but with $a = e$ *(the natural base)*.

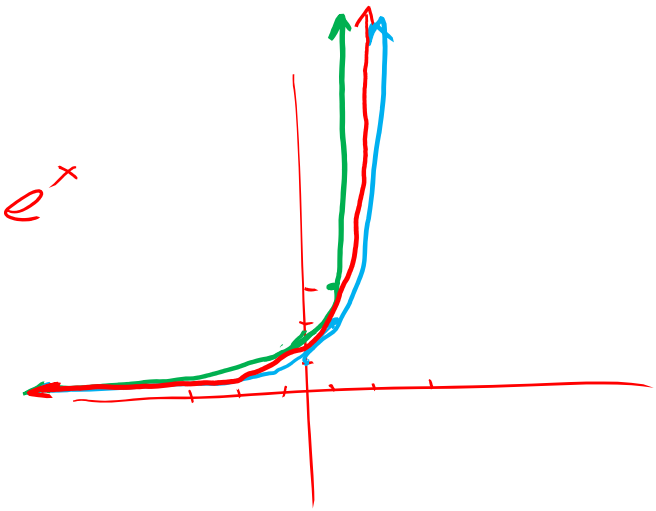
Definition: e is the "limiting value" of $\left(1 + \frac{1}{x}\right)^x$ as x grows to infinity.

$e \approx 2.718281282459$. It is an irrational number, like π. This means it cannot be written as a fraction nor as a terminating or repeating decimal.

$e \approx 2.7$

- $f(x) = 2^x$
- $g(x) = 3^x$

$\Big\}$ $h(x) = e^x$



In case you were wondering, the letter “e” is used for this particular irrational number because of the mathematician Euclid used this constant extensively in his work.

For instance:

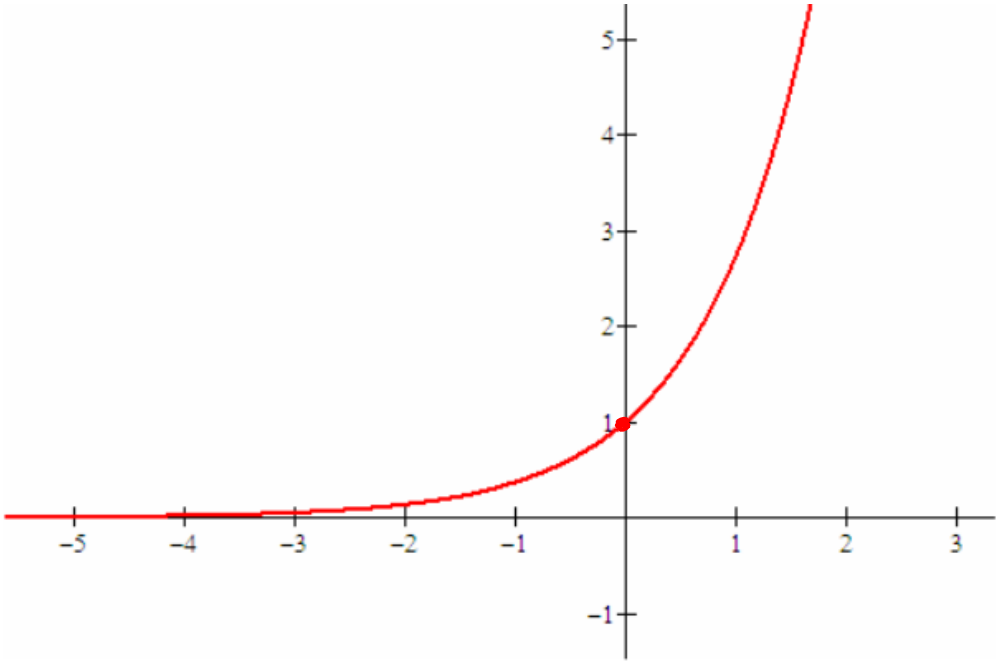
$$\cos(x) + i \sin(x) = e^{ix}$$

Meaning that:

$$e^{i\pi} = -1$$

Since $e > 1$, e can be the base of an exponential function. So everything we learned in Section 5.1 about graphing exponential functions will apply to graphing the function $f(x) = e^x$

The graph of $f(x) = e^x$ will have the following features:
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Key point: $(0, 1)$
Horizontal asymptote: $y = 0$ since $y \to 0$ as $x \to -\infty$



Example 1: Sketch the graph of the function of $f(x) = -e^{x+2} + 2$ using transformations. State the domain, range, asymptote and translation of the key point.

① Parent Function: $y = e^x$

② Horizontal: Left 2

$y = e^{x+2}$

③ x-axis reflection

$y = -e^{x+2}$

④ Up 2

$f(x) = -e^{x+2} + 2$

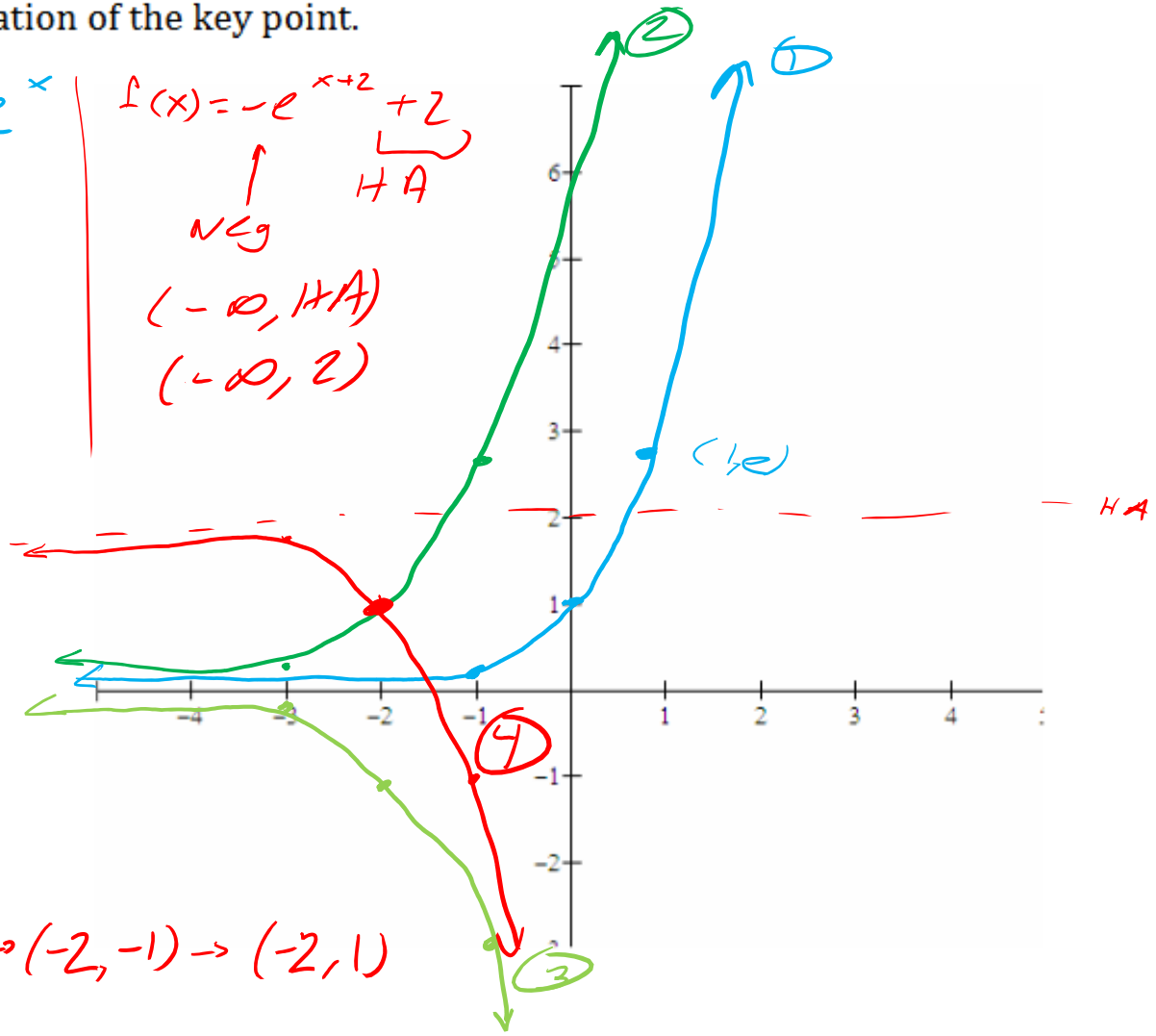
$f(x) = -e^{x+2} + 2$

Neg

HA

$(-\infty, HA)$

$(-\infty, 2)$



Domain: $(-\infty, \infty)$

Range: $(-\infty, 2)$

HA: $y = 2$

Key Point: $(0,1) \to (-2,1) \to (-2,-1) \to (-2,1)$

Example 2: Sketch the graph of the function of $f(x) = e^{-x-1} - 1$ using transformations. State the domain, range, asymptote and translation of the key point.

① Parent function $Y = e^x$

② Right 1 $Y = e^{x-1}$

③ Y-axis refl $Y = e^{-x-1}$

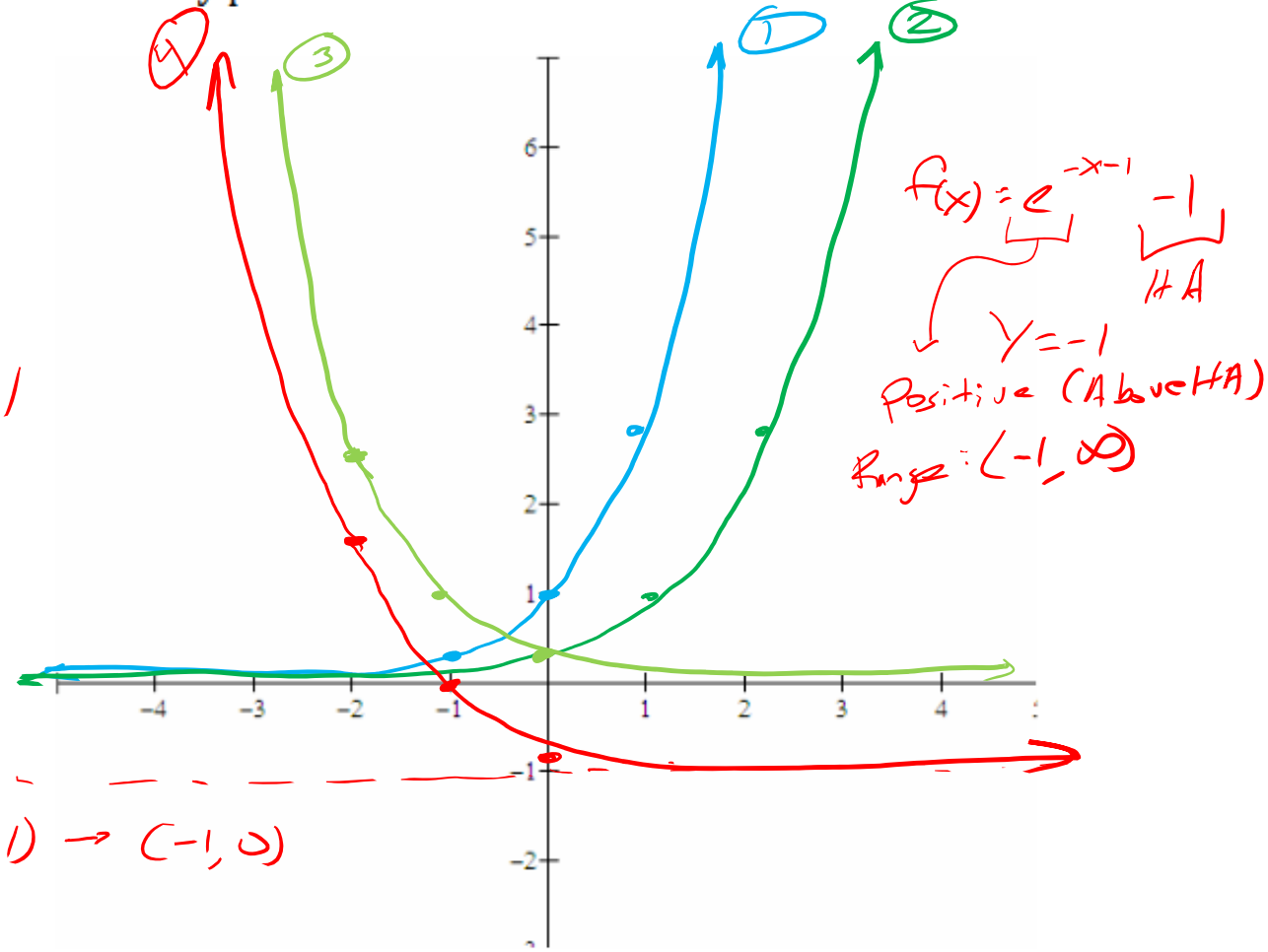
④ Down 1 $f(x) = e^{-x-1} - 1$

Domain: $(-\infty, \infty)$

Range: $(-1, \infty)$

Asymptote: $Y = -1$ (HA)

Key Point: $(0,1) \to (1,1) \to (-1,1) \to (-1,0)$



$f(x) = e^{-x-1} - 1$

HA

$Y = -1$

Positive (Above HA)

Range: $(-1, \infty)$

Example 3:

Base e

Common → Base 10

Write the equation of a natural exponential function that has been shifted left 3 units, down 1 unit and reflected in the x-axis.

$f(x) = e^x$

Left 3: $f(x) = e^{x+3}$

x-axis refl: $f(x) = -e^{x+3}$

Down 1: $f(x) = -e^{x+3} - 1$

Popper 22:

Consider the function: $f(x) = -e^{x-2} + 3$.

1. Determine the domain of the function:

a. $(-\infty, \infty)$ b. $(3, \infty)$ c. $[3, \infty)$ d. $(-2, \infty)$

2. Determine the range of the function:

a. $(-\infty, \infty)$ b. $(-3, \infty)$ c. $(-\infty, 3)$ d. $(-\infty, 2)$

3. Determine the y-intercept of the function:

a. 1 b. $e^{-2} + 3$ c. $3 - e^2$ d. $\frac{3e^2 - 1}{e^2}$

4. Determine the asymptote of the function:

a. $y = e$ b. $x = e$ c. $y = 3$ d. $x = 3$

5. Which transformation did not take place from $g(x) = e^x$?

a. Horizontal Shift b. Vertical Shift c. Horizontal Reflection d. Vertical Reflection

Neg HA: $y = 3$ Beneath the HA Range $(-\infty, 3)$

$f(0) = -e^{0-2} + 3$

$-e^{-2} + 3$

$\frac{-1}{e^2} + \frac{3e^2}{e^2}$

$\frac{-1 + 3e^2}{e^2}$

Right 2 up 3 x-axis refl y-axis $y = e^{-x-2} + 3$

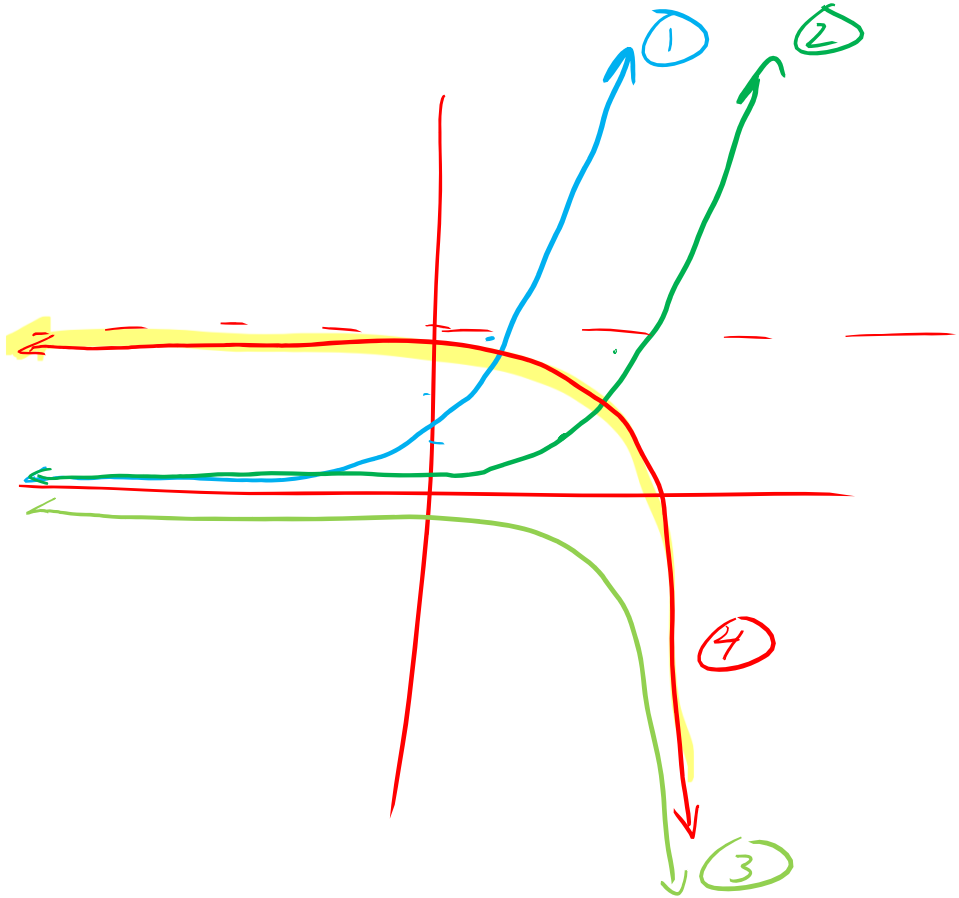
$$f(x) = -e^{x-2} + 3$$

① Parent Function $y = e^x$

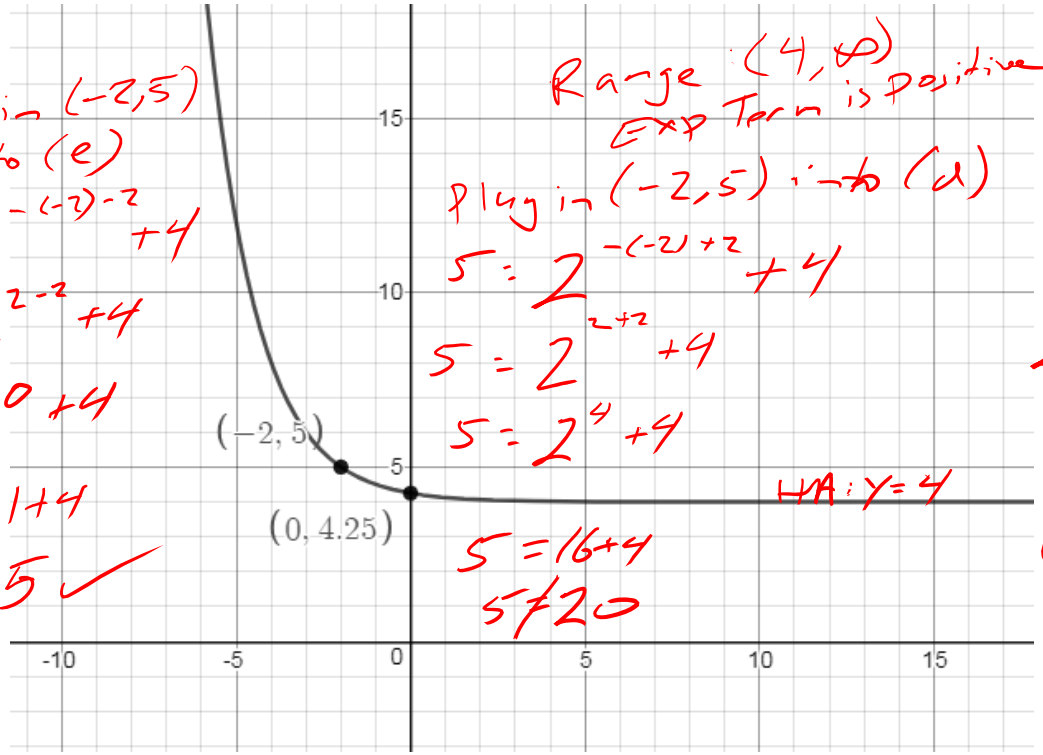
② Right 2 $y = e^{x-2}$

③ x-axis refl. $y = -e^{x-2}$

④ up 3 $f(x) = -e^{x-2} + 3$



Which of the following functions is displayed here?



Plug in (-2,5) into (e)
$5 = 2^{-(-2)-2} + 4$
$5 = 2^{2-2} + 4$
$5 = 2^{0} + 4$
$5 = 1 + 4$
$5 = 5$ ✓

Range $(4, \infty)$
Exp Term is positive
Plug in (-2,5) into (d)
$5 = 2^{-(-2)+2} + 4$
$5 = 2^{2+2} + 4$
$5 = 2^{4} + 4$
$5 = 16 + 4$
$5 \neq 20$

HA: $y = 4$

$a > 0, a \neq 1$
$a \neq 0$

~~a.~~ $f(x) = -2^{x} + 4$

~~b.~~ $f(x) = 2^{x-2} - 4$

~~c.~~ $f(x) = (-2)^{x-2} + 4$

~~d.~~ $f(x) = 2^{-x+2} + 4$

e. $f(x) = 2^{-x-2} + 4$

~~f.~~ $f(x) = 2^{-x-2} - 4$