

# MATH 1314

Section 5.2

# The number “e.”

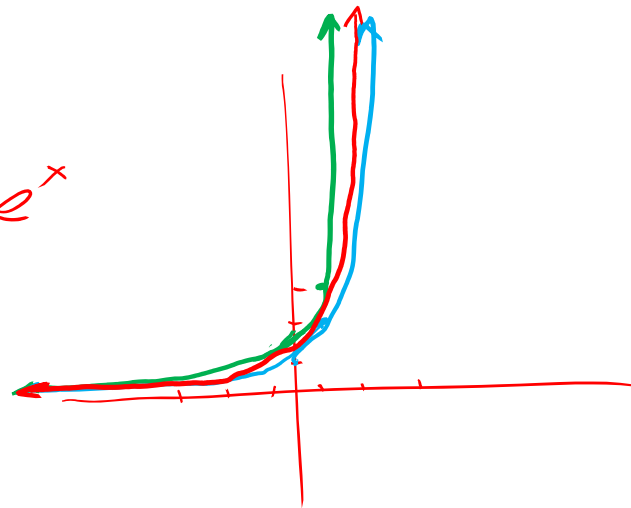
More on transformations of the exponential function  $f(x) = a^x$ , but with  $a = e$  (the natural base).

**Definition:**  $e$  is the “limiting value” of  $\left(1 + \frac{1}{x}\right)^x$  as  $x$  grows to infinity.

$e \approx 2.718281282459$ . It is an irrational number, like  $\pi$ . This means it cannot be written as a fraction nor as a terminating or repeating decimal.

$$e \approx 2.7$$

- $f(x) = 2^x$
  - $g(x) = 3^x$
- }  $h(x) = e^x$



In case you were wondering, the letter “e” is used for this particular irrational number because of the mathematician Euclid used this constant extensively in his work.

For instance:

$$\cos(x) + i \sin(x) = e^{ix}$$

Meaning that:

$$e^{i\pi} = -1$$

Since  $e > 1$ ,  $e$  can be the base of an exponential function. So everything we learned in Section 5.1 about graphing exponential functions will apply to graphing the function  $f(x) = e^x$

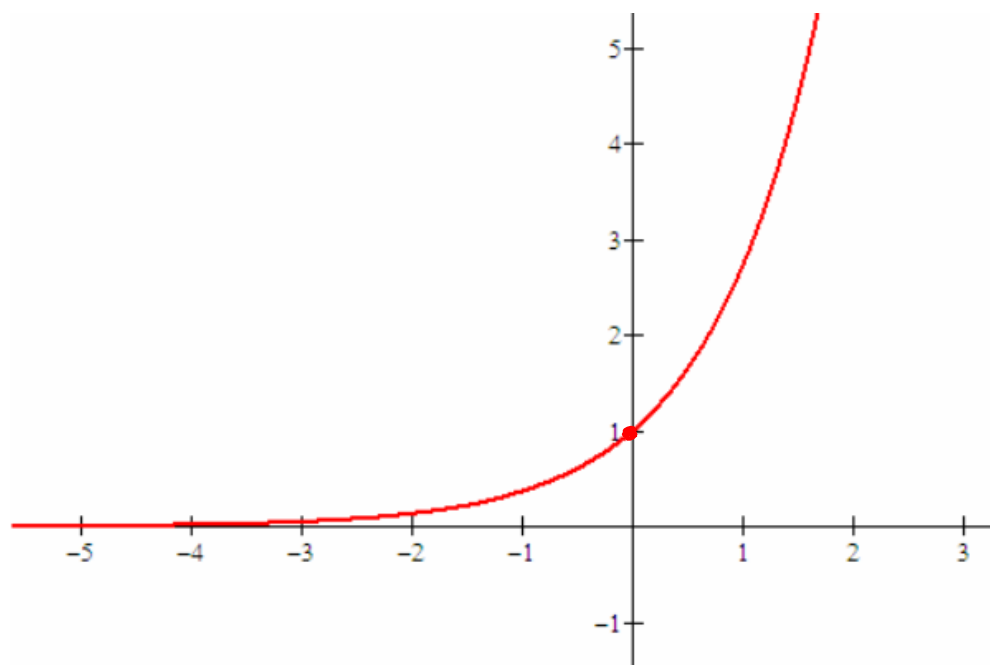
The graph of  $f(x) = e^x$  will have the following features:

Domain:  $(-\infty, \infty)$

Range:  $(0, \infty)$

Key point:  $(0, 1)$

Horizontal asymptote:  $y = 0$  since  $y \rightarrow 0$  as  $x \rightarrow -\infty$



**Example 1:** Sketch the graph of the function of  $f(x) = -e^{x+2} + 2$  using transformations. State the domain, range, asymptote and translation of the key point.

① Parent Function:  $y = e^x$

② Horizontal: Left +2  
 $y = e^{x+2}$

③ x-axis reflection  
 $y = -e^{x+2}$

④ UP 2  
 $f(x) = -e^{x+2} + 2$

Domain:  $(-\infty, \infty)$

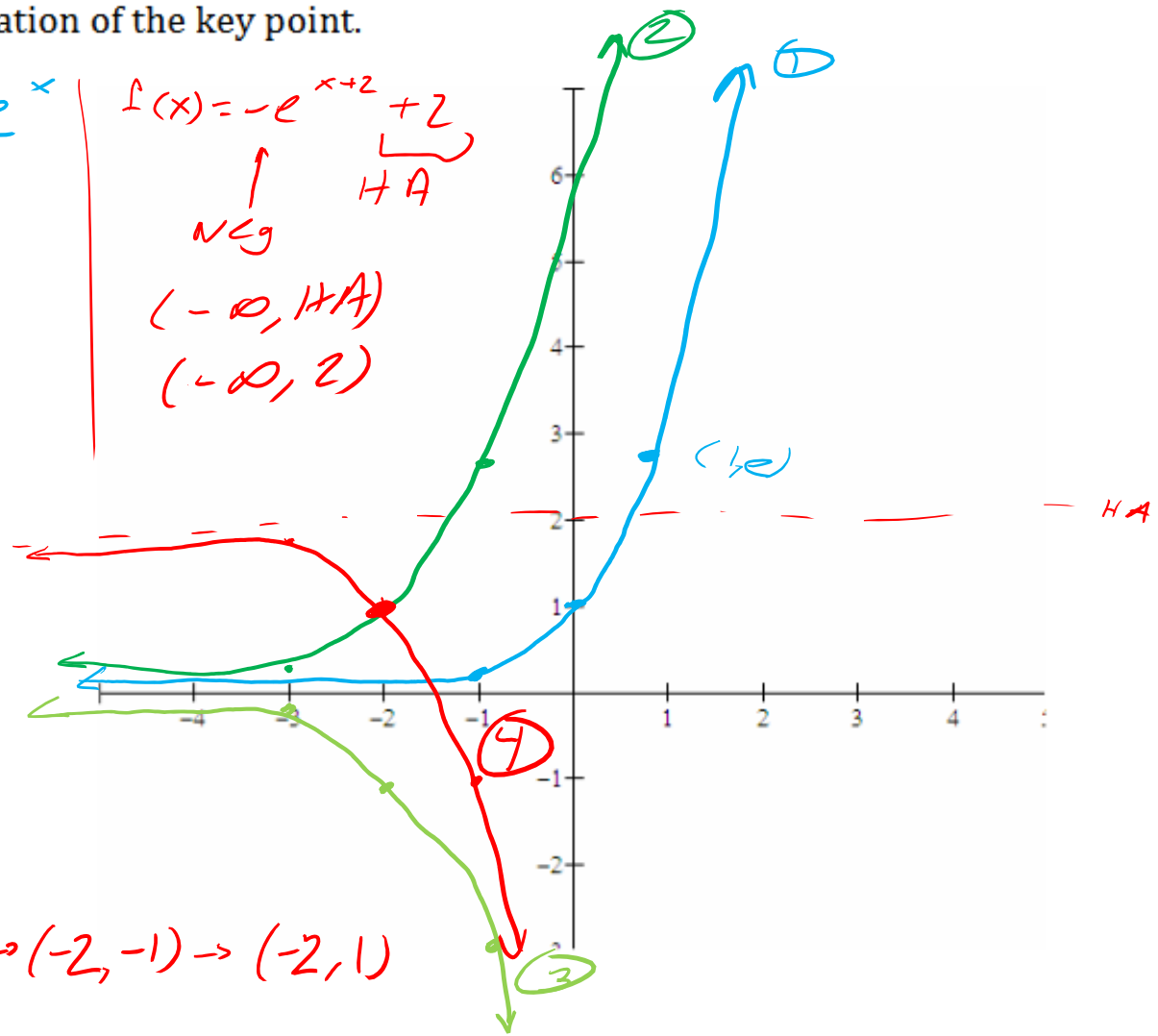
Range:  $(-\infty, 2)$

HA:  $y = 2$

Key Point:  $(0, 1) \rightarrow (-2, 1) \rightarrow (-2, -1) \rightarrow (-2, 1)$

$$f(x) = -e^{x+2} + 2$$

$\uparrow$  neg  
 $\uparrow$  HA  
 $(-\infty, HA)$   
 $(-\infty, 2)$



**Example 2:** Sketch the graph of the function of  $f(x) = e^{-x-1} - 1$  using transformations. State the domain, range, asymptote and translation of the key point.

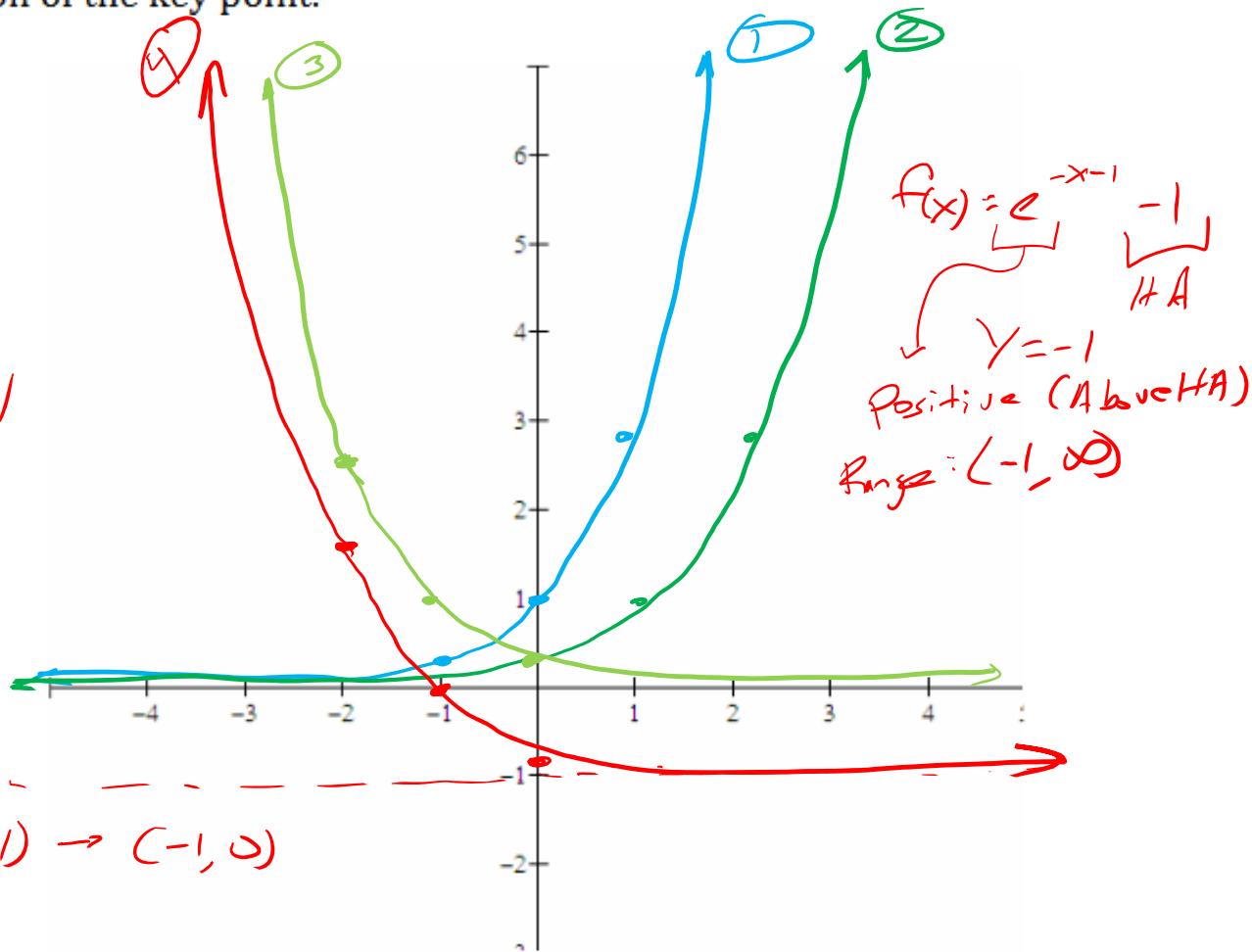
- ① Parent function  $Y = e^x$
- ② Right 1  $Y = e^{x-1}$
- ③ Y-axis refl.  $Y = e^{-x-1}$
- ④ Down 1  $f(x) = e^{-x-1} - 1$

Domain:  $(-\infty, \infty)$

Range:  $(-1, \infty)$

Asymptote:  $Y = -1$   
(HA)

Key Point:  $(0, 1) \rightarrow (1, 1) \rightarrow (-1, 1) \rightarrow (-1, 0)$



### Example 3:

→ Base  $e$

Common → Base 10

Write the equation of a natural exponential function that has been shifted left 3 units, down 1 unit and reflected in the x-axis.

$$f(x) = e^x$$

$$\text{Left } 3: f(x) = e^{x+3}$$

$$\text{x-axis refl: } f(x) = -e^{x+3}$$

$$\text{Down } 1: f(x) = -e^{x+3} - 1$$

# Popper 27:

Consider the function:  $f(x) = -e_{x-2} + 3$ .

*Neg*  
 $\downarrow$   
 HA:  $y=3$  Below the HA  
 Range:  $(-\infty, 3)$

1. Determine the domain of the function:

- a.  $(-\infty, \infty)$       b.  $(3, \infty)$       c.  $[3, \infty)$       d.  $(-2, \infty)$

2. Determine the range of the function:

- a.  $(-\infty, \infty)$       b.  $(-3, \infty)$       c.  $(-\infty, 3)$       d.  $(-\infty, 2)$

3. Determine the y-intercept of the function:

- a. 1      b.  $e^{-2} + 3$       c.  $3 - e^2$       d.  $\frac{3e^2 - 1}{e^2}$

4. Determine the asymptote of the function:

- a.  $y = e$       b.  $x = e$       c.  $y = 3$       d.  $x = 3$

5. Which transformation did not take place from  $g(x) = e^x$ ?

- ~~a. Horizontal Shift~~      ~~b. Vertical Shift~~      c. Horizontal Reflection      ~~d. Vertical Reflection~~

*Right 2*

*UP 3*

*x-axis refl*

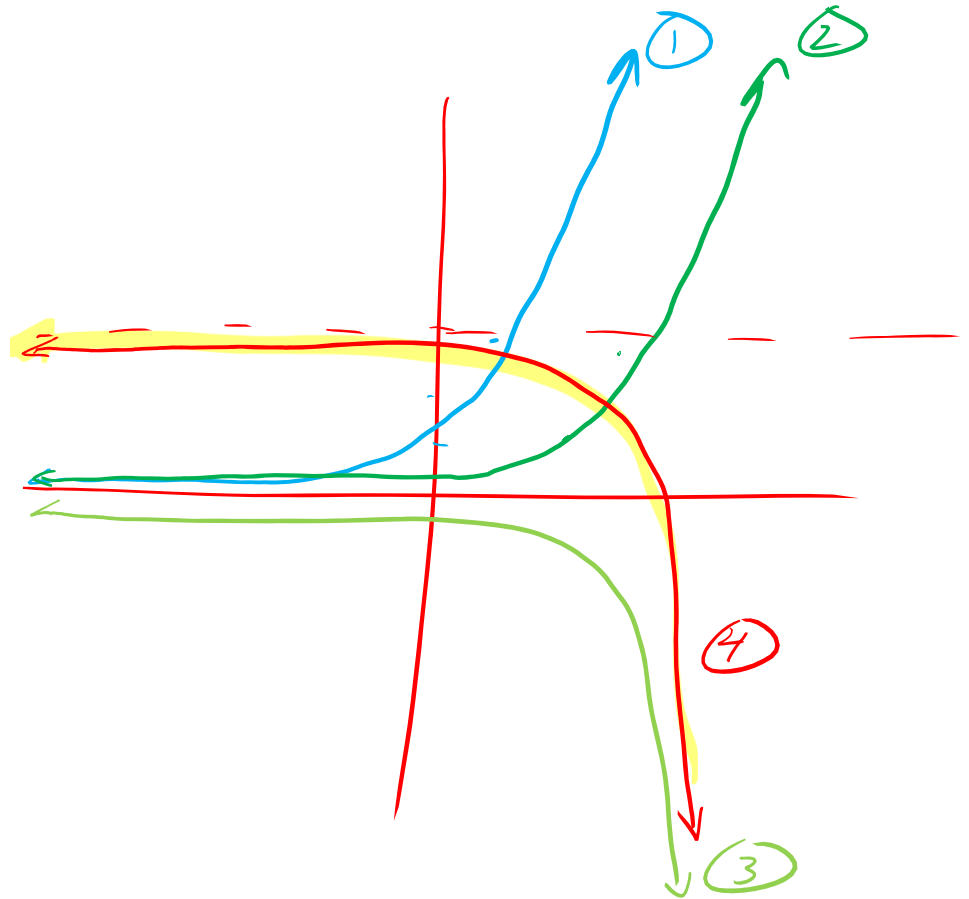
*y-axis*  
 $y = e^{-x-2} + 3$

*f(0) = -e^{0-2} + 3*  
 $-e^{-2} + 3$   
 $-\frac{1}{e^2} + \frac{3e^2}{e^2}$   
 $\frac{-1 + 3e^2}{e^2}$

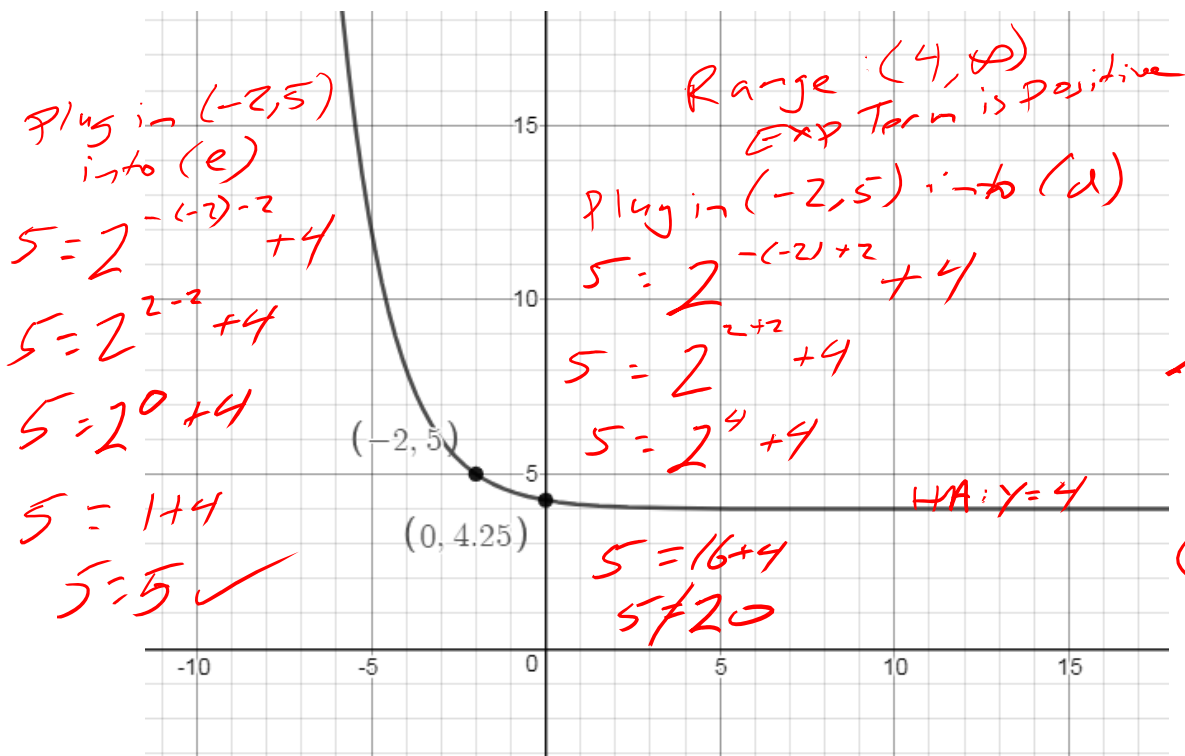


$$f(x) = -e^{x-2} + 3$$

- ① Parent Function  $y = e^x$
- ② Right 2  $y = e^{x-2}$
- ③ X-axis refl.  $y = -e^{x-2}$
- ④ CP 3  $f(x) = -e^{x-2} + 3$



Which of the following functions is displayed here?



~~a.~~  $f(x) = -2^x + 4$

~~b.~~  $f(x) = 2^{x-2} - 4$

~~c.~~  $f(x) = (-2)^{x-2} + 4$

$a > 0, a \neq 1$   
 $a < 0$

~~d.~~  $f(x) = 2^{-x+2} + 4$

**e.**  $f(x) = 2^{-x-2} + 4$

~~f.~~  $f(x) = 2^{-x-2} - 4$