MATH 1314

Section 5.2

The number "e."

More on transformations of the exponential function $f(x) = a^x$, but with $a = e$ *(the natural base)*.

Definition: e is the "limiting value" of $\left(1 + \frac{1}{x}\right)^x$ as x grows to infinity.

$e \approx 2.718281282459$. It is an irrational number, like π. This means it cannot be written as a fraction nor as a terminating or repeating decimal.

In case you were wondering, the letter “e” is used for this particular irrational number because of the mathematician Euclid used this constant extensively in his work.

For instance:

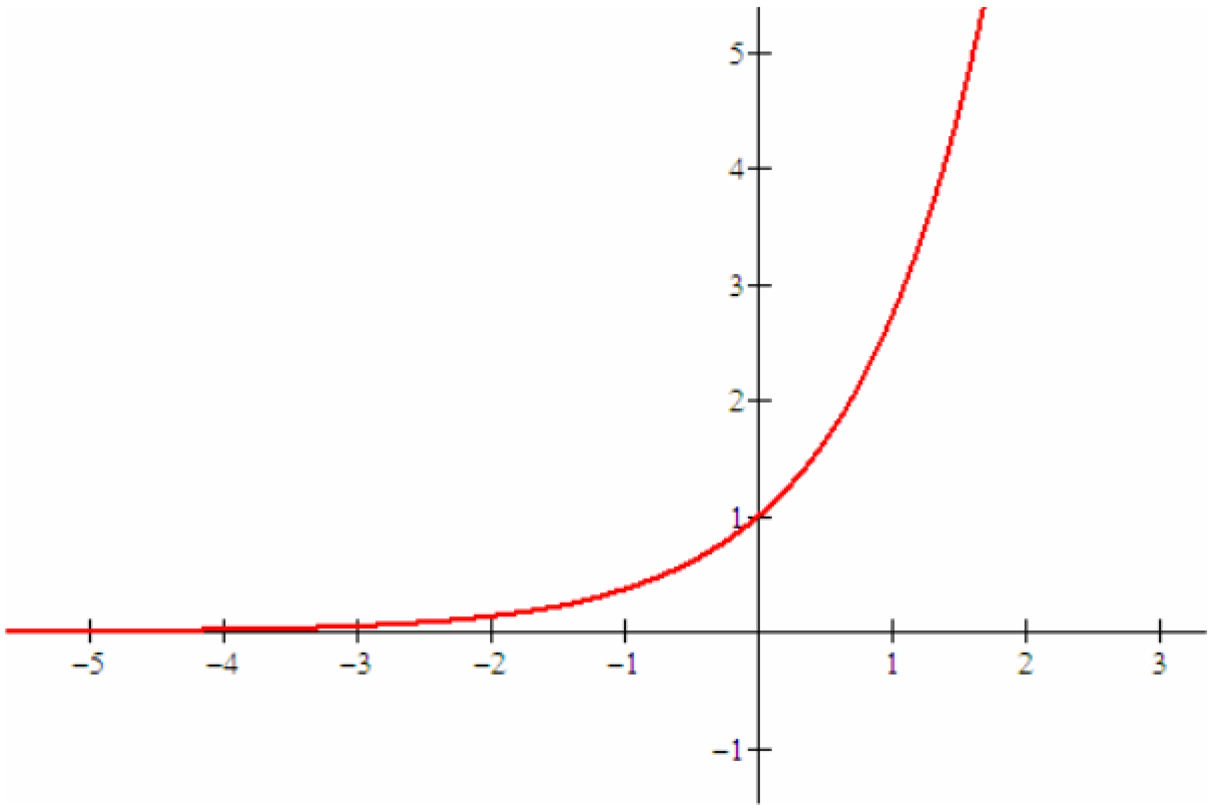
$\cos(x) + i \sin(x) = e^{ix}$

Meaning that:

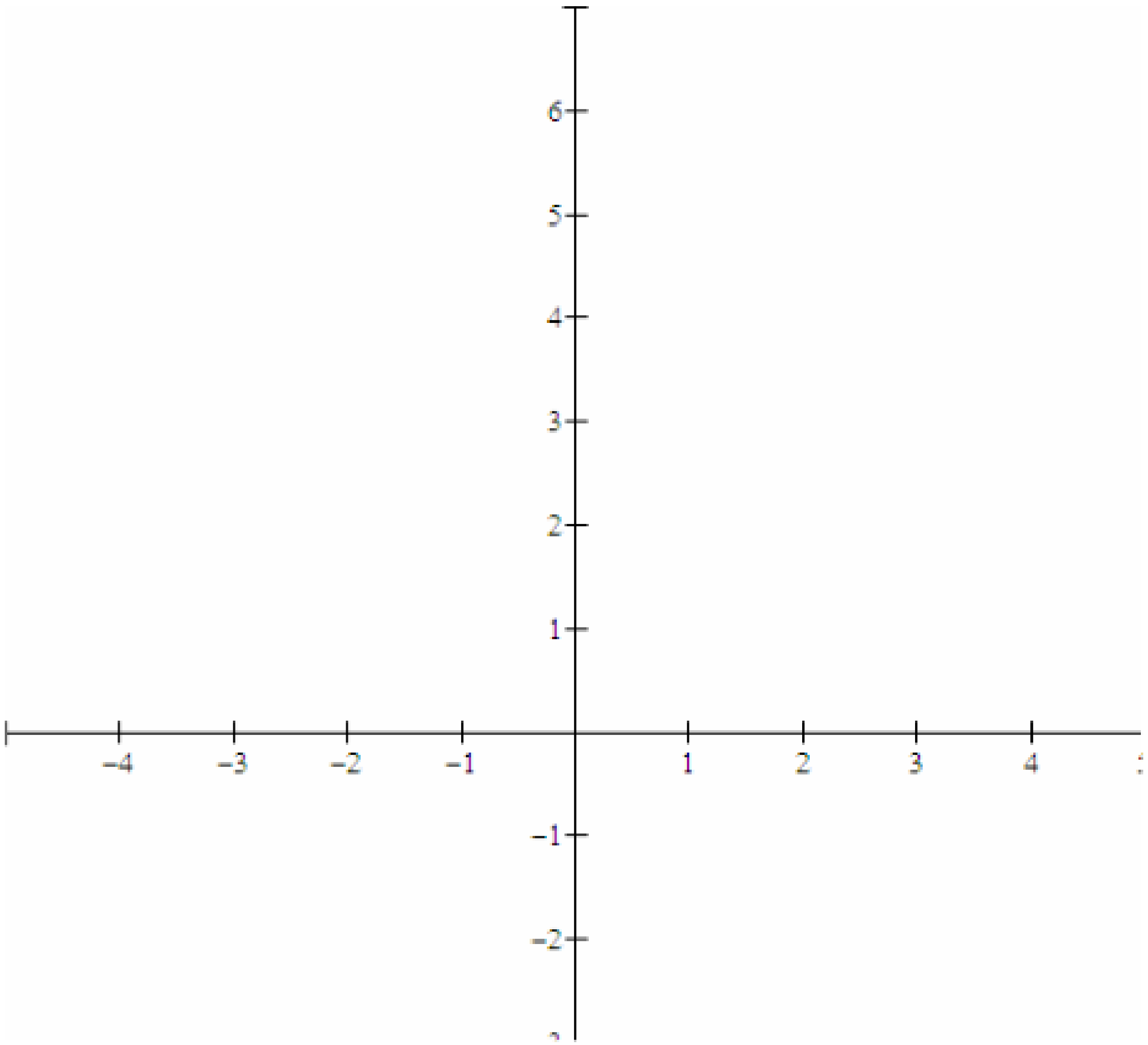
$e^{i\pi} = -1$

Since $e > 1$, e can be the base of an exponential function. So everything we learned in Section 5.1 about graphing exponential functions will apply to graphing the function $f(x) = e^x$

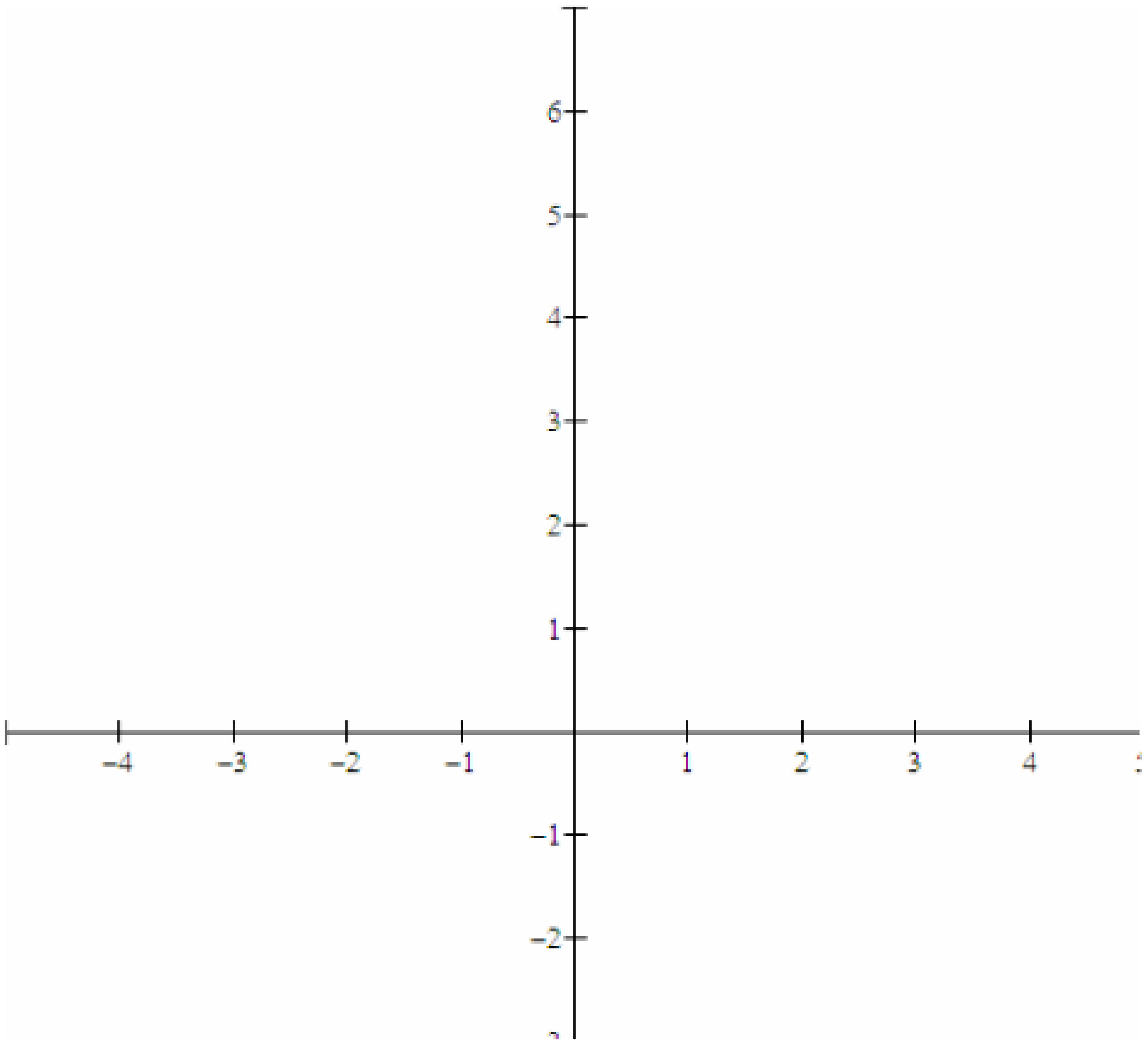
The graph of $f(x) = e^x$ will have the following features:
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Key point: $(0, 1)$
Horizontal asymptote: $y = 0$ since $y \to 0$ as $x \to -\infty$



Example 1: Sketch the graph of the function of $f(x) = -e^{x+2} + 2$ using transformations. State the domain, range, asymptote and translation of the key point.



Example 2: Sketch the graph of the function of $f(x) = e^{-x-1} - 1$ using transformations. State the domain, range, asymptote and translation of the key point.



Example 3:

Write the equation of a natural exponential function that has been shifted left 3 units, down 1 unit and reflected in the x-axis.

Consider the function: $f(x) = -e^{x-2} + 3$.

Determine the domain of the function:

Determine the range of the function:

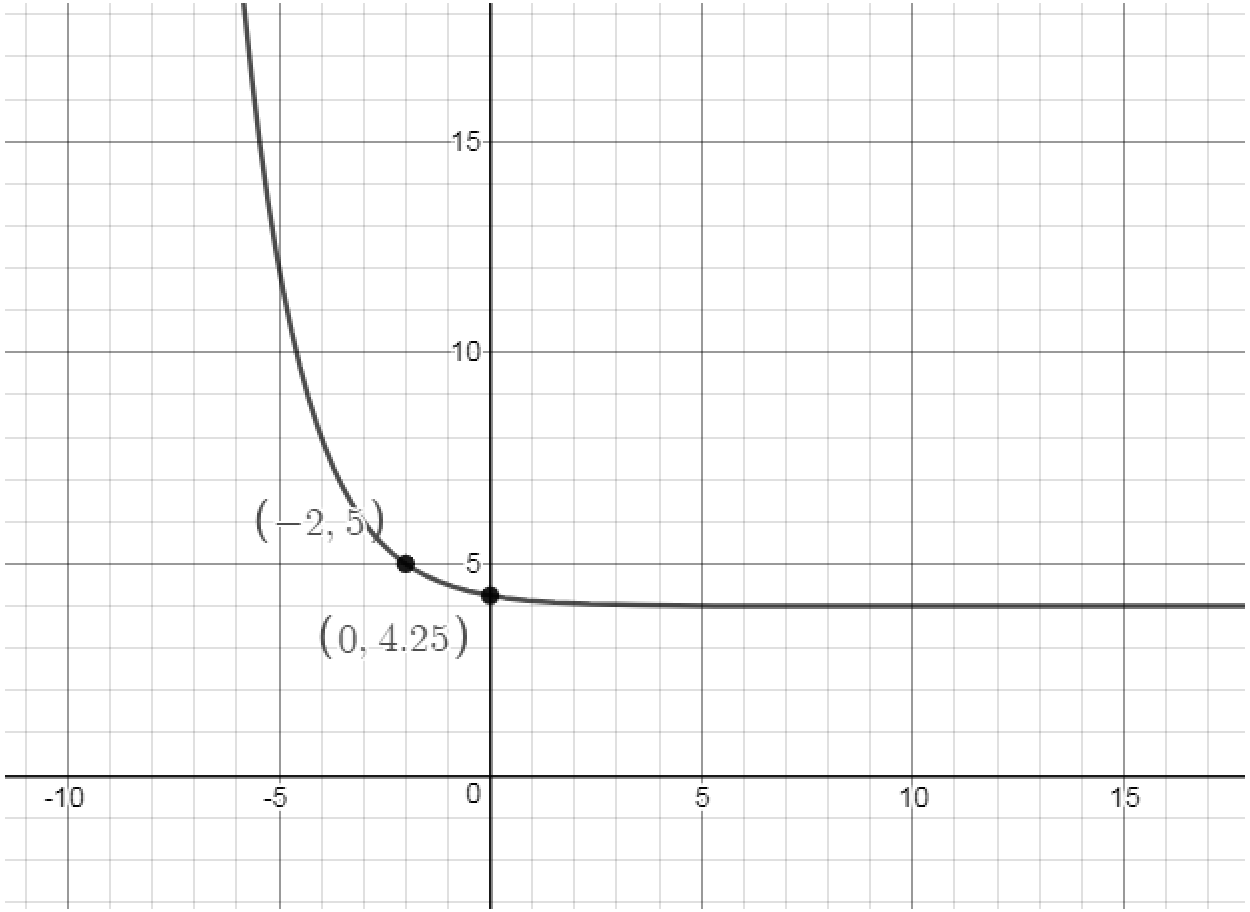
Determine the y-intercept of the function:

Determine the asymptote of the function:

Which transformations took place from $g(x) = e^x$?

$f(x) = -e^{x-2} + 3$ Sketch

Which of the following functions is displayed here?



a. $f(x) = -2^{x} + 4$

b. $f(x) = 2^{x-2} - 4$

c. $f(x) = (-2)^{x-2} + 4$

d. $f(x) = 2^{-x+2} + 4$

e. $f(x) = 2^{-x-2} + 4$

f. $f(x) = 2^{-x-2} - 4$