MATH 1310

Section 5.3

Logarithmic Functions

Base cannot be Neg
$\neq 0, \neq 1$

The exponential function is 1-1; therefore, it has an inverse function. The inverse function of the exponential function with base a is called the **logarithmic function with base a.**

$y = 2^x$

For $x > 0$ and $a > 0$ and a not equal to 1, $y = \log_a x$ is equivalent $a^y = x$

$x = 2^y$

The function $f(x) = \log_a x$ is the **logarithmic function with base a**

The **common logarithm** is the logarithm with base 10. We denote this as $\log_{10} x = \log x$
The **natural logarithm** is the logarithm with base e. We denote this as $\log_e x = \ln x$

$\log_2 x = y$

You will find both of these logarithms on a scientific calculator.

Note: We do not typically write either $\log_{10} x$ or $\log_e x$.

$\log_{10} x = \log x$
common logarithms

$\log_e x = \ln x$
natural logarithms

Never
$\ln_e x$

Example 1: Write each equation in its equivalent exponential form.

a. $3 = \log_6 x$

$6^3 = x$

b. $2 = \log_a 64$

$a^2 = 64 \quad (a = 8)$

c. $\log_3 27 = 3$

$3^3 = 27$

d. $\log_{10} 100000 = 5$

$10^5 = 100000$

e. $\ln \frac{1}{e^2} = -2 \rightarrow \log_e \left(\frac{1}{e^2}\right) = -2$

$e^{-2} = \frac{1}{e^2}$

Example 2: Write each equation in its equivalent logarithmic form.

a. $4^3 = 64$ $\quad \log_4 64 = 3$

b. $2^6 = 64$ $\quad \log_2 64 = 6$

c. $e^x = 25$ $\quad \log_e 25 = x \longrightarrow \ln 25 = x$

d. $10^x = 1000$ $\quad \log_{10} 1000 = x \longrightarrow \log 1000 = x$

Example 3: Evaluate, if possible.

$\log_6 36 = x$

$6^x = 36$

$x = 2$

$\log_2 \frac{1}{8} = x$

$2^x = \frac{1}{8}$

[Fraction → Neg
$2^3 = 8$]

$x = -3$

$\log_5 125 = x$

$5^{-x} = 1/25$

$x = 3$

$\log_3(\sqrt[3]{81}) = x$

$3^x = \sqrt[3]{81} = \sqrt[3]{3^4}$

$3^{\boxed{4}} = 81$

$x = 4/3 = \frac{\text{power}}{\text{root}}$

$\log_5 \sqrt[4]{125} = x$

$5^{-x} = \sqrt[4]{125}$

$125 = 5^{\boxed{3}}$

$5^x = \sqrt[4]{5^3} = 5^{3/4}$

$x = 3/4$

Popper 23:

1. $\log_{10} 100 \rightarrow 10^x = 100 \rightarrow 10^2 = 100$

   a. 3 b. 10 c. 2 d. 0.5

2. $\log_4 2 \rightarrow 4^x = 2 \rightarrow 4^{1/2} = 2$ ... $\sqrt{4} = 2$

   a. 16 b. -2 c. 2 d. 0.5

3. $\log_{10} 0.001 \rightarrow 10^x = .001 = \frac{1}{1000} \rightarrow 10^{-3} = \frac{1}{1000}$

   $\frac{1}{\text{something}} \rightarrow$ neg

   $10^3 = 1000$

   a. 1/1000 b. -1 c. -3 d. No Solution

4. $\log_4 (-2) \rightarrow 4^x = -2$

   a. -0.5 b. -16 c. 1/8 d. No Solution

Inverse Property of Logarithms

For $a > 0$ and $a \neq 1$

1. $\log_a a^x = x$

2. $a^{\log_a x} = x$

$$\cancel{\log_5 5}^3 = 3$$

$$\log_2 4^3 = \log_2 (2^2)^3 = \cancel{\log_2 2}^6 = 6$$

$$\cancel{7^{\log_7}}{}^{10} = 10$$

$$3^{2\log_3 5}$$

Doesn't simplify (yet)

Example 4: Evaluate.

a. $\cancel{\log_{14}} 14^3 = 3$

b. $\cancel{5^{\log_5}} 34 = 34$

c. $\cancel{e^{\ln}} 32 = 32$

d. $\cancel{\log_{47} 47}^{\pi} = \pi$

e. $\log_{-2}(-8)$

undefined

f. $\cancel{\log_5 5}^{-3} = -3$

g. $6^{\log_6(-7)}$

undefined

h. $\cancel{\ln e}^{-0.02} = -.02$

i. $\ln_4(4e)^5$

undefined.

j. $\cancel{\log_2 2}^{-5} = -5$

k. $2^{\log_2(-5)}$ undefined.

l. $3^{5\log_3 7}$ (can't evaluate yet)

Recall that for $x > 0$ (and $a > 0$ and a not equal to 1), we have $f(x) = \log_a x$. So the domain of $f(x) = \log_a x$ consist of all x for which $x > 0$.

$f(x) = \log_a x$ Domain: $x > 0$

$g(x) = \ln(x+7)$

inside > 0

$x + 7 > 0$

$x > -7 \to (-7, \infty)$

$h(x) = \log_2(x^2 - 4)$

$x^2 - 4 > 0$

$x^2 - 4 = 0$

$x = \pm 2$

Yes, No, Yes

-2 2

Test $x = 0$

$0^2 - 4 > 0$

D: $(-\infty, -2) \cup (2, \infty)$

Popper 23...continued

Example 5: Find the domain.

$x - 2 > 0 \rightarrow x > 2$

5. $f(x) = \log_2(x - 2)$

a. (2, ∞) b. [2, ∞) c. (-∞, 2) d. (-∞, ∞)

$7 - 2x > 0 \rightarrow -2x > -7 \rightarrow x < 7/2$

6. $f(x) = \ln(7 - 2x)$

a. (3.5, ∞) b. [3.5, ∞) c. (-∞, 3.5) d. (-∞, ∞)

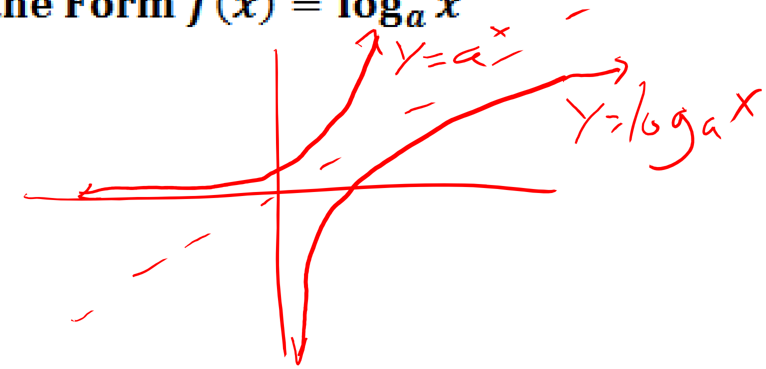
7. $f(x) = \log(x^2 + 1)$

$x^2 + 1 > 0$

a. (1, ∞) b. (-∞, -1) U (1, ∞) c. (-∞, -1) d. (-∞, ∞)

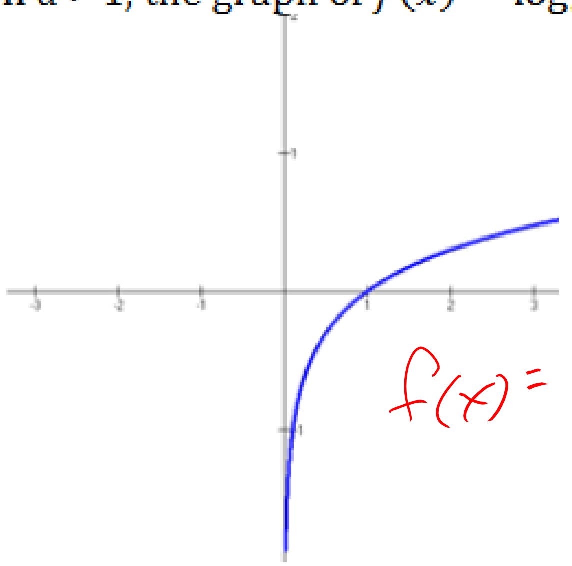
Characteristics of the Graphs of Logarithmic Functions of the Form $f(x) = \log_a x$

KeyPoint



1. The x-intercept is (1, 0) and there is no y-intercept.
2. The y-axis is a vertical asymptote.
3. The domain is all positive real numbers.
4. The range is all real numbers.

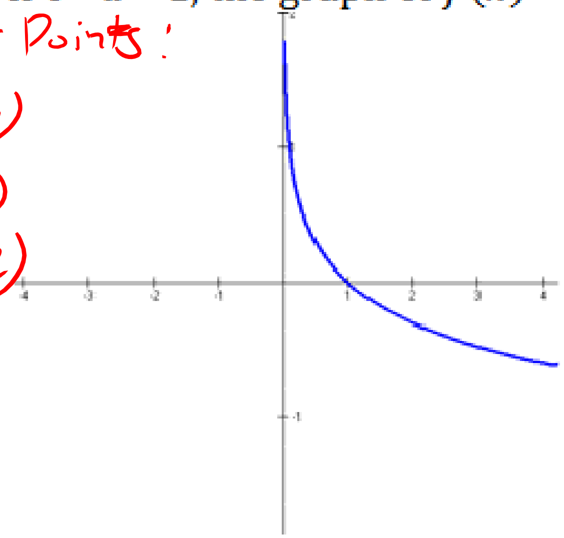
If $a > 1$, the graph of $f(x) = \log_a x$ looks like:



$f(x) = \log_2 x$

Major Points:
(1, 0)
(a, 1)
(a^2, 2)

If $0 < a < 1$, the graph of $f(x) = \log_a x$ looks like:



$g(x) = \log_{1/2} x$

Note: If a logarithmic function is translated to the left or to the right, the vertical asymptote is shifted by the amount of the horizontal shift.

Example 6: Sketch the graph of $f(x) = \log_4(x+2)$. State the domain, range, asymptote and key point.

① Parent Function: $y = \log_4 x$

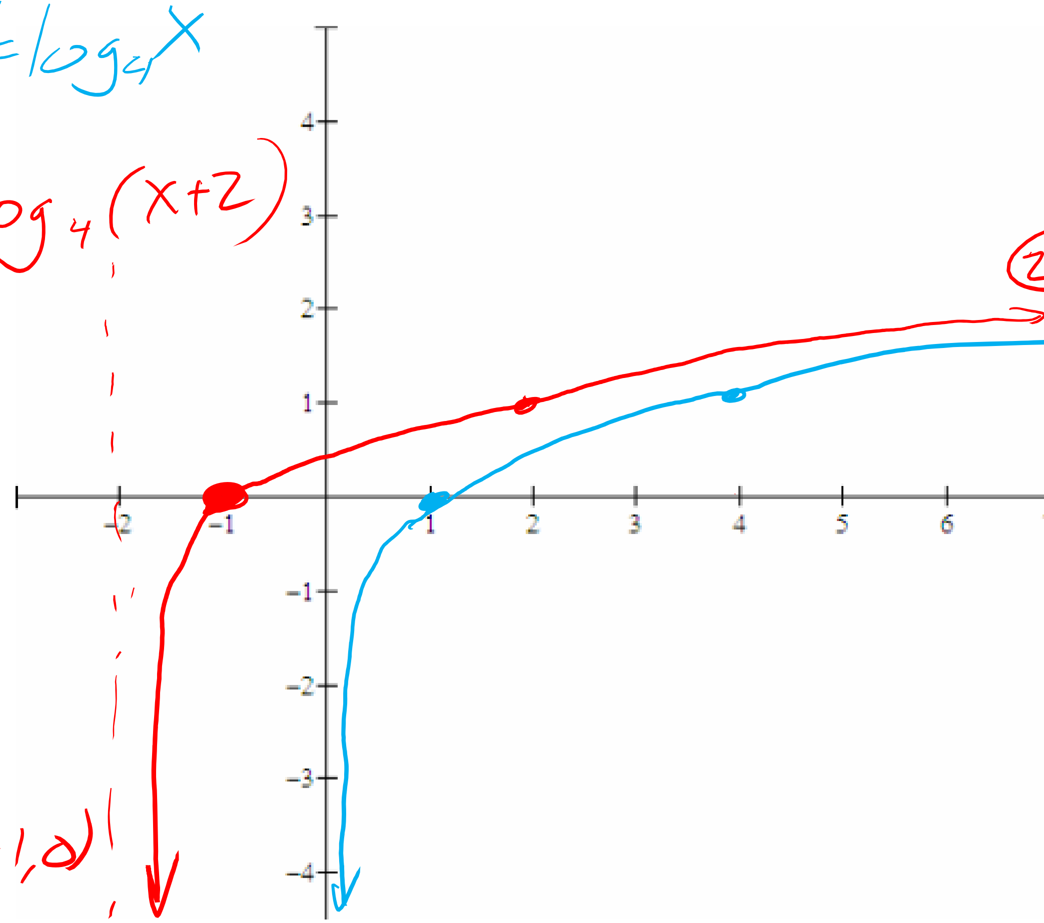
② Left 2 : $f(x) = \log_4(x+2)$

Domain: $x+2>0$
$x>-2$
$(-2, \infty)$

Range: $(-\infty, \infty)$

Asymptote: $x = -2$

Key Point: $(1,0) \to (-1,0)$



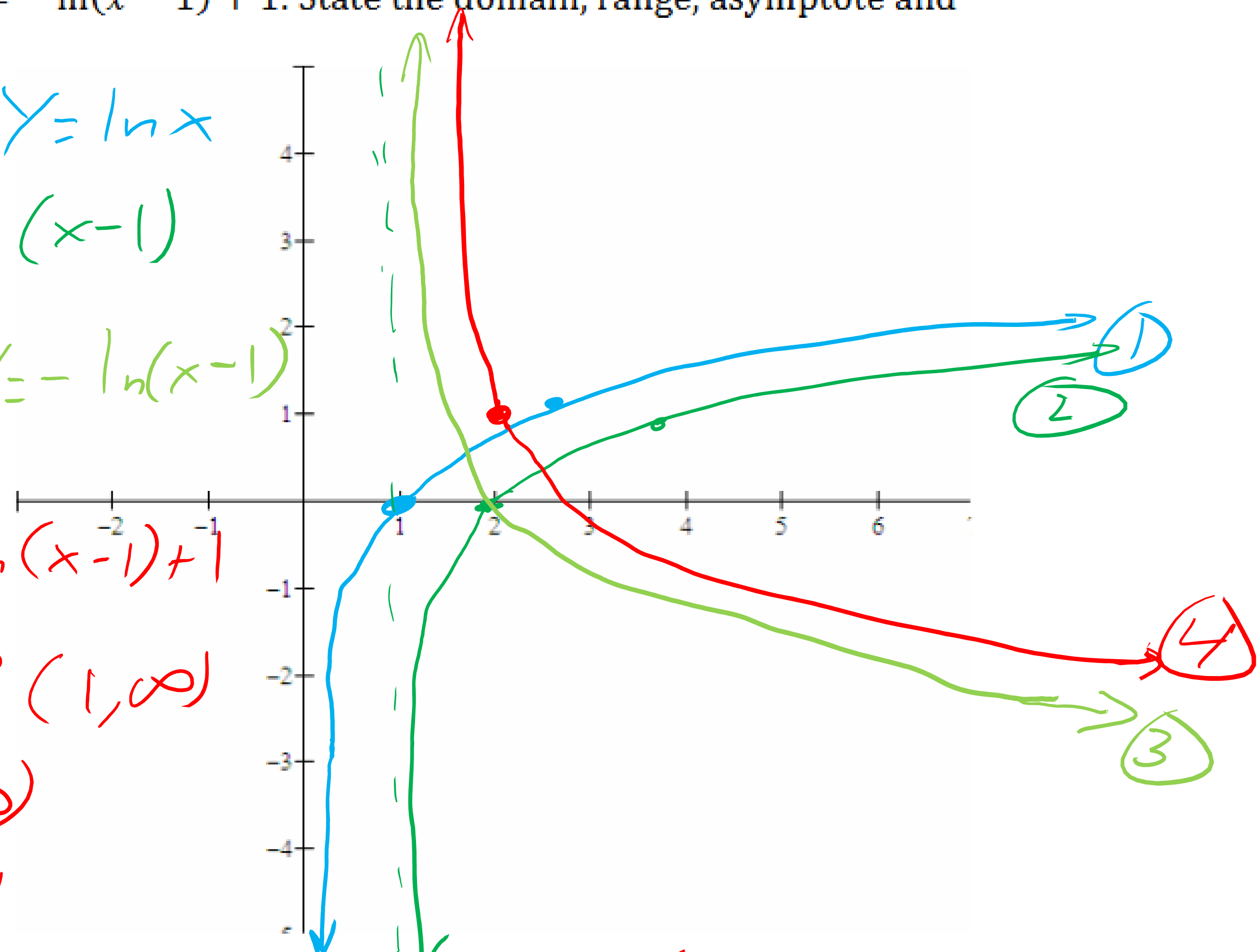
Example 7: Sketch the graph of $f(x) = -\ln(x-1) + 1$. State the domain, range, asymptote and key point.

① Parent Function: $y = \ln x$

② Right 1: $y = \ln(x-1)$

③ x-axis refl.: $y = -\ln(x-1)$

④ Up 1: $f(x) = -\ln(x-1)+1$



Domain: $x-1>0$, $x>1$, $(1,\infty)$

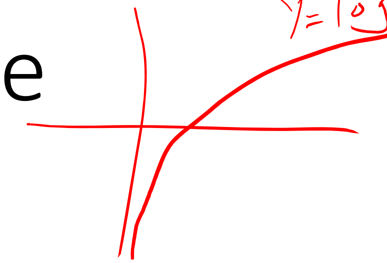
Range: $(-\infty,\infty)$

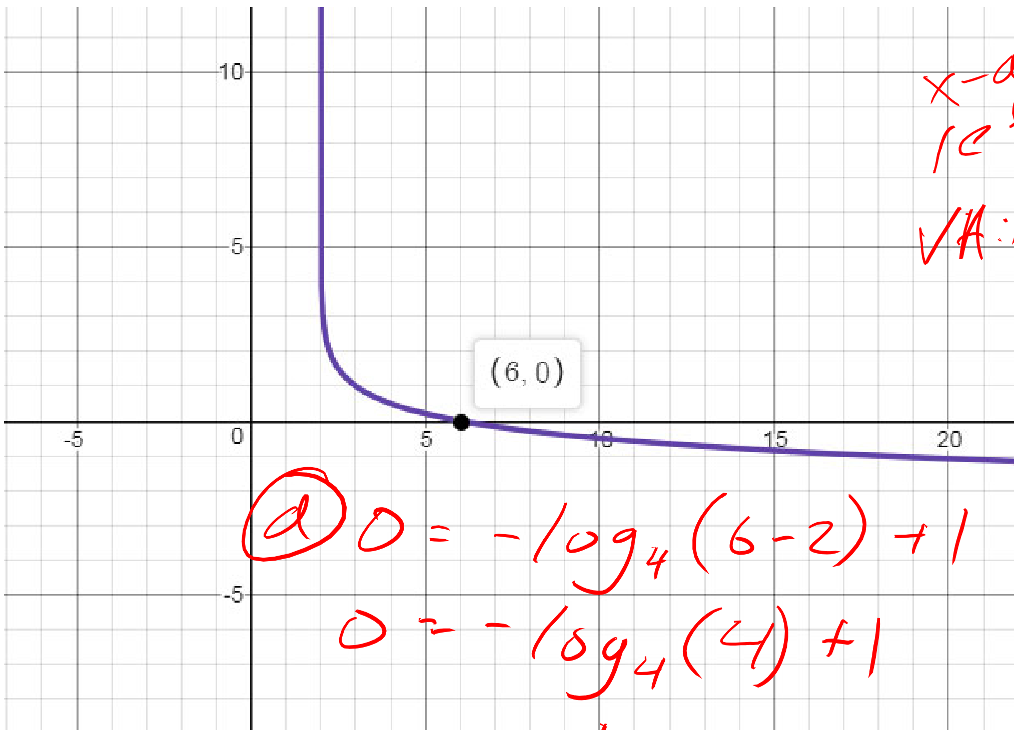
Asymptote: $x=1$

Key Point: $(1,0) \to (2,0) \to (2,0) \to (2,1)$

Determine which of the following is the illustrated function:

$y = \log_a x$





x-axis refl.

VA: x=2

a. ~~f(x) = $\log_4(x + 2) - 1$~~

b. ~~f(x) = $\log_4(x - 2) - 1$~~

c. ~~f(x) = - $\log_4(x + 2) + 1$~~

d. f(x) = - $\log_4(x - 2) + 1$

e. f(x) = - $\log_5(x - 2) - 1$

f. f(x) = - $\log_5(x - 2) + 1$

d

$$0 = -\log_4(6-2)+1$$

$$0 = -\log_4(4)+1$$

$$0 = -1+1$$

$$0 = 0$$