

MATH 1314

Section 5.3

Logarithmic Functions

Base cannot be Neg
 $\neq 0, \neq 1$

The exponential function is 1-1; therefore, it has an inverse function. The inverse function of the exponential function with base a is called the **logarithmic function with base a** .

For $x > 0$ and $a > 0$ and a not equal to 1, $y = \log_a x$ is equivalent $a^y = x$

$$y = 2^x$$

The function $f(x) = \log_a x$ is the **logarithmic function with base a**

$$x = 2^y$$

The **common logarithm** is the logarithm with base 10. We denote this as $\log_{10} x = \log x$

The **natural logarithm** is the logarithm with base e . We denote this as $\log_e x = \ln x$

You will find both of these logarithms on a scientific calculator.

$$\log_2 x = y$$

Note: We do not typically write either $\log_{10} x$ or $\log_e x$.

$$\log_{10} x = \log x$$

Common
logarithms

$$\log_e x = \ln x$$

Natural
logarithms

Never
 $\ln_e x$

Example 1: Write each equation in its equivalent exponential form.

a. $3 = \log_6 x$

$$6^3 = x$$

b. $2 = \log_a 64$

$$a^2 = 64 \quad (a=8)$$

c. $\log_3 27 = 3$

$$3^3 = 27$$

d. $\log_{10} 100000 = 5$

$$10^5 = 100000$$

e. $\ln \frac{1}{e^2} = -2 \rightarrow \log_e \left(\frac{1}{e^2}\right) = -2$

$$e^{-2} = \frac{1}{e^2}$$

Example 2: Write each equation in its equivalent logarithmic form.

a. $4^3 = 64$

$$\log_4 64 = 3$$

b. $2^6 = 64$

$$\log_2 64 = 6$$

c. $e^x = 25$

$$\log_e 25 = x \rightarrow \ln 25 = x$$

d. $10^x = 1000$

$$\log_{10} 1000 = x \rightarrow \log 1000 = x$$

Example 3: Evaluate, if possible.

$$\log_6 36 = x$$

$$6^x = 36$$

$$x = 2$$

$$\log_2 \frac{1}{8} = x$$

$$2^x = \frac{1}{8} \quad \left[\begin{array}{l} \text{Fraction} \rightarrow \text{Neg} \\ 2^3 = 8 \end{array} \right]$$

$$x = -3$$

$$\log_5 125 = x$$

$$5^x = 125$$

$$x = 3$$

$$\log_3 (\sqrt[3]{81}) = x$$

$$3^x = \sqrt[3]{81} = \sqrt[3]{3^4}$$

$$x = \frac{4}{3} = \frac{\text{power}}{\text{root}}$$

$$3^{\boxed{4}} = 81$$

$$\log_5 \sqrt[4]{125} = x$$

$$5^x = \sqrt[4]{125} \quad 125 = 5^{\boxed{3}}$$

$$5^x = \sqrt[4]{5^3} = 5^{3/4}$$

$$x = 3/4$$

Popper 28:

1. $\log_{10} 100 \rightarrow 10^x = 100 \rightarrow 10^2 = 100$

a. 3

b. 10

c. 2

d. 0.5

2. $\log_4 2 \rightarrow 4^x = 2 \rightarrow 4^{1/2} = 2$ $\sqrt{4} = 2$

a. 16

b. -2

c. 2

d. 0.5

3. $\log_{10} 0.001 \rightarrow 10^x = .001 = \frac{1}{1000} \rightarrow 10^{-3} = \frac{1}{1000}$

a. 1/1000

b. -1

c. -3

d. No Solution

$\frac{1}{1000} \rightarrow$
something like
 $10^3 = 1000$

4. $\log_4 (-2) \rightarrow 4^x = -2$

a. -0.5

b. -16

c. 1/8

d. No Solution

Inverse Property of Logarithms

For $a > 0$ and $a \neq 1$

1. $\log_a a^x = x$

2. $a^{\log_a x} = x$

$$\cancel{\log_5 5^3} = 3$$

$$\log_2 4^3 = \log_2 (2^2)^3 = \cancel{\log_2 2^6} = 6$$

$$\cancel{\log_7 10} = 10$$

$$3^{2 \log_3 5}$$

Doesn't
simplify
(yet)

Example 4: Evaluate.

a. ~~$\log_{14} 14^3 = 3$~~

b. ~~$5^{\log_5 34} = 34$~~

c. ~~$e^{\ln 32} = 32$~~

d. ~~$\log_{47} 47^\pi = \pi$~~

e. ~~$\log_{-2}(-8)$~~
undefined

f. ~~$\log_5 5^{-3} = -3$~~

g. ~~$6^{\log_6(-7)}$~~
undefined

h. ~~$\ln e^{-0.02} = -0.02$~~

i. ~~$\ln_4(4e)^5$~~
undefined.

j. ~~$\log_2 2^{-5} = -5$~~

k. ~~$2^{\log_2(-5)}$~~ undefined.

l. ~~$3^{5 \log_3 7}$~~ (can't evaluate yet)

Recall that for $x > 0$ (and $a > 0$ and a not equal to 1), we have $f(x) = \log_a x$. So the domain of $f(x) = \log_a x$ consist of all x for which $x > 0$.

$$f(x) = \log_a x$$

$$\text{Domain} = x > 0$$

$$g(x) = \ln(x+7)$$

$$\text{inside} > 0$$

$$x+7 > 0$$

$$x > -7 \rightarrow (-7, \infty)$$

$$h(x) = \log_2(x^2 - 4)$$

$$x^2 - 4 > 0$$

$$x^2 - 4 = 0$$

$$x = \pm 2$$

Yes, No, Yes

$$\text{Test } x = 0$$

$$0^2 - 4 > 0$$

$$D: (-\infty, -2) \cup (2, \infty)$$

Popper 28...continued

Example 5: Find the domain.

$$x - 2 > 0 \rightarrow x > 2$$

5. $f(x) = \log_2(x - 2)$

a. $(2, \infty)$

b. $[2, \infty)$

c. $(-\infty, 2)$

d. $(-\infty, \infty)$

$$7 - 2x > 0 \rightarrow -2x > -7 \rightarrow x < 7/2$$

6. $f(x) = \ln(7 - 2x)$

a. $(3.5, \infty)$

b. $[3.5, \infty)$

c. $(-\infty, 3.5)$

d. $(-\infty, \infty)$

7. $f(x) = \log(x^2 + 1)$

$$x^2 + 1 > 0$$

a. $(1, \infty)$

b. $(-\infty, -1) \cup (1, \infty)$

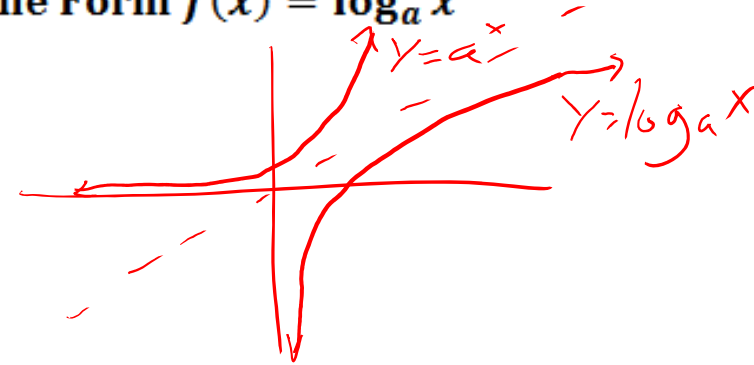
c. $(-\infty, -1)$

d. $(-\infty, \infty)$

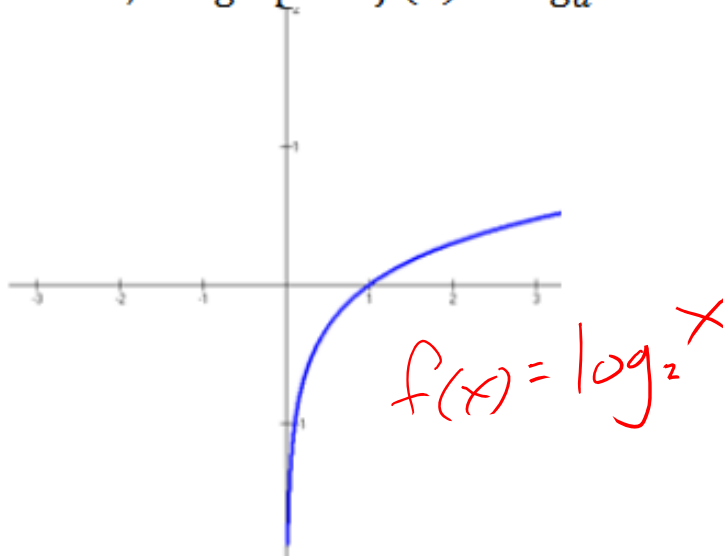
Characteristics of the Graphs of Logarithmic Functions of the Form $f(x) = \log_a x$

Key Point

1. The x-intercept is $(1, 0)$ and there is no y-intercept.
2. The y-axis is a vertical asymptote.
3. The domain is all positive real numbers.
4. The range is all real numbers.



If $a > 1$, the graph of $f(x) = \log_a x$ looks like:



$$f(x) = \log_2 x$$

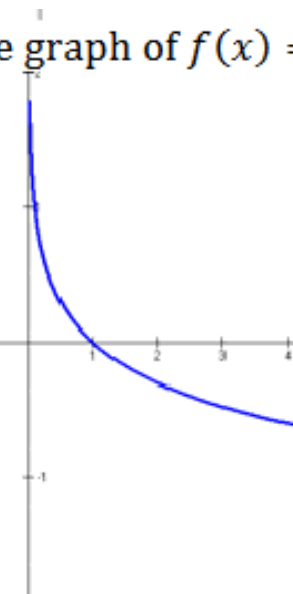
If $0 < a < 1$, the graph of $f(x) = \log_a x$ looks like:

Major Points!

$$(1, 0)$$

$$(a, 1)$$

$$(a^2, 2)$$



$$g(x) = \log_{1/2} x$$

Note: If a logarithmic function is translated to the left or to the right, the vertical asymptote is shifted by the amount of the horizontal shift.

Example 6: Sketch the graph of $f(x) = \log_4(x + 2)$. State the domain, range, asymptote and key point.

① Parent Function: $Y = \log_4 X$

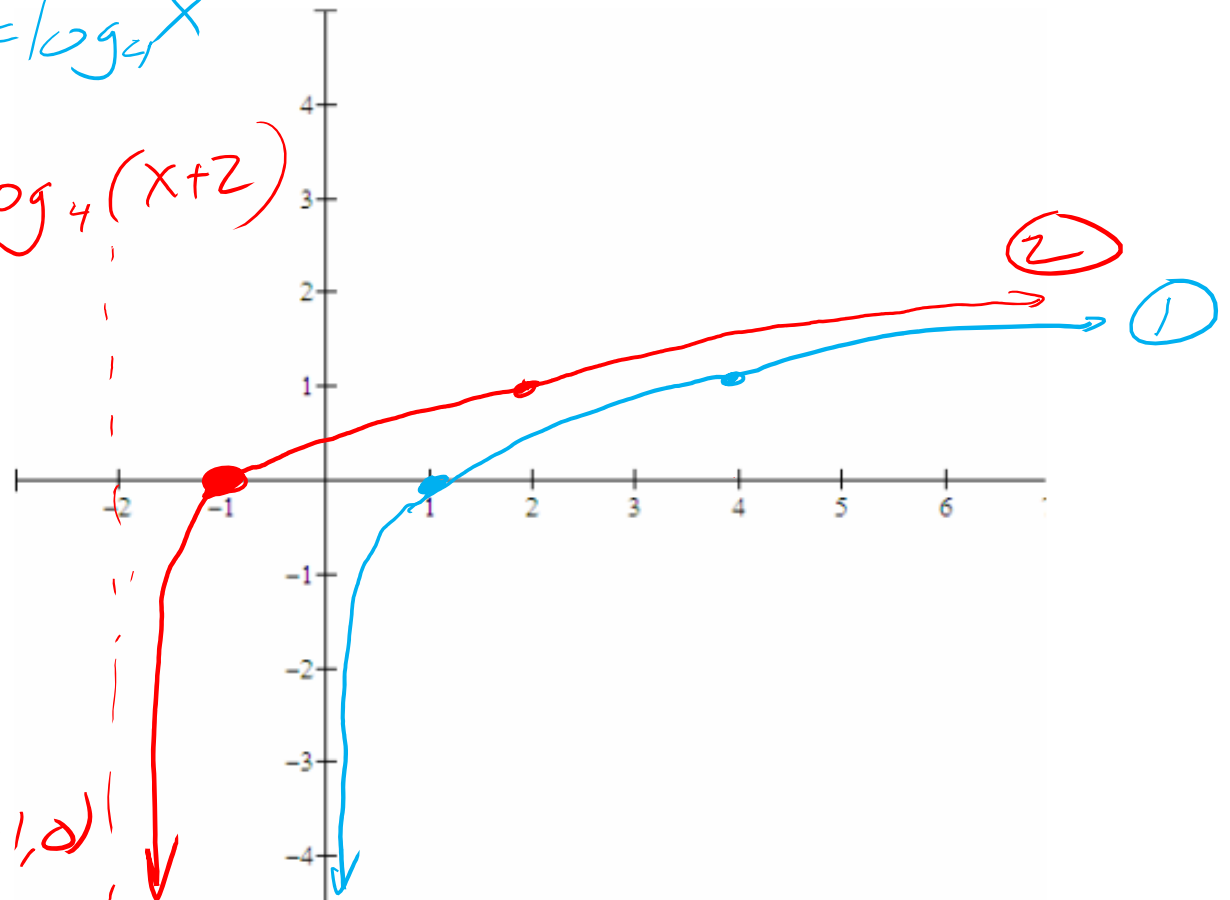
② Left 2 : $f(x) = \log_4(x + 2)$

Domain: $x + 2 > 0$
 $x > -2$
 $(-2, \infty)$

Range: $(-\infty, \infty)$

Asymptote: $x = -2$

Key Point: $(1, 0) \rightarrow (-1, 0)$



Example 7: Sketch the graph of $f(x) = -\ln(x - 1) + 1$. State the domain, range, asymptote and key point.

① Parent Function: $y = \ln x$

② Right 1: $y = \ln(x - 1)$

③ X-axis refl: $y = -\ln(x - 1)$

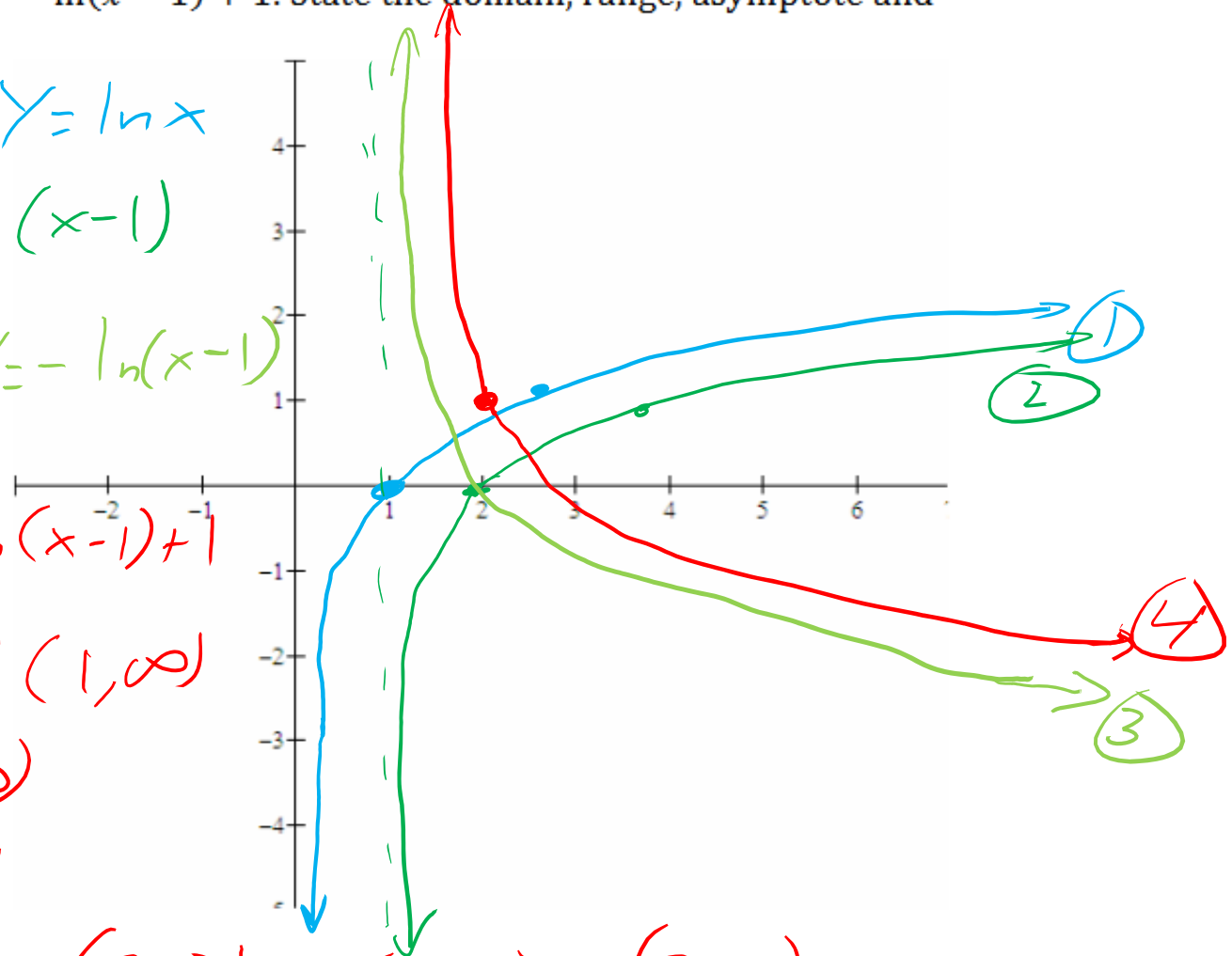
④ Up 1: $f(x) = -\ln(x - 1) + 1$

Domain: $x - 1 > 0$
 $x > 1$ $(1, \infty)$

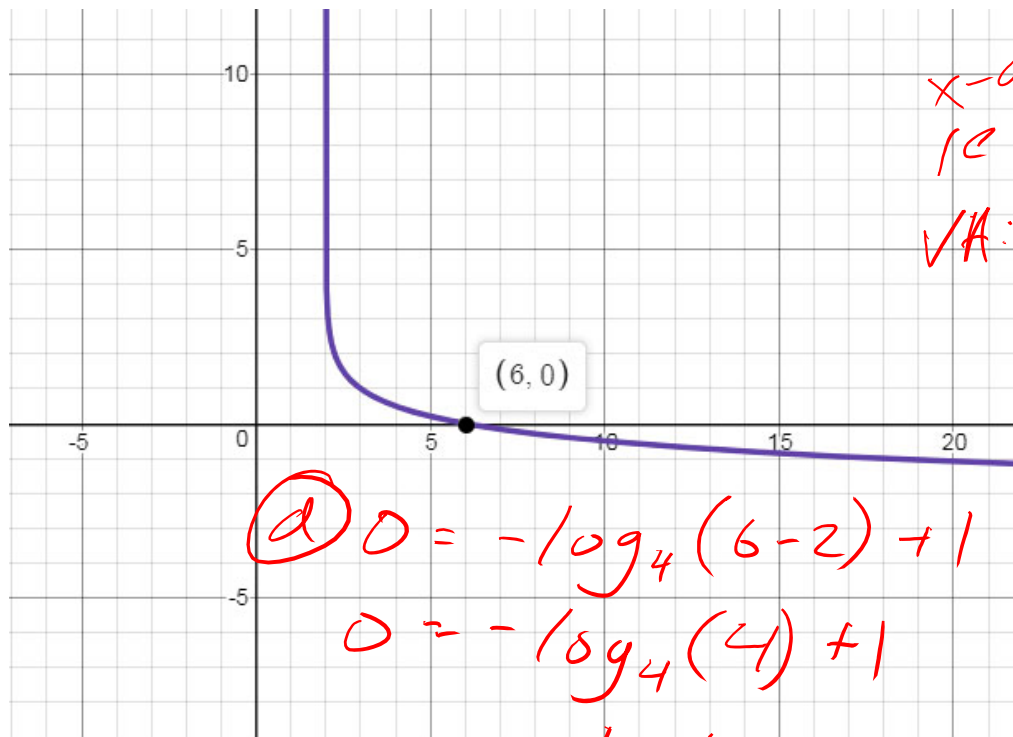
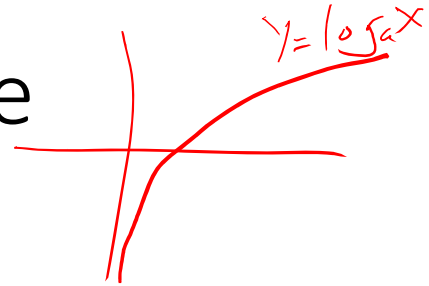
Range: $(-\infty, \infty)$

Asymptote: $x = 1$

Key Point: $(1, 0) \rightarrow (2, 0) \rightarrow (2, 1)$



Determine which of the following is the illustrated function:



*x-axis
refl.
VA: x=2*

~~a. $f(x) = \log_4(x + 2) - 1$~~

~~b. $f(x) = \log_4(x - 2) - 1$~~

~~c. $f(x) = -\log_4(x + 2) + 1$~~

d. $f(x) = -\log_4(x - 2) + 1$

e. $f(x) = -\log_5(x - 2) - 1$

f. $f(x) = -\log_5(x - 2) + 1$

d $0 = -\log_4(6-2) + 1$

$0 = -\log_4(4) + 1$

$0 = -1 + 1$

$0 = 0$