# MATH 1314

Section 5.3

# Logarithmic Functions

The exponential function is 1-1; therefore, it has an inverse function. The inverse function of the exponential function with base $a$ is called the **logarithmic function with base $a$**.

For $x > 0$ and $a > 0$ and $a$ not equal to 1, $y = \log_a x$ is equivalent $a^y = x$

The function $f(x) = \log_a x$ is the **logarithmic function with base $a$**

The **common logarithm** is the logarithm with base 10. We denote this as $\log_{10} x = \log x$
The **natural logarithm** is the logarithm with base e. We denote this as $\log_e x = \ln x$

You will find both of these logarithms on a scientific calculator.

Note: We do not typically write either $\log_{10} x$ or $\log_e x$.

**Example 1:** Write each equation in its equivalent exponential form.

a. $3 = \log_6 x$

b. $2 = \log_a 64$

c. $\log_3 27 = 3$

d. $\log 100000 = 5$

e. $\ln \frac{1}{e^2} = -2$

**Example 2:** Write each equation in its equivalent logarithmic form.

a. $4^3 = 64$

b. $2^6 = 64$

c. $e^x = 25$

d. $10^x = 1000$

**Example 3:** Evaluate, if possible.

$\log_6 36$

$\log_2 \frac{1}{8}$

$\log_5 125$

$\log_3\left(\sqrt[3]{81}\right)$

$\log_5 \sqrt[4]{125}$

$\log 100$

$\log_4 2$

$\log 0.001$

$\log_4 (-2)$

# Inverse Property of Logarithms

For $a > 0$ and $a \neq 1$

1. $\log_a a^x = x$

2. $a^{\log_a x} = x$

**Example 4:** Evaluate.

a. $\log_{14} 14^3$

b. $5^{\log_5 34}$

c. $e^{\ln 32}$

d. $\log_{47} 47^{\pi}$

e. $\log_{-2}(-8)$

f. $\log_5 5^{-3}$

g. $6^{\log_6(-7)}$

h. $\ln e^{-0.02}$

i. $\ln_4(4e)^5$

j. $\log_2 2^{-5}$

k. $2^{\log_2(-5)}$

l. $3^{5\log_3 7}$

Recall that for $x > 0$ (and $a > 0$ and $a$ not equal to 1), we have $f(x) = \log_a x$. So the domain of $f(x) = \log_a x$ consist of all $x$ for which $x > 0$.

**Example 5:** Find the domain.

a. $f(x) = \log_2(x - 2)$

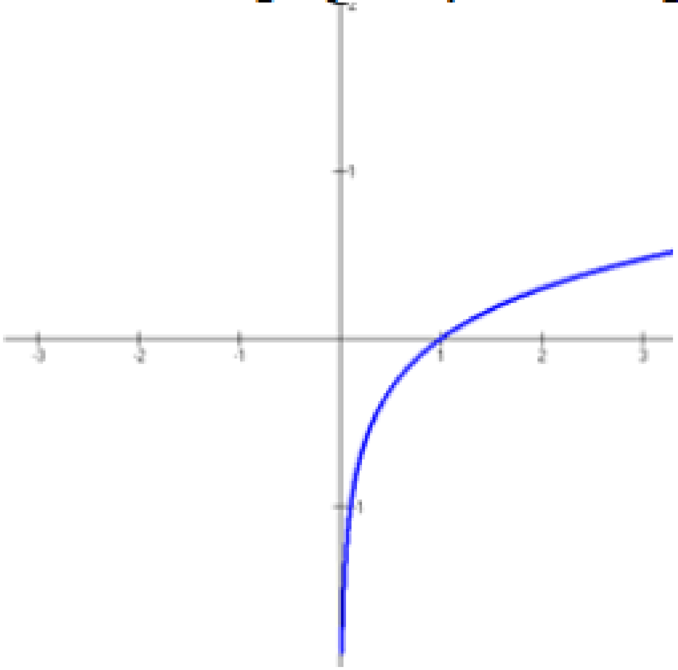
b. $f(x) = \ln(7 - 2x)$

c. $f(x) = \log(x^2 + 1)$

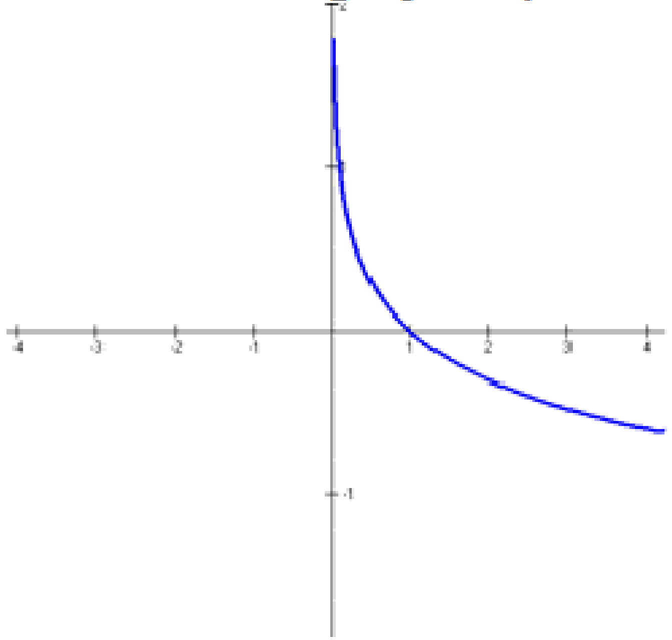
## Characteristics of the Graphs of Logarithmic Functions of the Form $f(x) = \log_a x$

1. The $x$-intercept is (1, 0) and there is no $y$-intercept.
2. The $y$-axis is a vertical asymptote.
3. The domain is all positive real numbers.
4. The range is all real numbers.

If $a > 1$, the graph of $f(x) = \log_a x$ looks like:

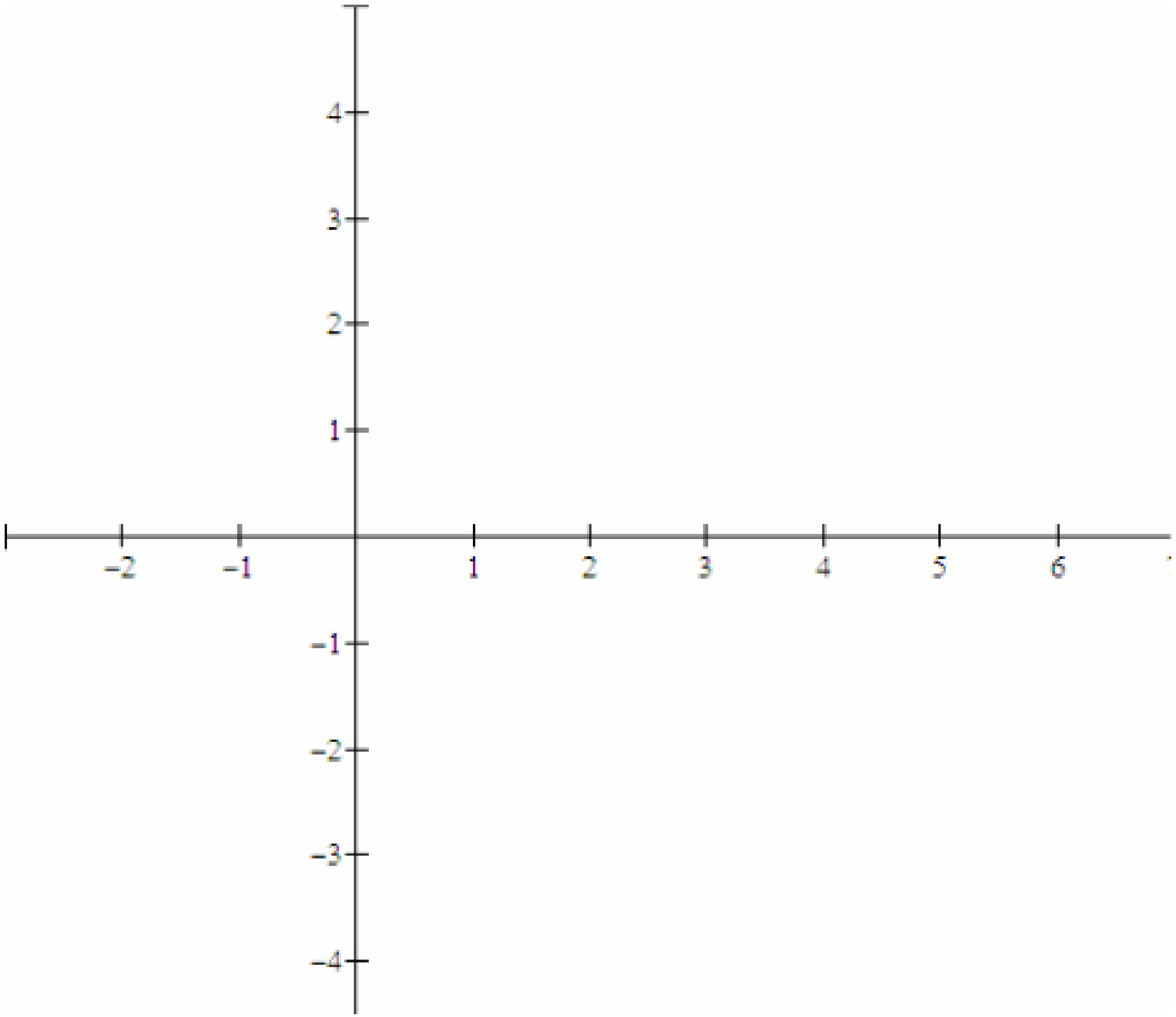


If $0 < a < 1$, the graph of $f(x) = \log_a x$ looks like:

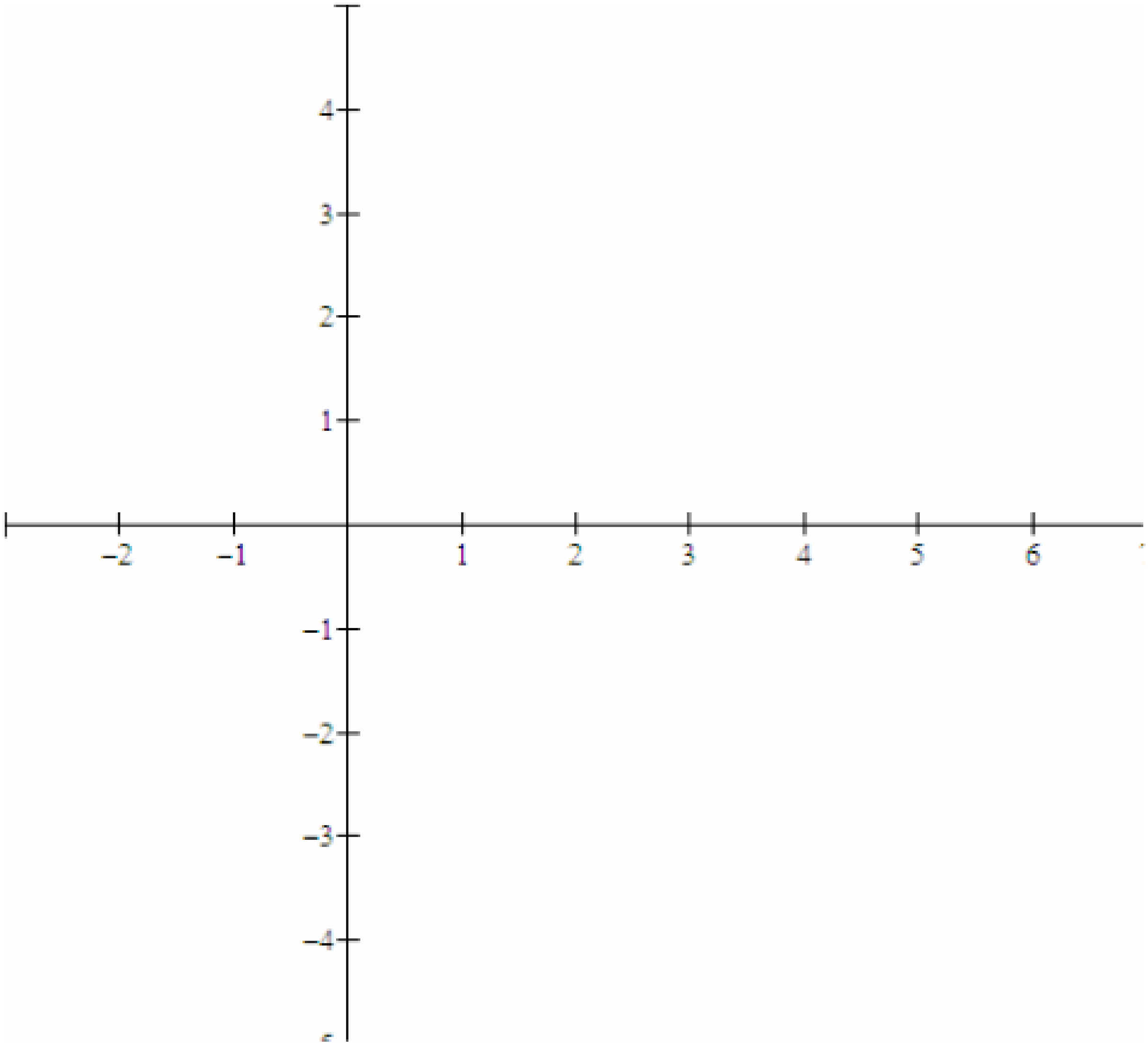


Note: If a logarithmic function is translated to the left or to the right, the vertical asymptote is shifted by the amount of the horizontal shift.

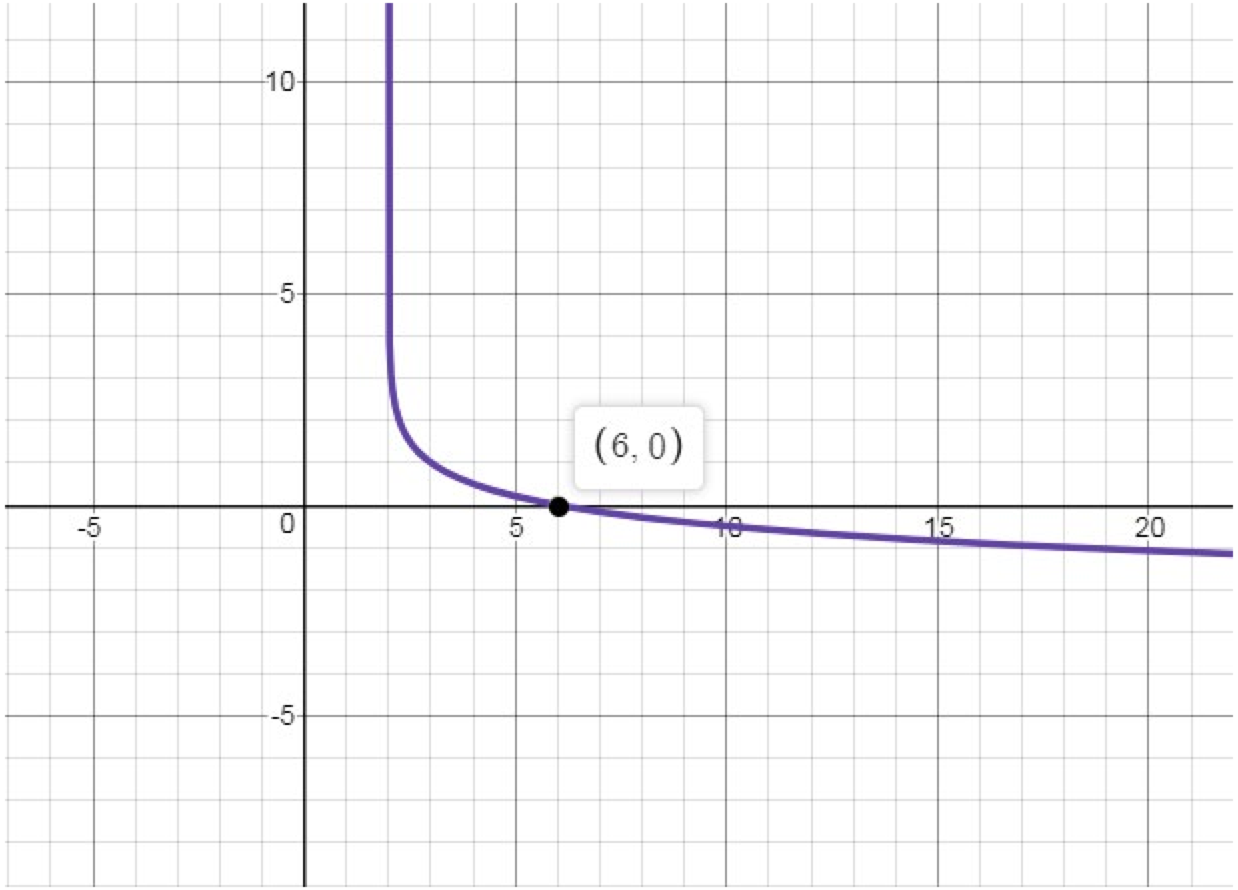
**Example 6:** Sketch the graph of $f(x) = \log_4(x + 2)$. State the domain, range, asymptote and key point.



**Example 7:** Sketch the graph of $f(x) = -\ln(x-1) + 1$. State the domain, range, asymptote and key point.



# Determine which of the following is the illustrated function:




a. $f(x) = \log_4(x + 2) - 1$
b. $f(x) = \log_4(x - 2) - 1$
c. $f(x) = -\log_4(x + 2) + 1$
d. $f(x) = -\log_4(x - 2) + 1$
e. $f(x) = -\log_5(x - 2) - 1$
f. $f(x) = -\log_5(x - 2) + 1$