MATH 1314

Section 5.4
In case you missed one:
Extra Popper:
Popper 34: Choice A for Questions 1 - 10

Properties of Logarithms

Bases must be $a>0,\ a\neq 1$

You will sometimes be asked to rewrite logarithmic expressions in either an expanded or contracted form. To do this, you will use the Laws of Logarithms. You must know all 8 of the Laws of Logarithms.

Laws of Logarithms
If m, n and a are positive numbers, $a \neq 1$, then

1. $\log_a mn = \log_a m + \log_a n$
2. $\log_a \frac{m}{n} = \log_a m - \log_a n$
3. $\log_a m^n = n\log_a m$
4. $\log_a 1 = 0$

$\log_8 1 = 0$

5. $\log_a a = 1$

(5) $\log_3 3 = 1$

6. $\log_a a^x = x$
7. $a^{\log_a x} = x$
8. $\log_a m = \frac{\log m}{\log a}$ (change of bases formula)

$\log_5 8 = \frac{\log_8 8}{\log_8 5} = \frac{1}{\log_8 5}$

(1) $\log_2(5x) = \log_2 5 + \log_2 x$

(2) $\ln\left(\frac{x}{3}\right) = \ln x - \ln 3$

(3) $\log(x^5) = 5\log x$

(6) $\log_4 4^x = x$

(7) $e^{\ln x} = x$

These properties are true for logs of any base, including common logs and natural logs.

Example 1: Rewrite each of the following expressions in a form that has no logarithms of a product, quotient or power. (All numerators will be positive logs; All denominator values will be negative logs.)

a. $\log\left(\frac{3x}{y}\right)$ = $\log(3x) - \log y = \log 3 + \log x - \log y$

b. $\ln(a^3bc)$ = $\ln(a^3) + \ln(b) + \ln(c) = 3\ln a + \ln b + \ln c$

c. $\log_4\left(\frac{4}{x}\right)$ = $\log_4 4 - \log_4 x = 1 - \log_4 x$

Example 1: Rewrite each of the following expressions in a form that has no logarithms of a product, quotient or power.

d. $\log\sqrt{xy} = \log(xy)^{1/2} = \frac{1}{2}\log(xy) = \frac{1}{2}(\log x + \log y) = \frac{1}{2}\log x + \frac{1}{2}\log y$

e. $\ln\left(\frac{\sqrt[3]{x+4}}{(x+2)^4(x-5)^3}\right) = \ln\sqrt[3]{x+4} - \ln(x+2)^4 - \ln(x-5)^3$

$= \frac{1}{3}\ln(x+4) - 4\ln(x+2) - 3\ln(x-5)$

f. $\log\sqrt{\frac{(x-1)(x+2)^3}{x^2(x-2)}} = \frac{1}{2}\left(\log(x-1) + \log(x+2)^3 - \log x^2 - \log(x-2)\right)$

$= \frac{1}{2}\left(\log(x-1) + 3\log(x+2) - 2\log x - \log(x-2)\right)$

$= \frac{1}{2}\log(x-1) + \frac{3}{2}\log(x+2) - \log x - \frac{1}{2}\log(x-2)$

Popper 31:

Example 2: Express each as a single logarithm:

1. $\log x - \underline{3} \log y = \log x - \log y^3 = \log\left(\frac{x}{y^3}\right)$

a. $\log(x - y^3)$ b. $\log\left(\frac{x}{y^3}\right)$ c. $\log_3(x - y)$ d. $\log\left(\frac{x}{y}\right)^3$

2. $2 \ln x + 3 \ln(x + 2) - 5 \ln y = \ln x^2 + \ln(x+2)^3 - \ln y^5 = \ln\left(\frac{x^2(x+2)^3}{y^5}\right)$

a. $\ln\left(\frac{x^5+8x^2}{y^5}\right)$ b. $\ln\left(\frac{(x^2+2x)^5}{5y}\right)$ c. $\ln\left(\frac{x+2}{xy}\right)^5$ d. $\ln\left(\frac{x^2(x+2)^3}{y^5}\right)$

3. $\frac{1}{2}\ln(x + 2) - 3\ln(x^3 + 1) = \ln(x+2)^{1/2} - \ln(x^3+1)^3 = \ln \frac{\sqrt{x+2}}{(x^3+1)^3}$

a. $\ln\left(\frac{\sqrt{x+2}}{(x^3+1)^3}\right)$ b. $\ln\sqrt{\frac{x+2}{(x^3+1)^3}}$ c. $\ln\left(\frac{\sqrt{x+2}}{x^9+1}\right)$ d. $\ln\left(\frac{x+2}{2(x^3+1)^3}\right)$

Example 2: Express each as a single logarithm:

d. $-2\log_4(x-5) - \log_4(x+1) + 2\log_4(x^2+1)$

$-\log_4(x-5)^2 - \log_4(x+1) + \log_4(x^2+1)^2 = \log_4 \dfrac{(x^2+1)^2}{(x-5)^2(x+1)}$

e. $\ln 18 - \ln 2 = \ln \dfrac{18}{2} = \ln 9 = \ln 3^2 = 2\ln 3$

Popper 31 (continued) Rewrite each as sums so that each logarithm contains a prime number. Simplify as much as possible

4. $\ln 72 = \ln(2^3 \cdot 3^2) = \ln 2^3 + \ln 3^2 = 3\ln 2 + 2\ln 3$

a. ln(2) ln(3) b. 6ln(2)ln(3) c. 3ln(2) + 2ln(3) d. 2ln(36)

5. $\log_2 96 = \log_2(2^5 \cdot 3) = \log_2 2^5 + \log_2 3 = 5\underbrace{\log_2 2}_{=1} + \log_2 3 = 5 + \log_2 3$

a. $5 + \log_2 3$ b. $\log_2 24 + 2$ c. $5\log_2 2 + \log_2 3$ d. $\log_2 24 + \log_2 4$

6. $\log_7 147 - \log_7 3 = \log_7 \frac{147}{3} = \log_7 49 \rightarrow 7^{\square} = 49$

$\frac{147}{3} = \frac{120+27}{3} = \frac{120}{3} + \frac{27}{3} = 40 + 9 = 49$

a. 49 b. $\log_7 144$ c. 7 d. 2

7. $\log_4 32 + \log_4 2 = \log_4(32 \cdot 2) = \log_4 64 \rightarrow 4^{\boxed{3}} = 64$

$\begin{array}{r} {}^{2}16 \\ \times\ 4 \\ \hline 64 \end{array}$

a. 2 b. 3 c. 4 d. 8

72

36 × 2

6 × 6

3 × 2 × 3 × 2

$72 = 2^3 \times 3^2$

96

48 × 2

24 × 2

4 × 6

2 × 2 × 3 × 2

$96 = 2^5 \times 3$

Example 4: Use the change of bases formula to solve $\log_8 12 = x$and write in simplest form. Then use a calculator to evaluate to the nearest thousandth.

$\log_8 12 = x$

$\log_a m = \dfrac{\log_n m}{\log_n a}$

change to base 2

$\log_8 12 = \dfrac{\log_2 12}{\log_2 8} = \dfrac{\log_2(4\cdot 3)}{\log_2(2^3)} = \dfrac{\log_2 4 + \log_2 3}{3\log_2 2}$

$\log_2 4 \to 2^{\square} = 4 \to 2$

$\log_2 2 = 1$

$= \dfrac{2 + \log_2 3}{3} = \boxed{\dfrac{2}{3} + \log_2 \sqrt[3]{3}}$

$\dfrac{2}{3} + \dfrac{\log_2 3}{3}$

$\hookrightarrow \frac{1}{3}\log_2 3$

Example 5: Simplify each.

a. $\log_4 16^3 = \log_4 (4^2)^3 = \cancel{\log_4} 4^6 = 6$

b. $7^{3\log_7 3} = 7^{\log_7 3^3} = 3^3 = 27$

If it was $7^{\log_7 3}$ we can use inverse properties