MATH 1314

Section 5.4

Properties of Logarithms

You will sometimes be asked to rewrite logarithmic expressions in either an expanded or contracted form. To do this, you will use the Laws of Logarithms. You must know all 8 of the Laws of Logarithms.

Laws of Logarithms

If m, n and a are positive numbers, $a \neq 1$, then

1. $\log_a mn = \log_a m + \log_a n$
2. $\log_a \frac{m}{n} = \log_a m - \log_a n$
3. $\log_a m^n = n \log_a m$
4. $\log_a 1 = 0$
5. $\log_a a = 1$
6. $\log_a a^x = x$
7. $a^{\log_a x} = x$
8. $\log_a m = \frac{\log m}{\log a}$ (change of bases formula)

These properties are true for logs of any base, including common logs and natural logs.

Example 1: Rewrite each of the following expressions in a form that has no logarithms of a product, quotient or power.

a. $\log\left(\frac{3x}{y}\right)$

b. $\ln\left(a^3bc\right)$

c. $\log_4\left(\frac{4}{x}\right)$

Example 1: Rewrite each of the following expressions in a form that has no logarithms of a product, quotient or power.

d. $\log \sqrt{xy}$

e. $\ln\left(\frac{\sqrt[3]{x+4}}{(x+2)^4(x-5)^3}\right)$

f. $\log \sqrt{\frac{(x-1)(x+2)^3}{x^2(x-2)}}$

Example 2: Express each as a single logarithm:

$\log x - 3 \log y$

$2 \ln x + 3 \ln(x + 2) - 5 \ln y$

$\frac{1}{2} \ln(x + 2) - 3 \ln(x^3 + 1)$

Example 2: Express each as a single logarithm:

d. $-2\log_4(x-5) - \log_4(x+1) + 2\log_4(x^2+1)$

e. $\ln 18 - \ln 2$

Example 3: Rewrite each as sums so that each logarithm contains a prime number.

Simplify as much as possible

a. $\ln 72$

b. $\log_2 96$

c. $\log_7 147 - \log_7 3$

d. $\log_4 32 + \log_4 2$

Example 4: Use the change of bases formula to solve $\log_8 12 = x$and write in simplest form. Then use a calculator to evaluate to the nearest thousandth.

Example 5: Simplify each.

a. $\log_4 16^3$

b. $7^{3\log_7 3}$