

MATH 1310

Section 5.5

Solving Exponential and Logarithmic Equations

We'll start with exponential equations.

An exponential equation is an equation in which the variable appears in the exponent. To solve these equations, isolate the exponential expression on one side of the equation, then take the logarithm of both sides of the equation to solve for the variable. You can use either natural logarithms or common logarithms. Read the directions carefully, as they may instruct you as to which to use.

$$\begin{array}{r} 3^x + 2 = 11 \\ -2 \quad -2 \\ \hline 3^x = 9 \end{array}$$

$$x = 2$$

$$(\text{since } 3^2 = 9)$$

$$\begin{array}{r} 3^x + 2 = 13 \\ -2 \quad -2 \\ \hline 3^x = 11 \end{array}$$

option 1: use Base in equation

$$3^x = 11$$

$$\log_3 3^x = \log_3 11$$

$$x = \log_3 11$$

$$\log_3 11 = \frac{\ln 11}{\ln 3}$$

(change of Base formula)

option 2: use Base e

$$3^x = 11$$

$$\ln 3^x = \ln 11$$

$$\frac{x \cdot \ln 3}{\ln 3} = \frac{\ln 11}{\ln 3}$$

$$x = \frac{\ln 11}{\ln 3}$$

Example 1: Solve for x : $5^{3x} = 9$. (a) Give the exact value using natural logarithms. (b) Use a calculator to approximate the solution to the nearest thousandth. (c) Rewrite the results from part (a) so that each logarithm contains a prime number.

$$5^{3x} = 9$$

$$\ln 5^{3x} = \ln 9$$

$$\frac{\cancel{3x} \cdot \ln 5}{\cancel{3} \ln 5} = \frac{\ln 9}{3 \ln 5}$$

$$x = \frac{\ln 9}{3 \ln 5} = \frac{1}{3} \cdot \frac{\ln 9}{\ln 5}$$

$$x \approx 0.455$$

$$x = \frac{1}{3} \cdot \frac{\ln 9}{\ln 5} = \frac{1}{3} \cdot \frac{\ln 3^2}{\ln 5} = \frac{2}{3} \frac{\ln 3}{\ln 5}$$

change of Base:

$$x = \frac{2}{3} \log_5 3$$

Popper 26:

Example 2: Solve for x : $4e^{(x+5)} + 5 = 7$. (a) Give the exact value using natural logarithms. (b) Use a calculator to approximate the solution to the nearest thousandth. (c) Rewrite the results from part (a) so that each logarithm contains a prime number.

1. Simplify the equation:

a. $e^{(x+5)} = 2$

b. $e^{(x+5)} = \frac{1}{2}$

c. $e^{(x+5)} = 3$

d. $e^{(x+5)} = 48$

2. Solve the equation:

a. $x = \ln(\frac{1}{2}) + 5$

b. $x = \log(\frac{1}{2}) - 5$

c. $x = \ln(\frac{1}{2}) - 5$

d. $x = \ln(\frac{1}{2})$

3. Approximate the answer (Calculator):

a. $x \approx 4.317$

b. $x \approx -5.301$

c. $x \approx -5.693$

d. $x \approx -0.693$

4. Simplify the logarithmic answer (Question 2):

a. $x = \ln(-3)$

b. $x = \ln(-4.5)$

c. $x = \ln(\frac{1}{2}) - 5$

d. $x = -\ln(2) - 5$

$$4e^{x+5} + 5 = 7$$

$$4e^{x+5} = 2$$

$$e^{x+5} = \frac{1}{2}$$

$$d. x = \ln(\frac{1}{2}) \quad \ln e^{x+5} = \ln(\frac{1}{2})$$

$$x+5 = \ln(\frac{1}{2})$$

$$x = \ln(\frac{1}{2}) - 5$$

$$x = \ln(1) - \ln(2) - 5$$

$$x = 0 - \ln(2) - 5$$

$$d. x = -\ln(2) - 5 \quad x = -\ln(2) - 5$$

$$4e^{x+5} + 5 = 7$$

Example 3: Solve for x : $e^{2x} - 9e^x + 20 = 0$. (a) Give the exact value using natural logarithms. (b) Use a calculator to approximate the solution to the nearest thousandth. (c) Rewrite the results from part (a) so that each logarithm contains a prime number.

$$e^{2x} - 9e^x + 20 = 0$$

$$x = \ln 4$$

$$x = \ln 5$$

$$u = e^x, \quad u^2 = (e^x)^2 = e^{2x}$$

$$x \approx 1.386$$

$$x \approx 1.609$$

$$u^2 - 9u + 20 = 0$$

$$x = \ln 4$$

$$(u - 4)(u - 5) = 0$$

$$x = \ln 2^2$$

$$u - 4 = 0$$

$$u - 5 = 0$$

$$x = 2 \ln 2$$

$$u = 4$$

$$u = 5$$

$$e^x = 4$$

$$e^x = 5$$

$$\ln e^x = \ln 4$$

$$\ln e^x = \ln 5$$

$$x = \ln 4$$

$$x = \ln 5$$

Popper 26 Continued:

Solve the following equation: $3(2)^{x-7} + 6 = 12$

5. Simplify the equation:

a. $(2)^{x-7} = 2$

b. $(2)^{x-7} = 6$

c. $(2)^{x-7} = -3$

d. $(2)^{x-7} = 12$

6. Solve in base 2:

a. $\log_2 2 - 7$

b. $\log_2 2 + 13$

c. $\log_2 2$

d. $\log_2 2 + 7$

7. Solve in base e:

a. $\ln 2 + 7$

b. $\ln 2 - 7$

c. $\frac{\ln 2}{\ln 2} + 7$

d. $\frac{\ln 7}{\ln 2} + 2$

8. Simplify either answer:

a. $7/2$

b. $2/7$

c. 5

d. 8

$$3(2)^{x-7} + 6 = 12$$

$$\frac{3(2)^{x-7}}{3} = \frac{6}{3}$$

$$2^{x-7} = 2^1$$

$$\begin{array}{r} x-7 = 1 \\ +7 \quad +7 \\ \hline x = 8 \end{array}$$

Solve in base e:

$$\ln 2^{x-7} = \ln 2$$

$$\frac{(x-7) \ln 2}{\ln 2} = \frac{\ln 2}{\ln 2}$$

$$x-7 = \frac{\ln 2}{\ln 2}$$

$$x = \frac{\ln 2}{\ln 2} + 7$$

simplify:

$$x = 1 + 7 = 8$$

Solve in base 2:

$$\log_2 2^{x-7} = \log_2 2$$

$$x-7 = \log_2 2$$

$$x = \log_2 2 + 7$$

simplify:

$$x = 1 + 7 = 8$$

Example 4: Solve for x : $25^{3x-2} = \frac{1}{(\sqrt{125})^x}$ (a) Give the exact value using natural

logarithms. (b) Use a calculator to approximate the solution to the nearest thousandth. (c) Rewrite the results from part (a) so that each logarithm contains a prime number.

$$25^{3x-2} = \frac{1}{\sqrt{125}}^x$$

$$(5^2)^{(3x-2)} = (5^{-\frac{3}{2}})^x$$

$$25^{3x-2} = \left(\frac{1}{\sqrt{125}}\right)^x$$

$$5^{6x-4} = 5^{-\frac{3x}{2}}$$

$$(5^2)^{3x-2} = \left(\frac{1}{\sqrt{5^3}}\right)^x$$

$$2 \cdot (6x - 4 = \frac{-3x}{2}) \cdot 2$$

$$= \left(\frac{1}{(5^3)^{\frac{1}{2}}}\right)^x$$

$$\begin{array}{r} 12x - 8 = -3x \\ -12x \qquad -12x \\ \hline \end{array}$$

$$= \left(\frac{1}{5^{3/2}}\right)^x$$

$$\begin{array}{r} -8 = -15x \\ -15 \qquad -15 \\ \hline \end{array}$$

$$(5^2)^{3x-2} = (5^{-3/2})^x$$

$$8/15 = x$$

$$x = 8/15$$

$$3^{x+5} = 5^x$$

$$\ln 3^{x+5} = \ln 5^x$$

$$(x+5) \ln 3 = x \ln 5$$

$$\cancel{x \ln 3} + 5 \ln 3 = x \ln 5$$

[Think of $3x + 5 = 5x$]

$$\cancel{-x \ln 3} \qquad -x \ln 3$$

$$5 \ln 3 = x \ln 5 - x \ln 3$$

$$5 \ln 3 = x (\ln 5 - \ln 3)$$

$$5 \ln 3 = x (\ln 5 - \ln 3)$$

$$\frac{5 \ln 3}{\ln 5/3} = x \frac{\ln(5/3)}{\ln 5/3}$$

$$\boxed{\frac{5 \ln 3}{\ln(5/3)} = x}$$

$$5 \cdot \frac{\ln 3}{\ln(5/3)} = x \quad (\text{change of base})$$

$$\boxed{5 \log_{(5/3)} 3 = x}$$