# MATH 1314

Section 6.1, 6.2

# Solving 2 x 2 Linear Systems

To solve a system of two linear equations

$$\begin{Bmatrix} ax + by = c \\ dx + ey = f \end{Bmatrix}$$

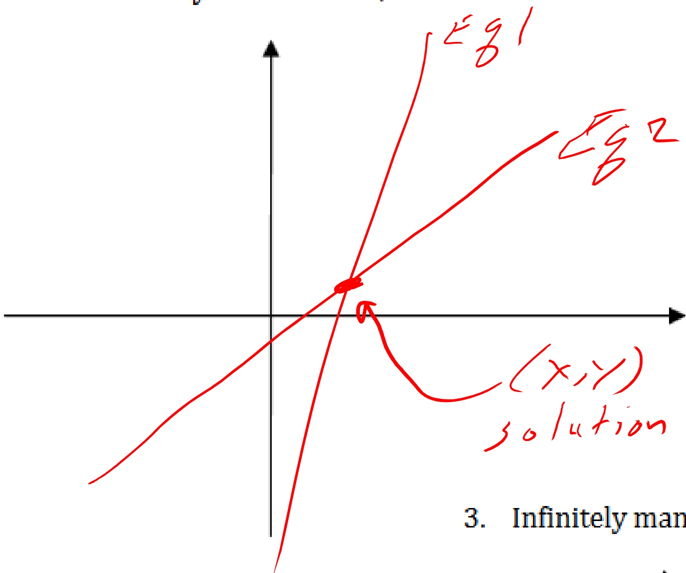
means to find values for $x$ and $y$ that satisfy both equations.
The system will have exactly one solution, no solution, or infinitely many solutions.

$$\begin{Bmatrix} 2x + 3y = 5 \\ x - y = -2 \end{Bmatrix}$$

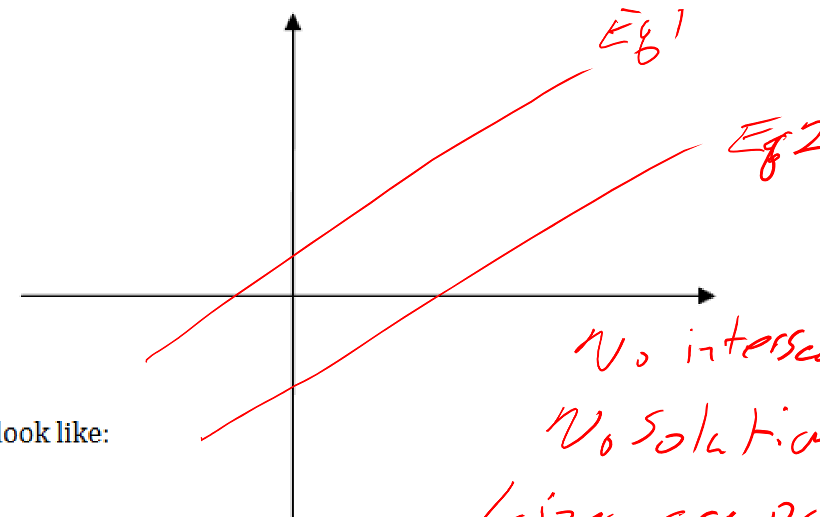
Goal: $(x, y)$ coordinate point as your answer.

solve the system for $x$

1. Exactly one solution, will look like:




2. No solution, will look like:



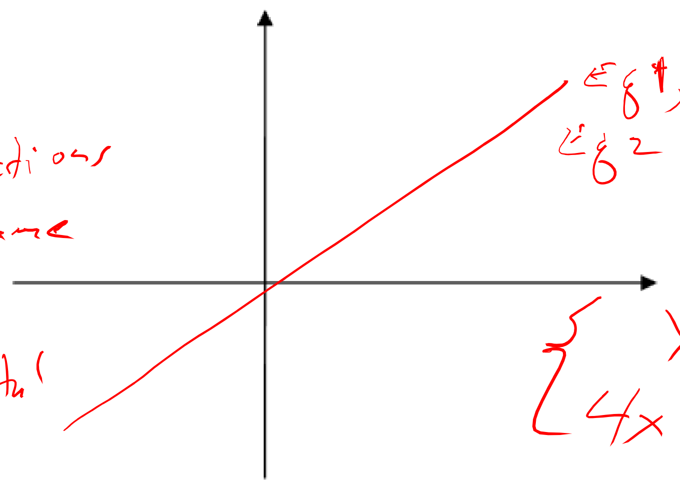

No intersection

No Solution.

Lines are parallel

(Disjoint System)

3. Infinitely many solutions, will look like:




Both equations have the same graph (coincidental lines)

$$\begin{cases} y = 2x - 3 \\ 4x - 2y = 6 \end{cases}$$

$$4x - 2y = 6$$
$$-4x \qquad -4x$$
$$\frac{-2y}{-2} = \frac{-4x}{-2} + \frac{6}{-2}$$

$y = 2x - 3$

Example 1: Solve the following systems of linear equations by the substitution method.

$2x - y = 5$
$5x + 2y = 8$

Eq 1: $2x - y = 5$
$-2x \qquad -2x$
$-y = -2x + 5$
$-1 \qquad -1 \quad -1$
$* \; y = 2x - 5 \; *$

Eq 2: $5x + 2y = 8$
$5x + 2(2x-5) = 8$
$5x + 4x - 10 = 8$
$9x - 10 = 8$

$9x - 10 = 8$
$+10 \quad +10$
$9x = 18$
$\frac{9x}{9} = \frac{18}{9}$
$\boxed{x = 2}$

Check (Eq 2)
$5x + 2y = 8$
$5(2) + 2(-1) = 8$
$10 - 2 = 8$
$8 = 8$ ✓

① Solve either Eq 1 or Eq 2 for either x or y.
② Substitute your answer into the other equation
③ Solve the resulting equation
④ Substitute into any x, y equation and solve.

$y = 2x - 5$
$y = 2(2) - 5$
$y = 4 - 5 = -1$
$\boxed{y = -1}$
$\boxed{(2, -1)}$

Example 2 : Solve the following systems of linear equations by the substitution method

$x - 2y = 3$
$2x - 4y = 7$

Eq1: $x - \cancel{2y} = 3$
$\cancel{+2y} \quad +2y$

$\boxed{x = 2y + 3}$

Eq2: $2x - 4y = 7$

$2(2y + 3) - 4y = 7$

$4y + 6 - 4y = 7$

$\cancel{4y} + 6 - \cancel{4y} = 7$

$6 = 7$ (Ruh, Oh)

- All variables have cancelled.
- Resulting Equation is False.

$\boxed{\text{No Solution}}$

Example 3: Solve the following systems by the Elimination Method.

$10\times(2x + 3y = -16)\times 10 \rightarrow 20x + 30y = -160$

$3\times(5x - 10y = 30)\times 3 \rightarrow 15x - \cancel{30y} = 90$

$\frac{35x}{35} = \frac{-70}{35}$

$x = -2$

① Multiply one or both equations by a constant so that either x or y are opposites.

② Add the two equations.

③ Solve the resulting equation

④ Substitute into one of the original equations to solve for other variable.

Check (Eq2)

$5(-2) - 10(-4) = 30$

$-10 + 40 = 30$

$30 = 30$ ✓

Eq1: $2(-2) + 3y = -16$

$-4 + 3y = -16$

$+4 \qquad\qquad +4$

$\frac{3y}{3} = \frac{-12}{3} \rightarrow y = -4$

$\boxed{(-2, -4)}$

Example 4: Solve the following systems by the Elimination Method.

$$x + 4y = 10 \longrightarrow \cancel{x} + \cancel{4y} = \cancel{10}$$

$$\left(\frac{1}{2}x + 2y = 5\right) \cdot -2 \longrightarrow -\cancel{x} - \cancel{4y} = -\cancel{10}$$

$$0 = 0$$

- All variables drop out of the system.
- Resulting equation is correct

Infinitely Many Solutions

$$x^2 + y = 120$$
$$x^2 - y = 80$$

Elimination since y's are already opposites

$$\frac{2x^2}{2} = \frac{200}{2}$$

$$\sqrt{x^2} = \sqrt{100}$$

$$x = \pm 10$$

using $x = +10$

$$x^2 + y = 120$$
$$10^2 + y = 120$$
$$100 + y = 120$$
$$-100 \qquad -100$$
$$y = 20$$

$$\boxed{(10, 20)}$$

check: $10^2 - 20 = 80$

$100 - 20 = 80$ ✓

using $x = -10$

$$x^2 + y = 120$$
$$(-10)^2 + y = 120$$
$$100 + y = 120$$
$$-100 \qquad -100$$
$$y = 20$$

$$\boxed{(-10, 20)}$$

$(-10)^2 - 20 = 80$

$100 - 20 = 80$ ✓

# Application Question:

Two integers have a sum of 35. The difference when subtracting the larger from twice the smaller is 10. What are the two numbers?

Larger: $x = 20$

smaller: $y = 15$

$$\boxed{20, 15}$$

$x + y = 35 \rightarrow x + y = 35$

$2y - x = 10 \rightarrow -x + 2y = 10$

$3y = 45$

$\frac{3y}{3} = \frac{45}{3}$

$y = 15$

Eq 1: $x + y = 35$

$x + 15 = 35$

$-15 \quad -15$

$x = 20$

# Popper 3

A parking garage for a concert venue needs to report to the local safety board the percent of motorcycles that were parked in the garage on the night a certain event. That night, the garage sold parking passes to a total of 340 vehicles (included cars: charged $10 each, seating 4; motorcycles: charged $5 each, seating 1; and busses: charged $25 each, seating 20). The parking garage earned $3275 for selling parking tags that evening, and was able to accommodate 1355 ticket holders. What percent of vehicles parked were motorcycles?

E 1. Create an equation *[Eq 1]* for the total number of vehicles parked? $c+m+b=340$

B 2. Create an equation *[Eq 2]* for the total money earned by the garage?

D 3. Create an equation *[Eq 3]* for the total number of ticket holders?

Answer choices for Questions 1, 2, 3: (c: cars, m: motorcycles, b: busses)

a. m+c+b=3275   b. 5m+10c+25b=3275   c. m+4c+20b=340

d. m+4c+20b=1355   e. m+c+b=340

#2 $10c+5m+25b=3275$   #3 $4c+m+20b=1355$

# Popper 3 continued:

$4c + m + 20b = 1355$
$-4c \quad -20b \quad -4c - 20b$
$m = 1355 - 4c - 20b$

$1355 \times 5 = 6775$

4. Solve *Eq 3* for m.

a. m=4c-20b-1355 b. m=1355-4c-20b c. m=1355-4c+20b

d. m=1335b-4c e. m=1355-24bc

5. Substitute your answer to Question 4 into *Eq 1* and *Eq 2* and simplify.

a. 1355-5c-21b=340; 6775-10c-75b=3275

b. 1355-5c-21b=340; 6525-10c-75b=3275

c. 1355-3c-19b=340; 6775-10c-75b=3275

d. 1355-3c-19b=340; 6525-10c-75b=3275

Eq 1: $m + b + c = 340$
$1355 - 4c - 20b + b + c = 340$
$1355 - 3c - 19b = 340$

Eq 2: $10c + 5m + 25b = 3275$
$10c + 5(1355 - 4c - 20b) + 25b = 3275$
$10c + 6775 - 20c - 100b + 25b = 3275$

6. Solve *Eq 2* for c.

a. c=350-7.5b b. c=350+7.5b c. c=-350+7.5b d. c=350-75b

$6775 - 10c - 75b = 3275$
$-6775 \quad +75b \quad -6775 + 75b$

$-10c = -3500 + 75b$
$\div -10 \quad \div -10$
$c = 350 - 7.5b$

$6775 - 10c - 75b = 3275$

# Popper 3 concluded:

$1355 - 3c - 19b = 340$

$1355 - 3(350 - 7.5b) - 19b = 340$

$1355 - 1050 + 22.5b - 19b = 340$

7. Substitute your answer to Question 6 into *Eq 1* and simplify:

a. ~~2405+3.5b=340~~  b. 305+41.5b=340

c. 305+3.5b=340  d. ~~2405+41.5b=340~~

8. Solve *Eq 1* for b.

$305 + 3.5b = 340$

$-305 \quad -305$

$\frac{3.5b}{3.5} = \frac{35}{3.5}$

$b = 10$

a. b=35  b. b=10  c. b=100  d. b=65  e. b=7

9. Solve for c and m.

a. ~~c=425, m=550~~  b. c=275, m=150  c. ~~c=315, m=55~~  d. c=275, m=55

10. What number should be reported to safety board, round to the nearest whole number, calculator is acceptable? (Look back at the original equation)

a. 55  b. 340  c. 16  d. 285  e. 41

$c = 350 - 7.5b$

$c = 350 - 7.5(10)$

$c = 350 - 75 = 275$

$\frac{\text{Motorcycles}}{\text{Total}} = \frac{55}{340} = \boxed{16}$

$m = 1355 - 4c - 20b$

$1355 - 4(275) - 20(10) = 1355 - 1100 - 200 = 55$