MATH 1314

Section 6.1, 6.2

Solving 2 x 2 Linear Systems

To solve a system of two linear equations

$$\begin{Bmatrix} ax + by = c \\ dx + ey = f \end{Bmatrix}$$

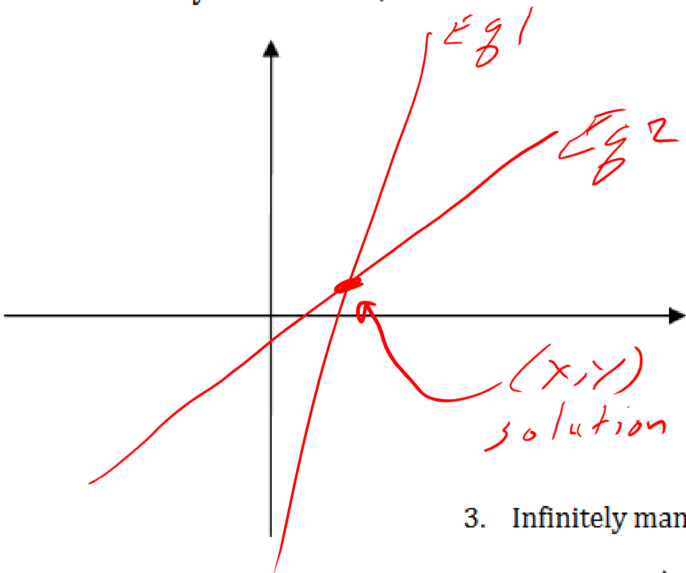
means to find values for x and y that satisfy both equations.
The system will have exactly one solution, no solution, or infinitely many solutions.

$$\begin{Bmatrix} 2x + 3y = 5 \\ x - y = -2 \end{Bmatrix}$$

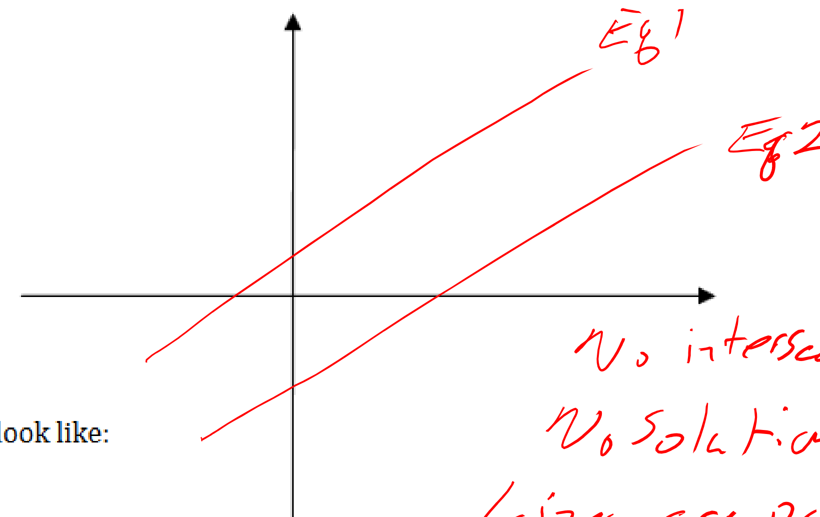
Goal: (x, y) coordinate point as your answer.

solve the system for x

1. Exactly one solution, will look like:



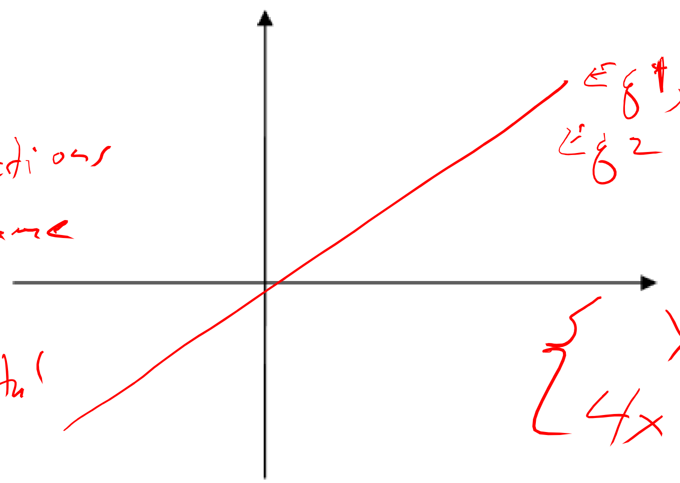
2. No solution, will look like:



No intersection
No Solution
Lines are parallel
(Disjoint System)

3. Infinitely many solutions, will look like:

Both equations have the same graph (coincidental lines)



$$\begin{cases} y = 2x - 3 \\ 4x - 2y = 6 \end{cases}$$

$$4x - 2y = 6$$
$$-4x \qquad -4x$$
$$\frac{-2y}{-2} = \frac{-4x}{-2} + \frac{6}{-2}$$

$y = 2x - 3$

Example 1: Solve the following systems of linear equations by the substitution method.

$2x - y = 5$
$5x + 2y = 8$

Eq 1: $2x - y = 5$
$-2x \qquad -2x$
$-y = -2x + 5$
$-1 \qquad -1 \quad -1$
* $\boxed{y = 2x - 5}$ *

Eq 2: $5x + 2y = 8$
$5x + 2(2x - 5) = 8$
$5x + 4x - 10 = 8$
$9x - 10 = 8$

$9x - 10 = 8$
$+10 \quad +10$
$9x = 18$
$\frac{9x}{9} = \frac{18}{9}$
$\boxed{x = 2}$

Check (Eq 2)
$5x + 2y = 8$
$5(2) + 2(-1) = 8$
$10 - 2 = 8$
$8 = 8$ ✓

① Solve either Eq 1 or Eq 2 for either x or y.
② Substitute your answer into the other equation
③ Solve the resulting equation
④ Substitute into any x, y equation and solve.

$y = 2x - 5$
$y = 2(2) - 5$
$y = 4 - 5 = -1$ $\quad \boxed{y = -1}$
$\boxed{(2, -1)}$

Example 2 : Solve the following systems of linear equations by the substitution method

$x - 2y = 3$
$2x - 4y = 7$

Eq 1: $x - \cancel{2y} = 3$
$\quad \cancel{+2y} \quad +2y$

$\boxed{x = 2y + 3}$

Eq 2: $2x - 4y = 7$

$2(2y + 3) - 4y = 7$

$4y + 6 - 4y = 7$

$\cancel{4y} + 6 - \cancel{4y} = 7$

$6 = 7$ (Ruh, Oh)

- All variables have cancelled.
- Resulting Equation is False.

$\boxed{\text{No Solution}}$

Example 3: Solve the following systems by the Elimination Method.

$$10\times(2x + 3y = -16)\times 10 \rightarrow 20x + 30y = -160$$
$$3\times(5x - 10y = 30)\times 3 \rightarrow 15x - 30y = 90$$

$$\frac{35x}{35} = \frac{-70}{35}$$

$$x = -2$$

Check (Eq 2)

$5(-2) - 10(-4) = 30$

$-10 + 40 = 30$

$30 = 30$ ✓

Eq 1: $2(-2) + 3y = -16$

$$-4 + 3y = -16$$
$$+4 \qquad\qquad +4$$

$$\frac{3y}{3} = \frac{-12}{3} \rightarrow y = -4$$

$$\boxed{(-2, -4)}$$

1. Multiply one or both equations by a constant so that either x or y are opposites.
2. Add the two equations.
3. Solve the resulting equation
4. Substitute into one of the original equations to solve for other variable.

Example 4: Solve the following systems by the Elimination Method.

$$x + 4y = 10 \longrightarrow \cancel{x} + \cancel{4y} = \cancel{10}$$

$$\left(\frac{1}{2}x + 2y = 5\right) \cdot -2 \longrightarrow -\cancel{x} - \cancel{4y} = -\cancel{10}$$

$$0 = 0$$

- All variables drop out of the system.
- Resulting equation is correct

Infinitely Many Solutions

$$x^2 + y = 120$$
$$x^2 - y = 80$$

Elimination since y's are already opposites

$$\frac{2x^2}{2} = \frac{200}{2}$$

$$\sqrt{x^2} = \sqrt{100}$$

$$x = \pm 10$$

using $x = +10$

$x^2 + y = 120$

$10^2 + y = 120$

$100 + y = 120$

$-100 \qquad -100$

$y = 20$

$\boxed{(10, 20)}$

check: $10^2 - 20 = 80$

$100 - 20 = 80$ ✓

using $x = -10$

$x^2 + y = 120$

$(-10)^2 + y = 120$

$100 + y = 120$

$-100 \qquad -100$

$y = 20$

$\boxed{(-10, 20)}$

$(-10)^2 - 20 = 80$

$100 - 20 = 80$ ✓

Application Question:

Two integers have a sum of 35. The difference when subtracting the larger from twice the smaller is 10. What are the two numbers?

Larger: $x = 20$

smaller: $y = 15$

$x + y = 35 \rightarrow x + y = 35$

$2y - x = 10 \rightarrow -x + 2y = 10$

$3y = 45$

$\frac{3y}{3} = \frac{45}{3}$

$y = 15$

Eq 1: $x + y = 35$

$x + 15 = 35$

$-15 \quad -15$

$x = 20$

$\boxed{20, 15}$

l: Lab
s: Studio
n: Lecture (Normal)

Application Problem:

At a certain university, most courses carry 3 credits , the exception being science courses that have a laboratory component, which carry 4 credits. All courses are billed at \$350 per credit. To defer the cost of materials used in class, studio art courses have an additional fee of \$150 and lab science courses have an additional fee of \$250. All enrolled students must pay a student activity fee of \$300, a technology fee of \$150, and a security fee of \$200.

$16 \cdot 350$
$(10+6)\,350$
$3500 + 2100$
5600

For the current semester, you enrolled in 5 courses, for a total of 16 credits, and received a bill of \$6800. How many lecture courses, lab courses, and studio courses did you enroll in?

Number of courses: $l + s + n = 5$

Credits: $4l + 3s + 3n = 16$

Cost: $(\text{credits}) \cdot 350 + 150s + 250l + 300 + 150 + 200 = 6800$

$16 \cdot 350 + 150s + 250l + 650 = 6800 \to 150s + 250l + 6250 = 6800$

$150s + 250l = 550$

$-3 \times (l + s + n = 5) \times 3 \rightarrow -3l - \cancel{3s} - \cancel{3n} = -15$

$4l + 3s + 3n = 16 \longrightarrow 4l + \cancel{3s} + \cancel{3n} = 16$

$$\boxed{l = 1}$$

$250l + 150s = 550$

$250(1) + 150s = 550$

$150s = 300$

$$\boxed{s = 2}$$

$l + s + n = 5$

$1 + 2 + n = 5$

$3 + n = 5$

$$\boxed{n = 2}$$

2 Lecture
2 Studio
1 Lab