MATH 1314

Test 2 Review

19 Multiple Choice Questions

Example 1: The length of a rectangle is twice its width. If the perimeter of the rectangle is 180 feet, find the dimensions of the rectangle.

$\ell = 2w$

$P = 2\ell + 2w$

$180 = 2(2w) + 2w$

$180 = 4w + 2w$

$\frac{180}{6} = \frac{\cancel{6}w}{\cancel{6}}$

$\ell = 2(30) = 60$

$30 = w$

$\boxed{60' \times 30'}$

Example 2: Solve the following system of equations for y:

$$2 \times (4x + y = 47) \times 2 \rightarrow 8x + \cancel{2y} = 94$$
$$6x - 2y = -10 \qquad 6x - \cancel{2y} = -10$$

$$\frac{\cancel{14}x}{\cancel{14}} = \frac{84}{14}$$

$$x = 6$$

special cases:

$0 = 0 \rightarrow$ Infinitely Many Solutions

$2 = 0 \rightarrow$ No Solutions

$$4(6) + y = 47$$
$$\cancel{24} + y = 47$$
$$-\cancel{24} \qquad -24$$
$$\boxed{y = 23}$$

Example 3: State all solutions to the equation:

$$x^2 + 40 = 0$$

$$\sqrt{x^2} = \sqrt{-40}$$

$$x = \pm i\sqrt{4}\sqrt{10}$$

$$x = \pm 2i\sqrt{10}$$

Example 4: Solve the following equation:

$$x^2 + 5x - 7 = 0$$

$a = 1 \quad b = 5 \quad c = -7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \longleftarrow \text{Must Memorize}$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(-7)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{25 + 28}}{2} = \frac{-5 \pm \sqrt{53}}{2}$$

$$x = \frac{-5}{2} + \frac{\sqrt{53}}{2}, \ \frac{-5}{2} - \frac{\sqrt{53}}{2}$$

Example 5: Solve the following by completing the square: *

$$x^2 - 8x + \cancel{20} = 0$$

$$-20 \quad -20$$

$x^2 - 8x = -20 \longrightarrow x^2 - 8x + 16 = -20 + 16$

$b = -8$

$\frac{b}{2} = -4$

$\left(\frac{b}{2}\right)^2 = 16$

$\sqrt{(x-4)^2} = \sqrt{-4}$

$x - 4 = \pm 2i$

$+4 \quad +4$

$x = 4 \pm 2i$

Example 6: Solve:

LCD: $30x$

$$\frac{3}{10x} - \frac{1}{6x} = 1$$

$$\frac{3}{10x} \cdot \frac{3}{3} - \frac{1}{6x} \cdot \frac{5}{5} = \frac{1}{1} \cdot \frac{30x}{30x}$$

$$\frac{9}{\cancel{30x}} - \frac{5}{\cancel{30x}} = \frac{30x}{\cancel{30x}}$$

$$9 - 5 = 30x$$

$$\frac{4}{30} = \frac{\cancel{30}x}{\cancel{30}}$$

$$\boxed{2/15 = x}$$

Example 7: Simplify $\frac{3-2i}{2+i}$

$\boxed{i^2 = -1}$

$$\frac{(3-2i)}{(2+i)} \cdot \frac{(2-i)}{(2-i)} = \frac{6-3i-4i+2i^2}{4-\cancel{2i}+\cancel{2i}-i^2} = \frac{4-7i}{5}$$

-2

$+1$

$$\frac{4}{5} - \frac{7}{5}i$$

a + bi form

Example 8: Simplify

A. $(5-4i)(-1-2i)$

FOIL

$-5-10i+4i+\underbrace{8i^2}_{\to -8} = \boxed{-13-6i}$

B. $(2-3i)-(-1+5i)$

$2-3i+1-5i$

$\boxed{3-8i}$

Example 9: Solve the following inequality and express your answer in interval notation.

$$-2 < 3 - 4x \le 7$$

$$\begin{array}{ccc} -3 & -3 & -3 \end{array}$$

$$-5 < -4x \le 4$$

$$\begin{array}{ccc} \overline{-4} & \overline{-4} & \overline{-4} \end{array}$$

$$\frac{5}{4} > x \geq -1$$

$$-1 \le x < 5/4$$

$$[-1, 5/4)$$

Never: $-1 \geq x < 5/4$

Popper 9:

Question # 1 – 5….Fill out answer choice A.

Example 10: Solve the following inequality and express your answer in interval notation.

$$|7x+8|-4<-3$$

$$+4\quad+4$$

$$|7x+8|<1$$

$$-1<7x+8<1$$

$$-8\qquad -8\qquad -8$$

$$-9<7x<-7$$

$$\frac{-9}{7}<\frac{7x}{7}<\frac{-7}{7}$$

$$-\tfrac{9}{7}<x<-1 \longrightarrow \left(-\tfrac{9}{7},-1\right)$$

special cases:

$|\text{stuff}| \le \text{Neg}$

No Solution

$|\text{stuff}| \ge \text{Neg}$

$(-\infty,\infty)$

Example 11: Find the solution of the equation

$2+3|4x-1|\geq 14$.

$$-2 \qquad -2$$

$$\frac{3|4x-1|}{3} \geq \frac{12}{3}$$

$$|4x-1| \geq 4$$

$$4x-1 \leq -4 \quad \text{or} \quad 4x-1 \geq 4$$

$$+1 \quad +1 \qquad\qquad +1 \quad +1$$

$$\frac{4x}{4} \leq \frac{-3}{4} \qquad\qquad \frac{4x}{4} \geq \frac{5}{4}$$

$$x \leq -3/4 \quad \text{or} \quad x \geq 5/4$$

$$(-\infty, -3/4] \cup [5/4, \infty)$$

Example 12: Find all solutions to the equation: *

$\cancel{|3-2x|=6}$

$$4|3-2x| - \cancel{3} = 21$$
$$\cancel{+3} \quad +3$$
$$\cancel{4}|3-2x| = 24$$
$$\cancel{4} \quad\quad 4$$
$$|3-2x| = 6$$

$$\cancel{3} - 2x = 6 \quad \text{or} \quad \cancel{3} - 2x = -6$$
$$\cancel{-3} \quad -3 \quad\quad \cancel{-3} \quad -3$$
$$\cancel{-2}x = 3 \quad\quad \cancel{-2}x = -9$$
$$\cancel{-2} \quad -2 \quad\quad \cancel{-2} \quad -2$$
$$x = -3/2 \quad\quad x = 9/2$$

$$\{-3/2, 9/2\}$$

Example 13: Tom has a drawer with dimes, nickels and pennies in it. He has an equal number of each kind of coin. Tom counted his money and found that he has a total of $2.40 in the drawer. How many nickels does Tom have?

$d = x$

$n = x$

$p = x$

$$100 \cdot (.10d + .05n + .01p = 2.40) \cdot 100$$

$$10d + 5n + 1p = 240$$

$$10x + 5x + x = 240$$

$$16x = 240$$

$$x = 15$$

$\boxed{\text{15 nickles}}$

Example 14: Solve the following for x:

$$x^6 - 9x^3 - 36 = 0$$

$u = x^3 \longrightarrow u^2 = (x^3)^2 = x^6$

$u^2 - 9u - 36 = 0$

$(u - 12)(u + 3) = 0$

$u - 12 = 0 \qquad u + 3 = 0$

$u = 12 \qquad u = -3$

$u = 12$

$\sqrt[3]{x^3} = \sqrt[3]{12}$

$x = \sqrt[3]{12}$

$u = -3$

$\sqrt[3]{x^3} = \sqrt[3]{-3}$

$x = \sqrt[3]{-3}$

Example 15: Solve the following for x: *

$$\sqrt{x+5} - \cancel{x} = 5$$

$$\qquad\qquad +x \qquad +x$$

Check: $x = -4$

$$\sqrt{-4+5} - (-4) = 5$$

$$\sqrt{1} + 4 = 5$$

$$1 + 4 = 5 \checkmark$$

Check $x = -5$

$$\sqrt{-5+5} - (-5) = 5$$

$$\sqrt{0} + 5 = 5$$

$$0 + 5 = 5 \checkmark$$

$$\left(\sqrt{x+5}\right)^2 = (x+5)^2$$

$$x+5 = (x+5)(x+5)$$

$$x+5 = x^2 + 5x + 5x + 25$$

$$x + 5 = x^2 + 10x + 25$$

$$-x - 5 \qquad\qquad -x \quad -5$$

$$0 = x^2 + 9x + 20$$

$$0 = (x+4)(x+5)$$

$$\boxed{x = -4} \qquad \boxed{x = -5}$$

$$\{-4, -5\}$$

Example 17: Solve the following for x: *

$$\frac{(x-8)(x+4)}{x-3} \le 0$$

$(-\infty, -4] \cup (3, 8]$

Yes | No | Yes | No

$-4 \quad 3 \quad 8$

Num:

$x-8=0 \to x=8$

$x+4=0 \to x=-4$

square bracket

Den:

$x-3=0 \to x=3$

Round parenthesis

Test $x=-5$: $\dfrac{(-5-8)(-5+4)}{(-5-3)} = \dfrac{N \cdot N}{N} = N$

Test $x=0$: $\dfrac{(0-8)(0+4)}{(0-3)} = \dfrac{N \cdot P}{N} = P$

Test $x=5$: $\dfrac{(5-8)(5+4)}{(5-3)} = \dfrac{N \cdot P}{P} = N$

Test $x=10$: $\dfrac{(10-8)(10+4)}{(10-3)} = \dfrac{P \cdot P}{P} = P$

Example 18: Solve the following inequality:

$$2x^2 + 5x - 3 \le \cancel{x^2} - \cancel{2x} - \cancel{15}$$

$$-x^2 + 2x + 15 \qquad -x^2 + 2x + 15$$

$$x^2 + 7x + 12 \le 0$$

$$(x+3)(x+4) = 0$$

$x + 3 = 0$ $\qquad$ $x + 4 = 0$

$x = -3$ $\qquad$ $x = -4$

No Yes No

-4 $\quad$ -3

Test $x = 0$

$$2(0)^2 + 5(0) - 3 \le 0^2 - 2(0) - 15$$

$$-3 \le -15$$

Not True

$$[-4, -3]$$