MATH 1314

Test 3 Review (Alternate)

18 Multiple Choice Questions

Find the following considering the function:

$$f(x) = \begin{cases} x^2 - 6x + 2 & x < -3 \\ 9 & x = -3 \\ x - 5 & x > -3 \end{cases}$$

Top f(-6) [-6 < -3] → f(-6) = (-6)² - 6(-6) + 2 = 36 + 36 + 2 = 74

Middle f(-3) [-3 = -3] = 9

Bottom f(5) [5 > -3] → f(5) = 5 - 5 = 0

Find an additional x-value that can be plugged into $x^2 - 6x + 2$ -4, -5, -10, etc.

Find an additional x-value that can be plugged into $x - 5$ -2, -1, 0, 1, 2, etc.

Assuming that f(x) is an even function passing though the point (5, -3), which of the following must be true:

❑ f(x) is symmetric about the ~~origin~~ y- axis

❑ f(x) passes through the point (-5, -3)

❑ f(-x) = ~~-~~ f(x)

❑ the equation of f(x) must have only even exponents

❑ the equation of f(x) may have a constant term

→ $3 = 3x^0$ → Even Exp

Assuming that g(x) is an odd function passing though the point (-3, 8), which of the following must be true:

- ☐ g(x) is symmetric about the origin
- ☐ g(x) passes through the point (3, -8)
- ☐ g(-x) = - g(x)
- ☐ the equation of g(x) must have only odd exponents
- ☐ the equation of g(x) may have a constant term

Identify which of the following is a function:

- ☑ $y = |x|$
- ☐ $y^2 + 5y + 6 = x^2 + 4x + 2$ Not a Function → y^{Even}
- ☐ $x = |y|$ → Solve for y: $y = +x$ $y = -x$ → $y = \pm x$
  ↑ 2 answers
- ☑ $x^2 + 4x + 7y = 9$
- ☐ $x = 8$ Vertical Line → Not a Function
- ☑ $y = -4$

Determine the domain of the functions:

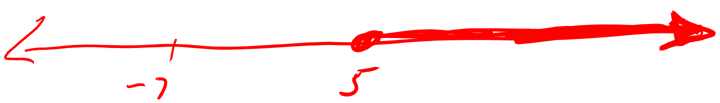
$$f(x) = \frac{\sqrt{x-5}}{x+7}$$

Den: $x+7 \neq 0$

$x \neq -7$

Radical: $x-5 \geq 0$

$x \geq 5$



$[5, \infty)$

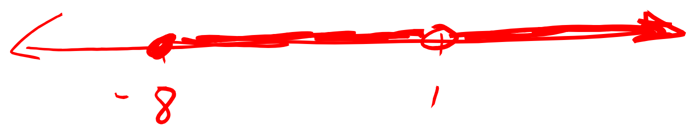
$$g(x) = \frac{\sqrt{x+8}}{x-1}$$

Den: $x-1 \neq 0$

$x \neq 1$

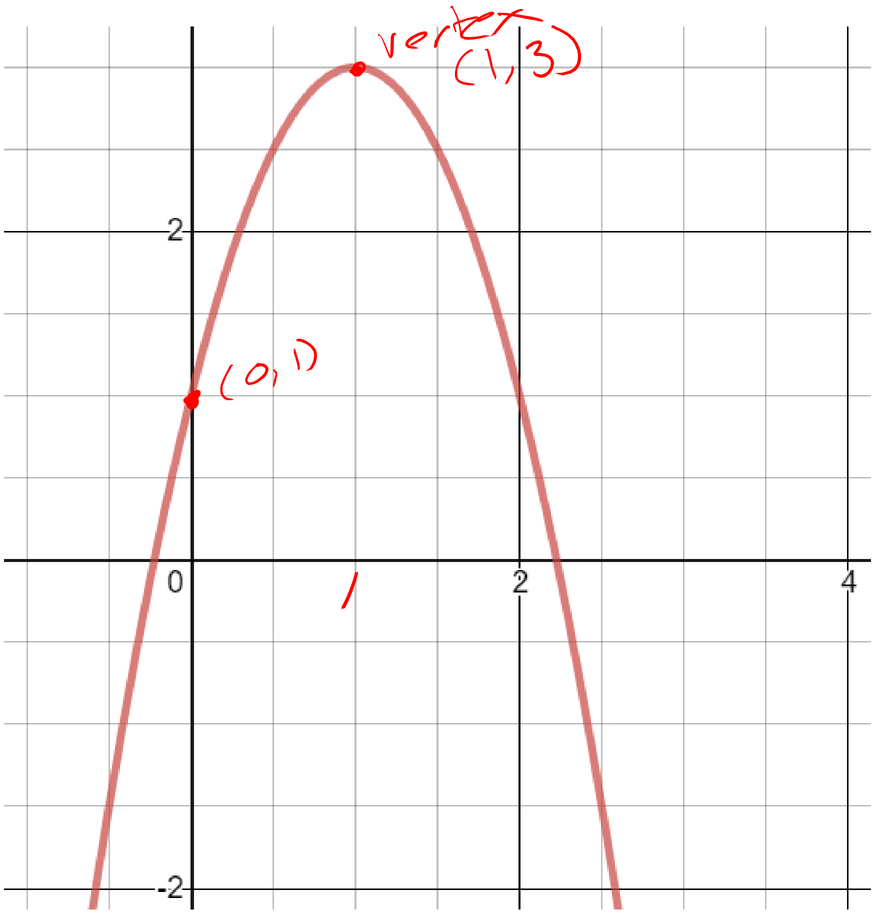
Radical: $x+8 \geq 0$

$x \geq -8$



$[-8, 1) \cup (1, \infty)$

Write the function of the following…



…in f(x) = a(x – h)^2 + k form

vertex: $(h,k) = (1,3)$

$f(x) = a(x-1)^2 + 3$

plug in (0,1) or any other point

$1 = a(0-1)^2 + 3$

$1 = a(-1)^2 + 3$

$1 = a + 3 \rightarrow a = -2$

$\boxed{f(x) = -2(x-1)^2 + 3}$

…in f(x) = ax^2 + bx + c form

$f(x) = -2(x-1)(x-1) + 3$

$= -2(\underbrace{x^2 - x - x + 1}_{(x^2-2x+1)}) + 3$

$= -2x^2 + 4x - 2 + 3$

$\boxed{f(x) = -2x^2 + 4x + 1}$

Consider the following function:

$f(x) = 2x^2 + 8x - 3$ *

$\hookrightarrow a$

Determine the direction opens up $2 > 0$

Find the vertex $x = \frac{-b}{2a} = \frac{-8}{2(2)} = \frac{-8}{4} = -2 \rightarrow h$

$f(-2) = 2(-2)^2 + 8(-2) - 3 = 2(4) + 8(-2) - 3 = 8 - 16 - 3 = -11 \rightarrow k$

$(-2, -11) \rightarrow (h, k)$

Write in standard form $f(x) = a(x-h)^2 + k \rightarrow f(x) = 2(x+2)^2 - 11$

Find the domain $(-\infty, \infty)$

Find the range since opening up $[k, \infty) \rightarrow [-11, \infty)$

Determine the minimum/maximum value

Minimum Value of -11

If f(x) is a one-to-one function and $g(x) = f^{-1}(x)$ and $f(-6) = 3$ and $f(9) = -6$

f(x)	g(x)
(-6, 3)	(3, -6)
(9, -6)	(-6, 9) ←
(a, b) →	(b, a)

Find g(-6)

$g(-6) = \boxed{9}$

Find (f ∘ g)(2) $= f(g(2)) = 2$

Rule for inverse functions

$f(g(x)) = g(f(x)) = x$

For the following function determine (and simplify) the difference quotient: *

$$\frac{f(x+h)-f(x)}{h}$$

$(x+h)^2$

$(x+h)(x+h)$

$x^2+xh+xh+h^2$

$x^2+2xh+h^2$

$$f(x) = -2x^2 + 7x - 3$$

$f(x+h) = -2(x+h)^2 + 7(x+h) - 3 = -2(x^2+2xh+h^2) + 7(x+h) - 3$

$= -2x^2 - 4xh - 2h^2 + 7x + 7h - 3$

$f(x+h) - f(x) = \cancel{-2x^2} - 4xh - 2h^2 \cancel{+7x} + 7h \cancel{-3} \cancel{+2x^2} \cancel{-7x} \cancel{+3}$

$= -4xh - 2h^2 + 7h$

$\frac{f(x+h)-f(x)}{h} = \frac{-4xh - 2h^2 + 7h}{h} = \frac{-4xh}{h} - \frac{2h^2}{h} + \frac{7h}{h} = \boxed{-4x-2h+7}$

For the following functions f and g:

$$f(x) = \frac{4}{x-3}$$

$$g(x) = \frac{2}{5x}$$

Determine the value of g(f(6)) $= g(4/3) = \frac{2}{5\left(\frac{4}{3}\right)} = \frac{2}{\left(\frac{20}{3}\right)} \cdot \frac{3}{3} = \frac{6 \div 2}{20 \div 2} = \boxed{\frac{3}{10}}$

$f(6) = \frac{4}{6-3} = \frac{4}{3}$

Find (f ∘ g)(x) $= f(g(x)) = f\left(\frac{2}{5x}\right) = \frac{4 \cdot 5x}{\frac{2}{5x} - 3 \cdot 5x} = \boxed{\frac{20x}{2-15x}}$

Determine the inverse function: *

$f(x) = 5x^2 - 4 \quad x \le 0$

$y = 5x^2 - 4$

Domain of $f(x)$

$(-\infty, 0]$

$x = 5y^2 - 4$

so

Range of $f^{-1}(x)$

$(-\infty, 0] \rightarrow$ Negs

$x + 4 = 5y^2$

$\frac{x+4}{5} = y^2$

$\pm\sqrt{\frac{x+4}{5}} = y$

$f^{-1}(x) = -\sqrt{\frac{x+4}{5}}$

$g(x) = \frac{2x+3}{x-7}$

$y = \frac{2x+3}{x-7}$

$\frac{x}{1} = \frac{2y+3}{y-7}$

$2y + \cancel{3} = \cancel{xy} - 7x$

$-xy \quad -3 \qquad -\cancel{xy} \quad -3$

$2y - xy = -7x - 3$

$\frac{y\cancel{(2-x)}}{\cancel{2-x}} = \frac{-7x-3}{2-x}$

$y = \left(\frac{-7x-3}{2-x}\right)\frac{\cdot -1}{\cdot -1} \longrightarrow g^{-1}(x) = \frac{7x+3}{x-2}$

For the following transformed function: *

- State the parent function $y = |x|$, Absolute Value Function 1
- List the transformations that appear
- Sketch the function (showing all transformations)
- State and label the transformed key point.

$$f(x) = |-x + 2| - 3$$

Left 2 $y = |x+2|$ 2

Y-axis Refl: $y = |-x+2|$ 3

Down 3 : $f(x) = |-x+2| - 3$ 4

Key Point: $(0,0) \rightarrow (-2,0) \rightarrow (2,0) \rightarrow (2,-3)$