MATH 1314

Test 3 Review

18 Multiple Choice Questions

1. Find the domain:

a. $f(x) = \dfrac{x}{7x - 14}$

$7x - 14 \neq 0$

$7x \neq 14$

$\frac{7x}{7} \neq \frac{14}{7}$

$x \neq 2$

$(-\infty, 2) \cup (2, \infty)$

b. $f(x) = \sqrt{5x - 1}$

$5x - 1 \geq 0$

$5x \geq 1$

$x \geq 1/5$

$[1/5, \infty)$

c. $f(x) = \sqrt{5 - 4x}$

$5 - 4x \geq 0$

$-4x \geq -5$

$\frac{-4x}{-4} \quad \frac{-5}{-4}$

$x \leq 5/4$

$(-\infty, 5/4]$

d. $f(x) = \dfrac{\sqrt{x+4}}{x-8}$

sqroot:

$x + 4 \geq 0$

$x \geq -4$

den: $x - 8 \neq 0$

$x \neq 8$

$[-4, 8) \cup (8, \infty)$

e. $f(x) = \dfrac{\sqrt{x+4}}{x+8}$

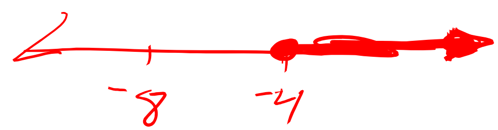
sqroot:

$x + 4 \geq 0$

$x \geq -4$

den: $x + 8 \neq 0$

$x \neq -8$



$[-4, \infty)$

2.

a. **Calculate $f(-2)$ if $f(x) = x^2 + x$**

x

$f(-2) = (-2)^2 + (-2)$

$= 4 - 2 = 2$ $(-2, 2)$

y

b. **Calculate $f(-2)$ if $f(x) = \begin{cases} x^2 + 2x & x \le -1 \\ x & x > -1 \end{cases}$** $\longrightarrow$ $f(0) = 0$

$[-2 \le -1]$ $[0 > -1]$

$f(-2) = (-2)^2 + 2(-2)$

x $= 4 - 4 = 0$

$(-2, 0)$ y

c. **Which point below is on the graph of f(x).**

$$f(x) = \begin{cases} 2 & x < -1 \\ 4 & x = -1 \\ x^2 - 1 & x > -1 \end{cases}$$

$\cancel{(-2,0)}$ or $\boxed{(1,0)}$

x y x y

$(-2,0)$:

$f(-2) = 2 \rightarrow (-2,2)$

$[-2 < -1]$

$(1,0)$

$f(1) = 1^2 - 1 = 1 - 1 = 0 \rightarrow (1,0)$

$[1 > -1]$

3. Determine which of the following is on the graph.

a. $f(x) = -\frac{1}{2}x - 3$

$\cancel{(-1,1)}$

$(-1, 1)$: $f(-1) = \frac{-1}{2}(-1) - 3 = \frac{1}{2} - \frac{3}{1}\cdot\frac{2}{2} = \frac{1}{2} - \frac{6}{2} = \frac{-5}{2}$

$(-1, -5/2)$

$\boxed{(0,-3)}$

$(0, -3)$: $f(0) = \frac{-1}{2}(0) - 3 = 0 - 3 = -3 \Rightarrow (0, -3)$

b. $f(x) = 2x^2 - 3x - 1$

$\boxed{(1,-2)}$

$(1, -2)$: $f(1) = 2(1)^2 - 3(1) - 1$

$= 2(1) - 3(1) - 1$

$= 2 - 3 - 1 = -2$

$(1, -2)$

$\cancel{(-1,-1)}$

$(-1, -1)$: $f(-1) = 2(-1)^2 - 3(-1) - 1$

$= 2(1) - 3(-1) - 1$

$= 2 + 3 - 1 = 4$

$(-1, 4)$

4. Determine if the following is a function:

a. $x^2 + y^2 = 25$

Any even exponents of y → Not a function

Not a function

b. $y = x^3 + 2x^2 + 5x - 1$

Yes

c. $|y| = x$

Solve for y

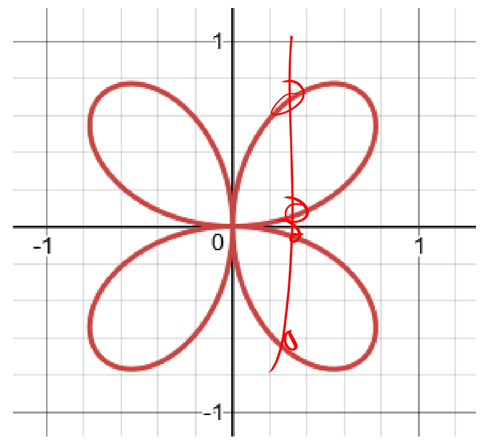
$y = +x$ $y = -x$

$y = \pm x$

Not a Function

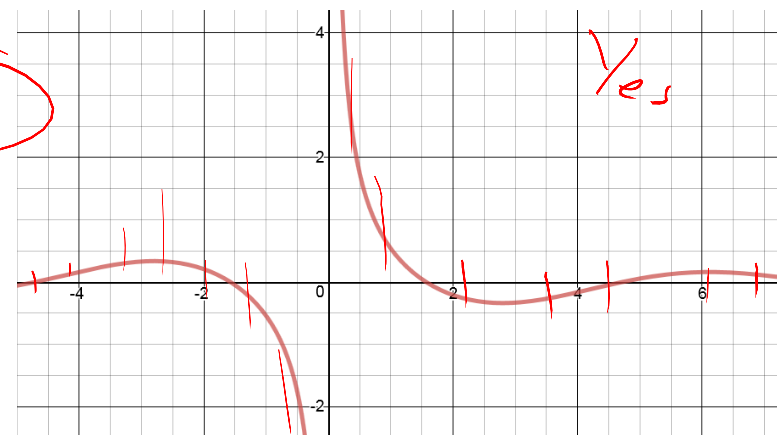
one x-value has 2 answers

d.



Not a Function

e.



Yes

5.

a. Sketch the graph $f(x) = -\sqrt{x-1}$ *

① Parent Function

$y = \sqrt{x}$ or Radical Function

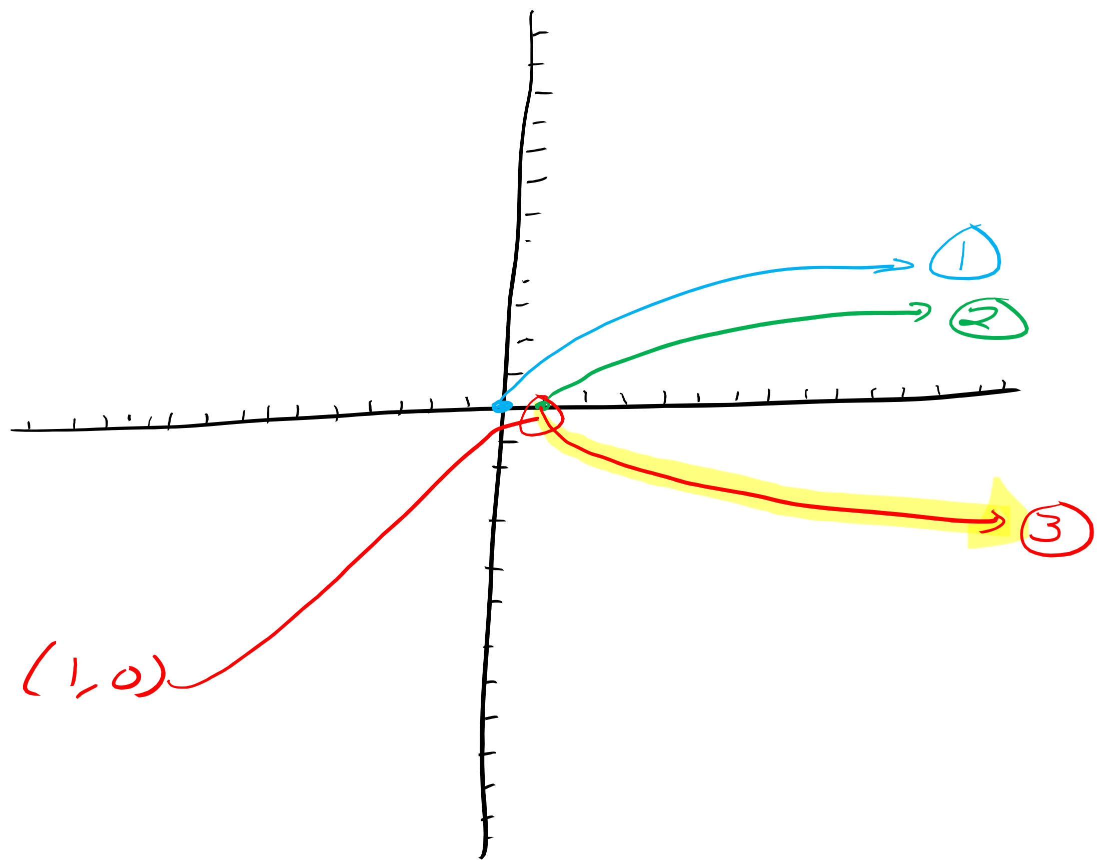
② Right 1

$y = \sqrt{x-1}$

③ x-axis reflection

$f(x) = -\sqrt{x-1}$

Key Point: $(0,0) \to (1,0) \to (1,0)$



b. Sketch the graph $f(x) = -(x+2)^2 - 1$ *

① Parent Function

$y = x^2$ or Quadratic Function

② Left 2

$y = (x+2)^2$

③ x-axis reflection

$y = -(x+2)^2$

④ Down 1

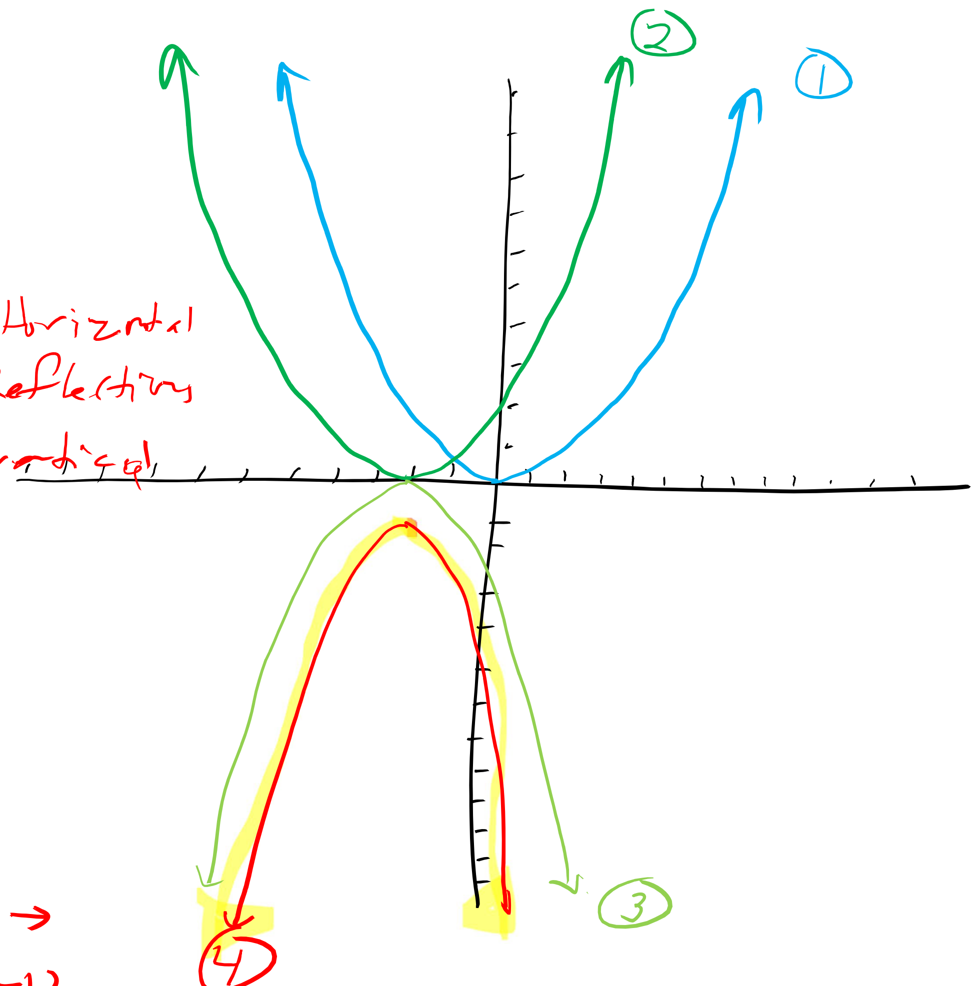
$f(x) = -(x+2)^2 - 1$

Key Point: $(0,0) \rightarrow (-2,0) \rightarrow (-2,0) \rightarrow (-2,-1)$

Heart → Horizontal

Rates → Reflections

Vary → vertical



6.

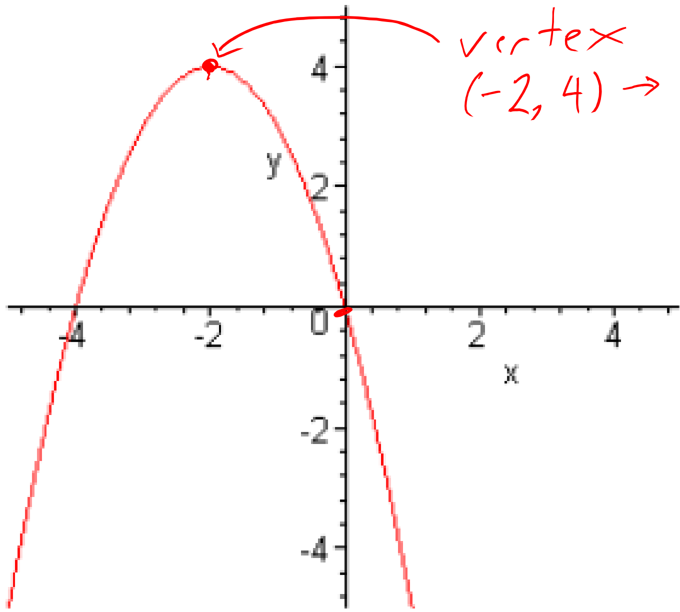
a. What are the necessary transformations $f(x) = (x + 3)^3 - 2$

Cubic function $y = x^3$

Left 3 $y = (x+3)^3$

Down 2 $y = (x+3)^3 - 2$

b. What is the function?



vertex

$(-2, 4) \to (h, k)$

$f(x) = a(x-h)^2 + k$

$f(x) = a(x+2)^2 + 4$

Pick any other (x, y) point on the graph.

Plug in $(0,0)$

$0 = a(0+2)^2 + 4$

$0 = a(2)^2 + 4$

$0 = 4a + 4$

$-4 = 4a$

$-1 = a$

$$\boxed{f(x) = -(x+2)^2 + 4}$$

If needed:

$f(x) = -(x+2)(x+2) + 4$

$f(x) = -(x^2 + 2x + 2x + 4) + 4$

$f(x) = -x^2 - 4x \cancel{-4+4}$

$$\boxed{f(x) = -x^2 - 4x}$$

7.

$a=2 \quad b=-4 \quad c=21$

a. Find the vertex $f(x) = 2x^2 - 4x + 21$ *

$$x = \frac{-b}{2a} = \frac{+4}{2(2)} = \frac{4}{4} = 1 \rightarrow h$$

$$f(1) = 2(1)^2 - 4(1) + 21 = 2(1) - 4(1) + 21 = 2 - 4 + 21 = 19 \rightarrow k$$

vertex: $(h, k) \rightarrow (1, 19)$

If standard form: $f(x) = a(x-h)^2 + k \rightarrow f(x) = 2(x-1)^2 + 19$

b. Find the maximum or minimum value of the function $f(x) = x^2 - 16x + 8$

$a=1$
$b=-16$
$c=8$

a is positive
open up
vertex is a minimum

$$x = \frac{-b}{2a} = \frac{16}{2(1)} = \frac{16}{2} = 8$$

$$f(8) = 8^2 - 16(8) + 8 = 64 - 128 + 8 = -64 + 8 = -56$$

$(10+6)(8)$
$80 + 48$
128

$\boxed{\text{Minimum Value: } -56}$

what is min/max?
y-value
where is the min/max?
x-value

9. **Put in standard form f(x) = –x² – 6x + 2** *

$(b/2)^2 \quad -a(b/2)^2$

$f(x) = -(x^2+6x)+2 = \overset{a}{-}(x^2+6x+9) - (-1)(9) + 2$

$+9+2$

$b = 6$

$\frac{b}{2} = 3$

$\left(\frac{b}{2}\right)^2 = 9$

$\boxed{f(x) = -(x+3)^2 + 11}$

what is min/max value?

a is negative

opens down

$\boxed{\text{Maximum Value of } 11}$

Popper 16

Fill out Choice D for Questions 1 – 5

10. If $f(x) = \sqrt{x+1}$ and $g(x) = x^2$, find $(g \circ f)(x)$ and $(f \circ g)(-1)$

$(g \circ f)(x) = g(f(x)) = g(\sqrt{x+1}) = (\sqrt{x+1})^2 = x+1$

inside

$(f \circ g)(-1) = f(g(-1)) = f(1) = \sqrt{1+1} = \sqrt{2}$

$g(-1) = (-1)^2 = 1$

11. If $f(x) = \dfrac{1}{2x}$ and $g(x) = x^2 - 1$, find $(f \circ g)(2)$.

$(f \circ g)(2) = f(g(2)) = f(3) = \dfrac{1}{2(3)} = \boxed{\dfrac{1}{6}}$

$g(2) = 2^2 - 1 = 4 - 1 = 3$

12. If $f(x) = -2x + 2$ and $g(x) = x^2 + x$, find $(f \circ g)(2)$.

$(f \circ g)(2) = f(g(2)) = f(6) = -2(6)+2 = -12 + 2 = \boxed{-10}$

$g(2) = 2^2 + 2 = 4 + 2 = 6$

13. Find the inverse:

*

a. $f(x) = -2x + 2$

$y = -2x + 2$

$x = -2y + \cancel{2}$

$-2 \qquad\qquad -\cancel{2}$

$x - 2 = -2y$

$\frac{x-2}{-2} = \frac{\cancel{-2}y}{\cancel{-2}}$

$\frac{x-2}{-2} = y$

$f^{-1}(x) = -\frac{x-2}{2} \rightarrow \frac{-x+2}{2} = \frac{-x}{2} + \frac{2}{2}$

$f^{-1}(x) = -\frac{1}{2}x + 1$

b. $f(x) = \frac{1}{x+2}$

$y = \frac{1}{x+2}$

$\frac{x}{1} = \frac{1}{y+2}$

$1 = xy + 2x$

$-xy - 1 \qquad -xy \qquad -1$

$\frac{-xy}{-x} = \frac{2x-1}{-x}$

$y = -\frac{2x-1}{x}$

$f^{-1}(x) = -\frac{2x-1}{x}$

$$c.\ f(x) = \frac{3x+2}{x-1}$$

$$y = \frac{3x+2}{x-1}$$

$$\frac{x}{1} = \frac{3y+2}{y-1}$$

$$3y+2 = xy - x$$

$$-xy - 2 \qquad -xy \quad -2$$

$$3y - xy = -x - 2$$

$$\frac{y(3-x)}{3-x} = \frac{-x-2}{3-x}$$

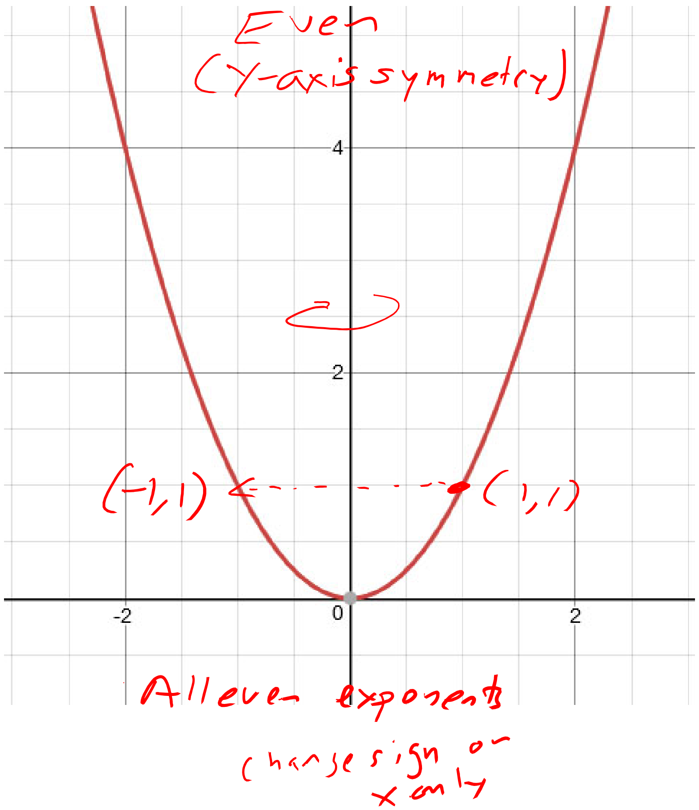
$$y = \frac{-x-2}{3-x}$$

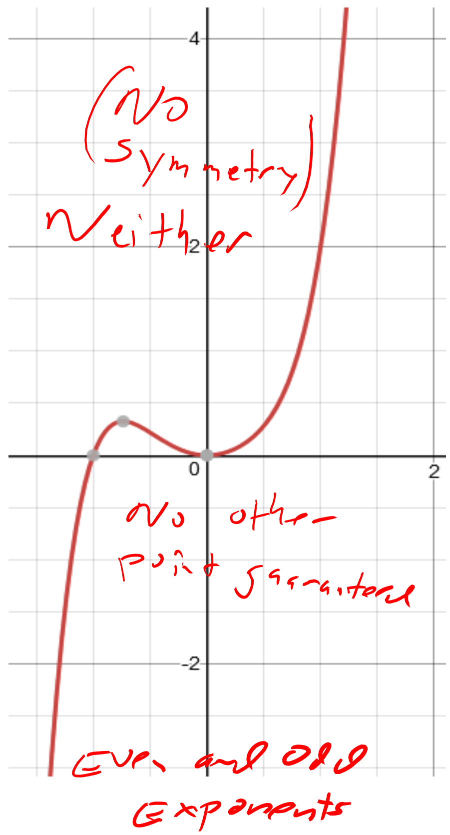
$$y = \frac{-x-2}{3-x} \begin{matrix} \cdot -1 \\ \cdot -1 \end{matrix}$$

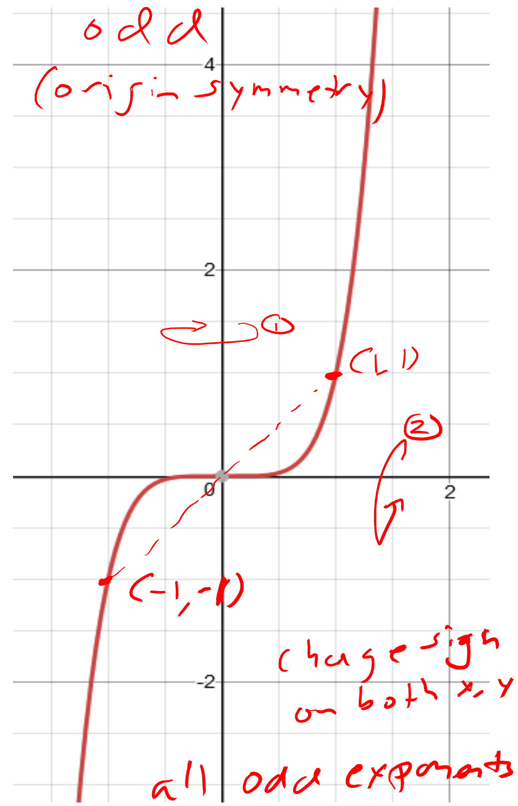
$$y = \frac{x+2}{x-3}$$

$$f^{-1}(x) = \frac{x+2}{x-3}$$

14. Classify the function as Even, Odd or Neither. Given the point (1, 1), what other point is guaranteed?







15. Evaluate the difference quotient for the function: $f(x) = 2x^2 + 8$ *

$$\frac{f(x+h) - f(x)}{h}$$

$(x+h)^2$

$(x+h)(x+h)$

$x^2 + xh + xh + h^2$

$x^2 + 2xh + h^2$

$f(x+h) = 2(x+h)^2 + 8 = 2(x^2 + 2xh + h^2) + 8 = 2x^2 + 4xh + 2h^2 + 8$

$f(x+h) - f(x) = \underbrace{\cancel{2x^2} + 4xh + 2h^2 \cancel{+8}}_{f(x+h)} \underbrace{\cancel{-2x^2} \cancel{-8}}_{-f(x)} = 4xh + 2h^2$

[① only h-terms remaining
② -f(x) should have cancelled]

$\frac{f(x+h) - f(x)}{h} = \frac{4xh + 2h^2}{h} = \frac{4x\cancel{h}}{\cancel{h}} + \frac{2h^{\cancel{2}}}{\cancel{h}}$

[denominator should cancel completely]

$\boxed{4x + 2h}$

16. Given the following table, determine the value of $(f \circ g)(4)$.

x	f(x)	g(x)
0	3	2
1	-1	8
2	7	5
3	8	4
4	2	3

step 2 (3, 8); step 1 (4, 3)

$f(3) = 8$

$g(4) = 3$

$(f \circ g)(4) = f(g(4)) = f(3) = \boxed{8}$

step 1: $g(4) = 3$

step 2: $f(3) = 8$

17. If f(x) and g(x) are inverse functions, and f(3) = 5, f(5) = 8, determine:

x x

x	f(x)
3	5
5	8

f(x) g(x)

$\rightarrow (3,5)$ $(5,3) \rightarrow$ (reverse x, y)

$\rightarrow (5,8)$ $(8,5) \rightarrow$ (reverse x, y)

a. g(5)

$g(5) = 3$

based on (5, 3)

b. f(g(2)) $= 2$

$f(g(x)) = g(f(x)) = x$

If f(x) = 3x + 5, find $f\left(\frac{3}{a+2}\right)$

$$f\left(\frac{3}{a+2}\right) = \frac{3}{1}\left(\frac{3}{a+2}\right) + 5 = \frac{9}{a+2} + \frac{5}{1}\,\frac{(a+2)}{(a+2)}$$

$$= \frac{9}{a+2} + \frac{5a+10}{a+2} = \boxed{\frac{5a+19}{a+2}}$$

Determine the difference quotient for $f(x) = 3x^2 + 6x - 8$ *

$$\frac{f(x+h)-f(x)}{h}$$

$(x+h)^2$

$(x+h)(x+h)$

$x^2 + xh + xh + h^2$

$x^2 + 2xh + h^2$

$$f(x+h) = 3(x+h)^2 + 6(x+h) - 8 = 3(x^2+2xh+h^2) + 6(x+h) - 8$$

$$= 3x^2 + 6xh + 3h^2 + 6x + 6h - 8$$

$$f(x+h) - f(x) = \cancel{3x^2} + 6xh + 3h^2 + \cancel{6x} + 6h \cancel{-8} \cancel{-3x^2} \cancel{-6x} \cancel{+8} = 6xh + 3h^2 + 6h$$

$$\frac{f(x+h)-f(x)}{h} = \frac{6xh+3h^2+6h}{h} = \frac{6xh}{h} + \frac{3h^2}{h} + \frac{6h}{h}$$

$$\boxed{6x + 3h + 6}$$