MATH 1314

Test 3 Review

18 Multiple Choice Questions

1. Find the domain:

a. $f(x) = \dfrac{x}{7x - 14}$

$7x - 14 \neq 0$

$7x \neq 14$

$\frac{7x}{7} \neq \frac{14}{7}$

$x \neq 2$

$(-\infty, 2) \cup (2, \infty)$

b. $f(x) = \sqrt{5x - 1}$

$5x - 1 \geq 0$

$5x \geq 1$

$x \geq 1/5$

$[1/5, \infty)$

c. $f(x) = \sqrt{5 - 4x}$

$5 - 4x \geq 0$

$-4x \geq -5$

$\frac{-4x}{-4} \quad \frac{-5}{-4}$

$x \leq 5/4$

$(-\infty, 5/4]$

d. $f(x) = \dfrac{\sqrt{x+4}}{x-8}$

sqroot:

$x + 4 \geq 0$

$x \geq -4$

den: $x - 8 \neq 0$

$x \neq 8$

$[-4, 8) \cup (8, \infty)$

e. $f(x) = \dfrac{\sqrt{x+4}}{x+8}$

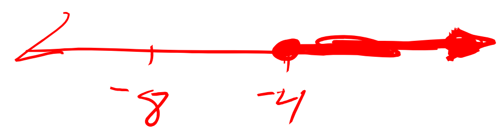
sqroot:

$x + 4 \geq 0$

$x \geq -4$

den: $x + 8 \neq 0$

$x \neq -8$



$[-4, \infty)$

2.

a. **Calculate $f(-2)$ if $f(x) = x^2 + x$**

x

$f(-2) = (-2)^2 + (-2)$

$= 4 - 2 = 2$

$(-2, 2)$

y

b. **Calculate $f(-2)$ if $f(x) = \begin{cases} x^2 + 2x & x \le -1 \\ x & x > -1 \end{cases}$**

$f(0) = 0$

$[0 > -1]$

$[-2 \le -1]$

$f(-2) = (-2)^2 + 2(-2)$

x

$= 4 - 4 = 0$

$(-2, 0)$

y

c. **Which point below is on the graph of f(x).**

$$f(x) = \begin{cases} 2 & x < -1 \\ 4 & x = -1 \\ x^2 - 1 & x > -1 \end{cases}$$

$\cancel{(-2,0)}$ or $\boxed{(1,0)}$

x y   x y

$(-2, 0)$:

$f(-2) = 2 \rightarrow (-2, 2)$

$[-2 < -1]$

$(1, 0)$

$f(1) = 1^2 - 1 = 1 - 1 = 0 \rightarrow (1, 0)$

$[1 > -1]$

3. Determine which of the following is on the graph.

a. $f(x) = -\frac{1}{2}x - 3$

~~(-1,1)~~ $(-1,1): f(-1) = \frac{-1}{2}(-1) - 3 = \frac{1}{2} - \frac{3}{1}\cdot\frac{2}{2} = \frac{1}{2} - \frac{6}{2} = \frac{-5}{2}$

$(-1, -5/2)$

$\boxed{(0,-3)}$ $(0,-3): f(0) = \frac{-1}{2}(0) - 3 = 0 - 3 = -3 \Rightarrow (0,-3)$

b. $f(x) = 2x^2 - 3x - 1$

$\boxed{(1,-2)}$ $(1,-2): f(1) = 2(1)^2 - 3(1) - 1$

$= 2(1) - 3(1) - 1$

$= 2 - 3 - 1 = -2$

$(1,-2)$

~~(-1,-1)~~ $(-1,-1): f(-1) = 2(-1)^2 - 3(-1) - 1$

$= 2(1) - 3(-1) - 1$

$= 2 + 3 - 1 = 4$

$(-1,4)$

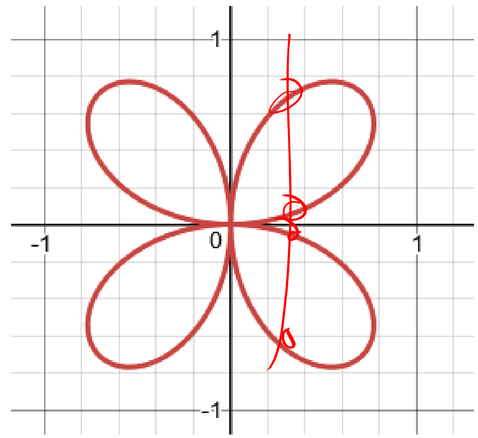
4. Determine if the following is a function:

a. $x^2 + y^2 = 25$

Any even exponents of y → Not a function

Not a function

d.



Not a Function

b. $y = x^3 + 2x^2 + 5x - 1$

Yes

e.

Yes

c. $|y| = x$

Solve for y

$y = +x$ $y = -x$

$y = \pm x$

one x-value has 2 answers

Not a Function

5.

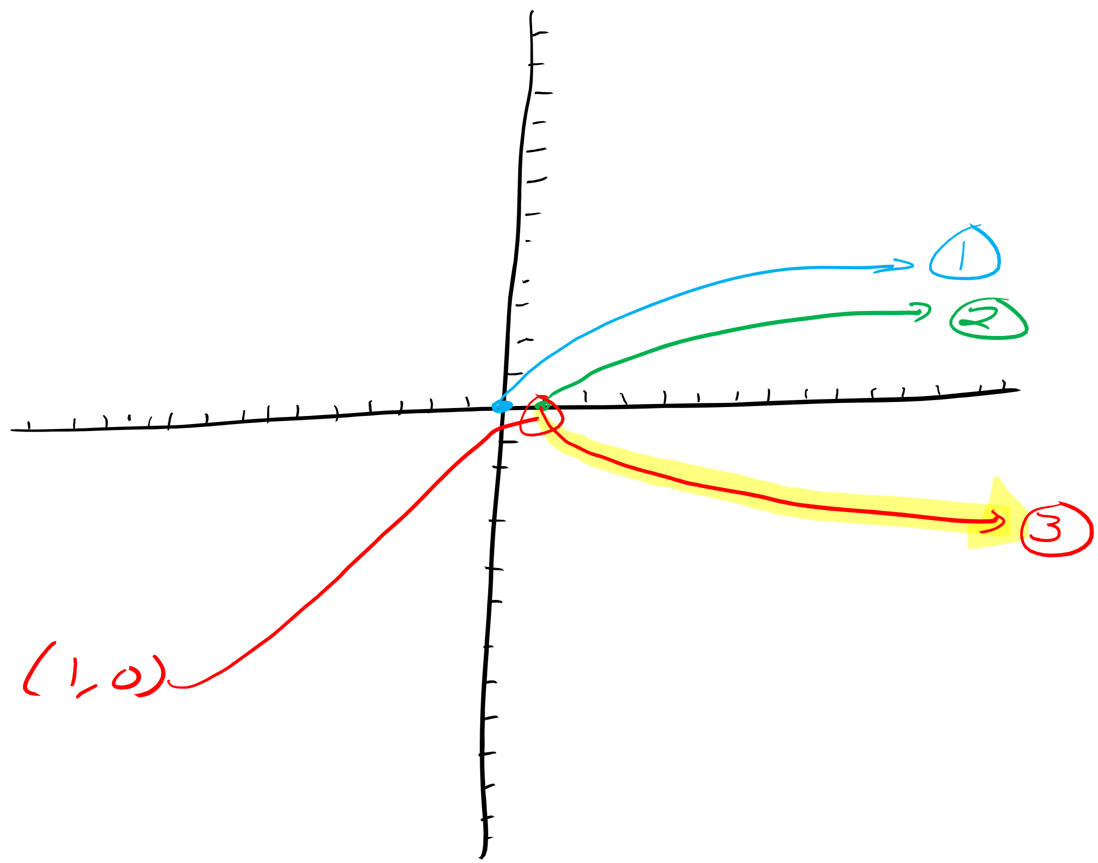
a. **Sketch the graph $f(x) = -\sqrt{x-1}$** *

① Parent Function
$y = \sqrt{x}$ or Radical Function

② Right 1
$y = \sqrt{x-1}$

③ x-axis reflection
$f(x) = -\sqrt{x-1}$

Key Point: $(0,0) \rightarrow (1,0) \rightarrow (1,0)$



b. Sketch the graph $f(x) = -(x+2)^2 - 1$ *

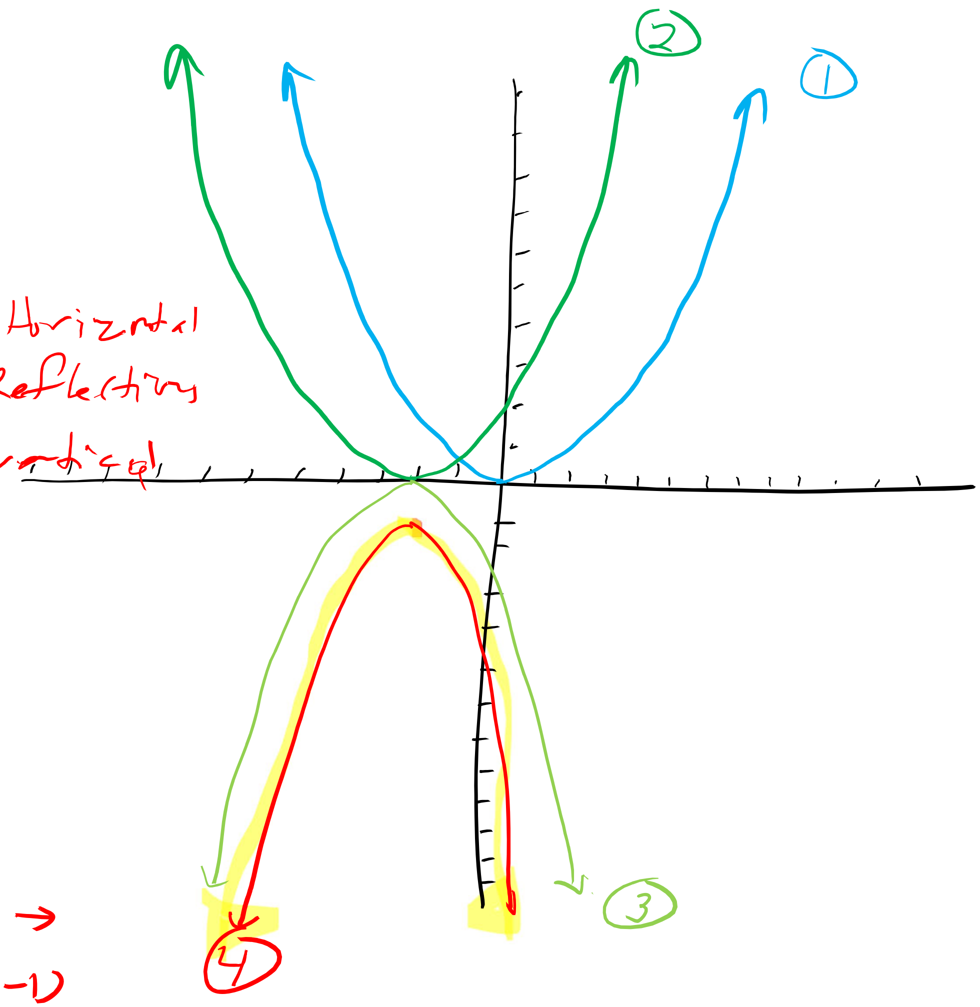
1 Parent Function
$y = x^2$ or Quadratic Function

2 Left 2
$y = (x+2)^2$

3 x-axis reflection
$y = -(x+2)^2$

4 Down 1
$f(x) = -(x+2)^2 - 1$

Heart → Horizontal
Rates → Reflections
Vary → vertical



Key Point: $(0,0) \to (-2,0) \to (-2,0) \to (-2,-1)$

Popper 18

Fill out Choice D for Questions 1 – 5

6.

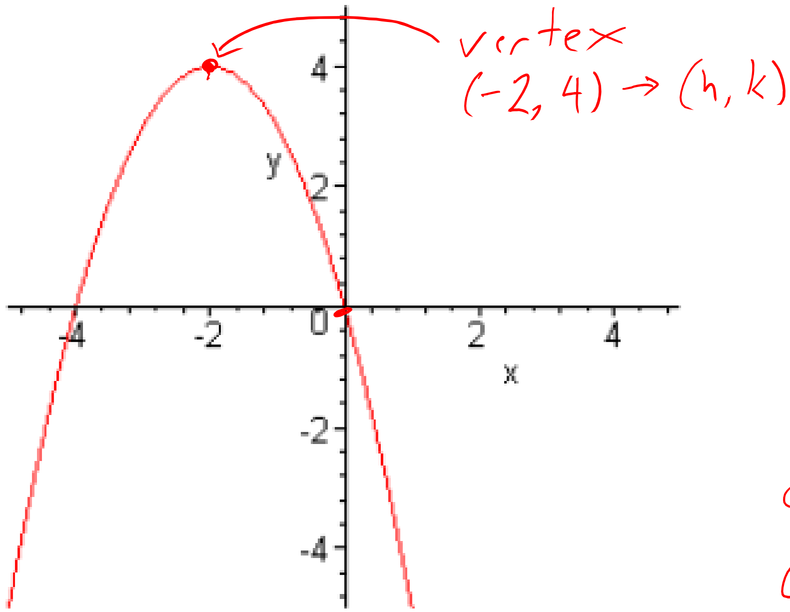
a. **What are the necessary transformations** $f(x) = (x + 3)^3 - 2$

Cubic function $y = x^3$

Left 3 $y = (x+3)^3$

Down 2 $y = (x+3)^3 - 2$

b. What is the function?



$f(x) = a(x-h)^2 + k$

$f(x) = a(x+2)^2 + 4$

Pick any other (x, y) point on the graph.

Plug in $(0, 0)$

$0 = a(0+2)^2 + 4$

$0 = a(2)^2 + 4$

$0 = 4a + 4$

$-4 = 4a$

$-1 = a$

$\boxed{f(x) = -(x+2)^2 + 4}$

If needed:

$f(x) = -(x+2)(x+2) + 4$

$f(x) = -(x^2 + 2x + 2x + 4) + 4$

$f(x) = -x^2 - 4x \cancel{-4+4}$

$\boxed{f(x) = -x^2 - 4x}$

7.

$a=2 \quad b=-4 \quad c=21$

a. Find the vertex $f(x) = 2x^2 - 4x + 21$ *

$$x = \frac{-b}{2a} = \frac{+4}{2(2)} = \frac{4}{4} = 1 \rightarrow h$$

$$f(1) = 2(1)^2 - 4(1) + 21 = 2(1) - 4(1) + 21 = 2 - 4 + 21 = 19 \rightarrow k$$

vertex: $(h,k) \rightarrow (1, 19)$

If standard form: $f(x) = a(x-h)^2 + k \rightarrow f(x) = 2(x-1)^2 + 19$

b. Find the maximum or minimum value of the function

$f(x) = x^2 - 16x + 8$

$a=1$
$b=-16$
$c=8$

a is positive, opens up, vertex is a minimum

what is min/max? y-value

where is the min/max? x-value

$$x = \frac{-b}{2a} = \frac{16}{2(1)} = \frac{16}{2} = 8$$

$$f(8) = 8^2 - 16(8) + 8 = 64 - 128 + 8 = -64 + 8 = -56$$

$(10+6)(8)$
$80 + 48$
128

$\boxed{\text{Minimum Value: } -56}$

9. **Put in standard form $f(x) = -x^2 - 6x + 2$** *

$(b/2)^2 \quad -a(b/2)^2$

$f(x) = -(x^2+6x)+2 = -(x^2+6x+9) - (-1)(9) + 2$

$+9+2$

$b = 6$

$\frac{b}{2} = 3$

$\left(\frac{b}{2}\right)^2 = 9$

$\boxed{f(x) = -(x+3)^2 + 11}$

what is min/max value?

a is negative

opens down

$\boxed{\text{Maximum Value of } 11}$

10. If $f(x) = \sqrt{x+1}$ and $g(x) = x^2$, find $(g \circ f)(x)$ and $(f \circ g)(-1)$

$(g \circ f)(x) = g(f(x)) = g(\sqrt{x+1}) = (\sqrt{x+1})^2 = x+1$

inside

$(f \circ g)(-1) = f(g(-1)) = f(1) = \sqrt{1+1} = \sqrt{2}$

$g(-1) = (-1)^2 = 1$

11. If $f(x) = \dfrac{1}{2x}$ and $g(x) = x^2 - 1$, find $(f \circ g)(2)$.

$(f \circ g)(2) = f(g(2)) = f(3) = \dfrac{1}{2(3)} = \boxed{\dfrac{1}{6}}$

$g(2) = 2^2 - 1 = 4 - 1 = 3$

12. If $f(x) = -2x + 2$ and $g(x) = x^2 + x$, find $(f \circ g)(2)$.

$(f \circ g)(2) = f(g(2)) = f(6) = -2(6)+2 = -12 + 2 = \boxed{-10}$

$g(2) = 2^2+2 = 4+2 = 6$

13. Find the inverse: *

a. $f(x) = -2x + 2$

$$y = -2x + 2$$

$$x = -2y + \cancel{2}$$

$$x - 2 = -2y$$

$$\frac{x-2}{-2} = y$$

$$f^{-1}(x) = -\frac{x-2}{2} \rightarrow \frac{-x+2}{2} = \frac{-x}{2} + \frac{2}{2}$$

$$f^{-1}(x) = \frac{-1}{2}x + 1$$

b. $f(x) = \dfrac{1}{x+2}$

$$y = \frac{1}{x+2}$$

$$\frac{x}{1} = \frac{1}{y+2}$$

$$1 = xy + 2x$$

$$-xy - 1 \qquad -xy - 1$$

$$\frac{-xy}{-x} = \frac{2x-1}{-x}$$

$$y = -\frac{2x-1}{x}$$

$$f^{-1}(x) = -\frac{2x-1}{x}$$

$$c.\ f(x) = \frac{3x+2}{x-1}$$

$$y = \frac{3x+2}{x-1}$$

$$\frac{x}{1} = \frac{3y+2}{y-1}$$

$$3y + 2 = xy - x$$

$$-xy - 2 \qquad -xy \quad -2$$

$$3y - xy = -x - 2$$

$$\frac{y(3-x)}{3-x} = \frac{-x-2}{3-x}$$

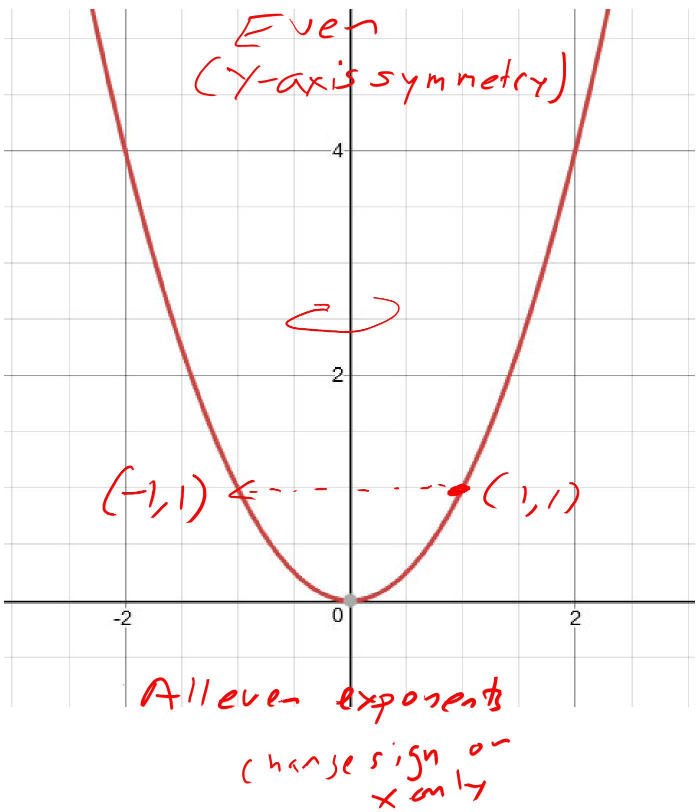
$$y = \frac{-x-2}{3-x}$$

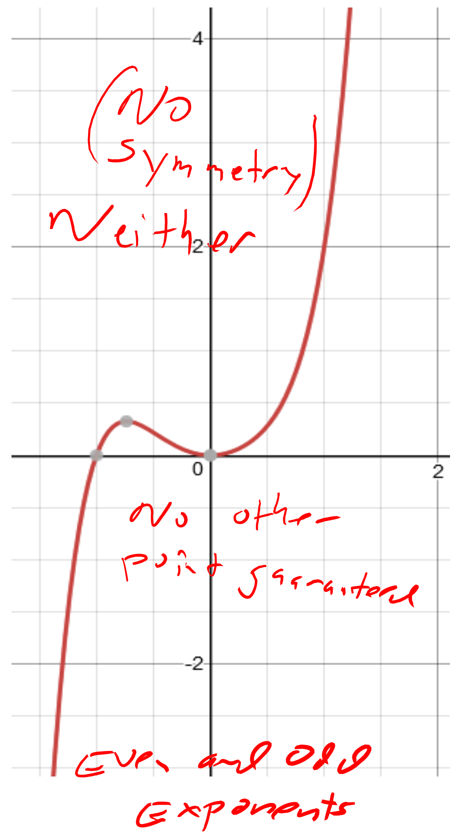
$$y = \frac{-x-2}{3-x} \begin{matrix} \cdot -1 \\ \cdot -1 \end{matrix}$$

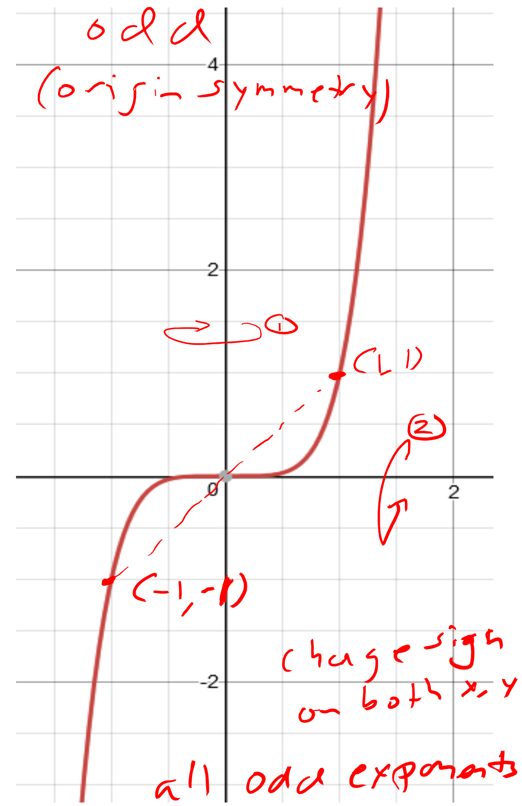
$$y = \frac{x+2}{x-3}$$

$$f^{-1}(x) = \frac{x+2}{x-3}$$

14. Classify the function as Even, Odd or Neither. Given the point (1, 1), what other point is guaranteed?







15. Evaluate the difference quotient for the function: $f(x) = 2x^2 + 8$ *

$$\frac{f(x+h) - f(x)}{h}$$

$(x+h)^2$

$(x+h)(x+h)$

$x^2 + xh + xh + h^2$

$x^2 + 2xh + h^2$

$f(x+h) = 2(x+h)^2 + 8 = 2(x^2 + 2xh + h^2) + 8 = 2x^2 + 4xh + 2h^2 + 8$

$f(x+h) - f(x) = \underbrace{\cancel{2x^2} + 4xh + 2h^2 \cancel{+8}}_{f(x+h)} \underbrace{\cancel{-2x^2} \cancel{-8}}_{-f(x)} = 4xh + 2h^2$

[① only h-terms remaining
② -f(x) should have cancelled]

$\frac{f(x+h) - f(x)}{h} = \frac{4xh + 2h^2}{h} = \frac{4x\cancel{h}}{\cancel{h}} + \frac{2h^{\cancel{2}}}{\cancel{h}}$

[denominator should cancel completely]

$\boxed{4x + 2h}$

16. Given the following table, determine the value of $(f \circ g)(4)$.

x	f(x)	g(x)
0	3	2
1	-1	8
2	7	5
step 2 3	8	4
step 1 4	2	3

$f(3)=8$

$g(4)=3$

$(f \circ g)(4) = f(g(4)) = f(3) = \boxed{8}$

step 1: $g(4)=3$

step 2: $f(3) = 8$

17. If f(x) and g(x) are inverse functions, and f(3) = 5, f(5) = 8, determine:

x x

x	f(x)
3	5
5	8

f(x) → (3,5) → (5,8)

g(x) (5,3) → (reverse x, y) (8,5) → (reverse x, y)

a. g(5)

$g(5) = 3$

based on (5,3)

b. f(g(2)) = 2

$f(g(x)) = g(f(x)) = x$

If f(x) = 3x + 5, find $f\left(\frac{3}{a+2}\right)$

$$f\left(\frac{3}{a+2}\right) = \frac{3}{1}\left(\frac{3}{a+2}\right) + 5 = \frac{9}{a+2} + \frac{5}{1}\frac{(a+2)}{(a+2)}$$

$$= \frac{9}{a+2} + \frac{5a+10}{a+2} = \boxed{\frac{5a+19}{a+2}}$$

Determine the difference quotient for $f(x) = 3x^2 + 6x - 8$ *

$$\frac{f(x+h)-f(x)}{h}$$

$(x+h)^2$

$(x+h)(x+h)$

$x^2+xh+xh+h^2$

$x^2+2xh+h^2$

$$f(x+h) = 3(x+h)^2+6(x+h)-8 = 3(x^2+2xh+h^2)+6(x+h)-8$$

$$= 3x^2+6xh+3h^2+6x+6h-8$$

$$f(x+h)-f(x) = \cancel{3x^2}+6xh+3h^2\cancel{+6x}+6h\cancel{-8}\cancel{-3x^2}\cancel{-6x}\cancel{+8} = 6xh+3h^2+6h$$

$$\frac{f(x+h)-f(x)}{h} = \frac{6xh+3h^2+6h}{h} = \frac{6xh}{h}+\frac{3h^2}{h}+\frac{6h}{h}$$

$$\boxed{6x+3h+6}$$

Popper 19

Fill out Choice B for Questions 1 – 5