MATH 1314

Test 4 Review (Alternate)

18 Multiple Choice Questions

Find all real and complex zeros of the following polynomial: $p(x) = (x^3 - 4x^2) + (27x - 108)$

$x^2(x-4) + 27(x-4)$

$(x-4)(x^2+27)$

$x - 4 = 0$

$x = 4$

$x^2 + 27 = 0$

$\sqrt{x^2} = \sqrt{-27}$

$x = \pm\sqrt{-27} = \pm\sqrt{9}\sqrt{3}\sqrt{-1}$

$x = \pm 3i\sqrt{3}$

Determine the domain and range of the following: $f(x) = -2 \cdot 3^{x-7} \underbrace{- 5}_{HA}$ Exponential Function

Domain: $(-\infty, \infty)$

Range: HA: $y = -5$

Exponential Term: Neg } Below the HA

$(-\infty, -5)$

Determine the domain and range of the following: $f(x) = \log_5(2x+3) - 5$

Logarithmic Function

Domain: inside > 0

$2x+3>0$

$2x>-3$

$x>-3/2$

$(-3/2, \infty)$

Range: $(-\infty, \infty)$

For the following polynomial function, determine the following: $p(x) = (x - 3)(x + 2)^3(x + 5)^2$ *

- Degree

Leading Term: $(x)^1(x)^3(x)^2 = x^1 \cdot x^3 \cdot x^2 = x^6$

Deg: 6

- End Behavior (Left and Right)

Left: ↑ , Right: ↑

Even Degree, Leading Term: Positive

- All x-intercepts and their multiplicities

$x - 3 = 0$, $x = 3$, M: 1

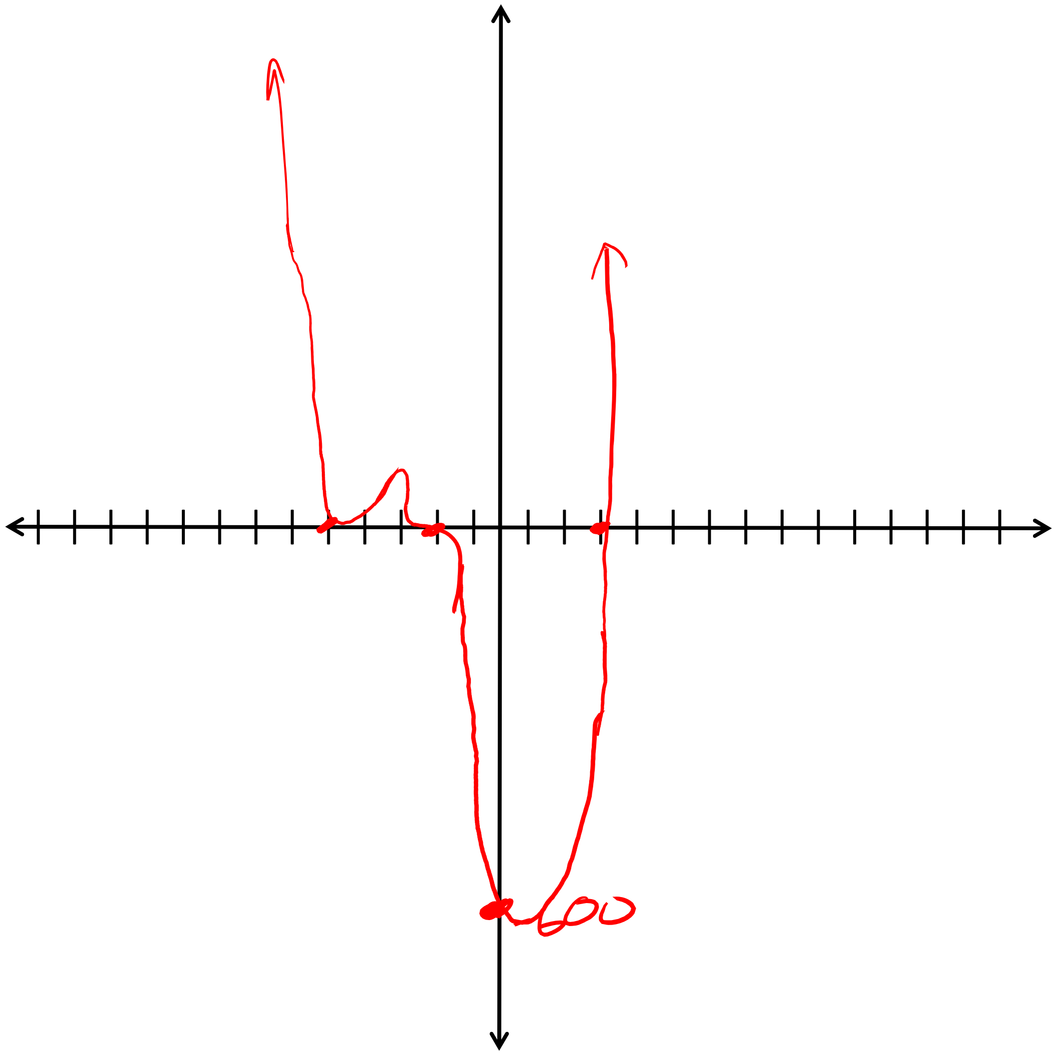
$x + 2 = 0$, $x = -2$, M: 3

$x + 5 = 0$, $x = -5$, M: 2

- The y-intercept

$p(0) = (0-3)(0+2)^3(0+5)^2 = (-3)(2)^3(5)^2 = -3 \cdot \underbrace{8 \cdot 25}_{200} = -600$

- Draw a rough sketch of the function (Next Slide)



X-int:

X=3 M:1
(Linear)

X=-2 M:3
(cubic)

X=-5 M:2
(Quadratic)

Y-int: -600

End Beh:

L↑ R↑

Determine the vertical asymptote of the following: $f(x) = \frac{x^2-4x-21}{x^2+11x+24} = \frac{(x-7)\cancel{(x+3)}}{(x+8)\cancel{(x+3)}}$

VA: (Den Only): $x+8=0$

$x=-8$

Determine the equation of the polynomial function with the following characteristics:

- Degree: 3
- Zeros located at: 8 and 5i (and $-5i$)
- Constant Coefficient: 800

$(x - \text{zero})$

$$P(x) = a(x-8)(x-5i)(x+5i)$$

$$P(x) = a(x-8)(x^2 + \cancel{5ix} - \cancel{5ix} \underbrace{-25i^2}_{+25})$$

$$P(x) = a(x-8)(x^2+25)$$

$$P(x) = a(x^3 + 25x - 8x^2 - 200)$$

$$P(x) = a(x^3 - 8x^2 + 25x \underline{-200})$$

$$P(x) = -4x^3 + 32x^2 - 100x + 800$$

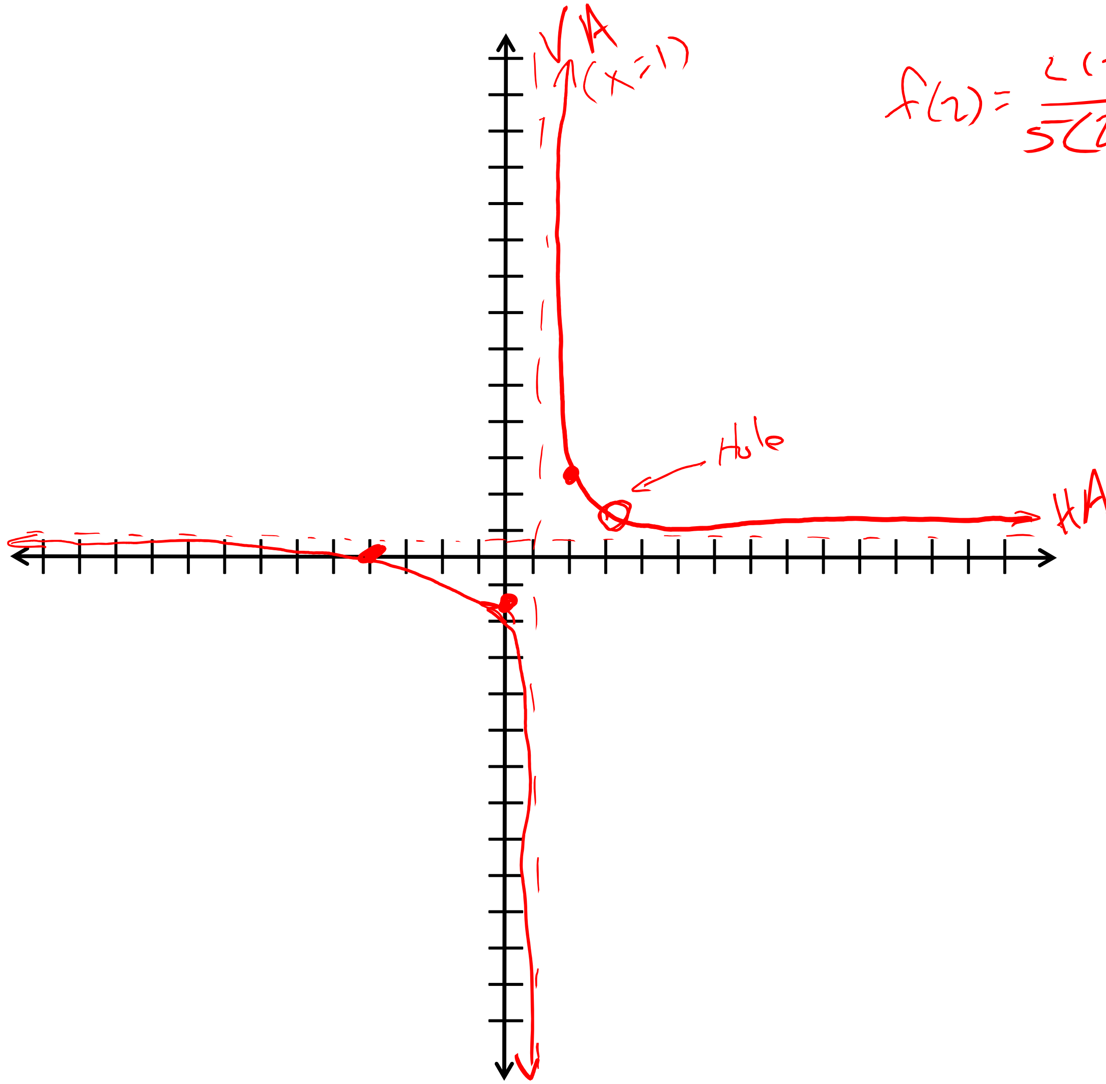
$$-200 \times \boxed{} = 800$$

$$a = -4$$

For the given function, find the following information:

$$f(x) = \frac{\boxed{2x^2}+2x-24}{\boxed{5x^2}-20x+15} = \frac{2(x^2+x-12)}{5(x^2-4x+3)} = \frac{2(x+4)(x-3)}{5(x-3)(x-1)}$$ *

- Location of the holes (if any) $x-3=0 \rightarrow x=3$
  (Num & Den)
- Location of the x-intercepts (if any) $x+4=0 \rightarrow x=-4$
  (Num Only)
- Location of the y-intercepts (if any) $f(0)=\frac{-24}{15}=-\frac{8}{5}$
- The equation of the vertical asymptote (if any) $x-1=0 \rightarrow x=1$
  (Den Only)
- The equation of the horizontal asymptote (if any) $y=\frac{2}{5}$
  Compare degrees
- Sketch the graph (next slide).



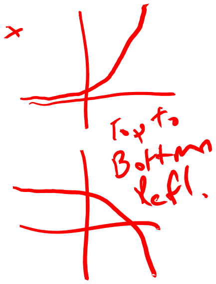
$$f(2) = \frac{2(2+4)}{5(2-1)} = \frac{2(6)}{5(1)} = \frac{12}{5}$$

Identify the function from the listed options.

(Horizontal Asymptote and translated key point are provided)

$f(x) = 3^{x+a} \underbrace{+b}_{HA}$

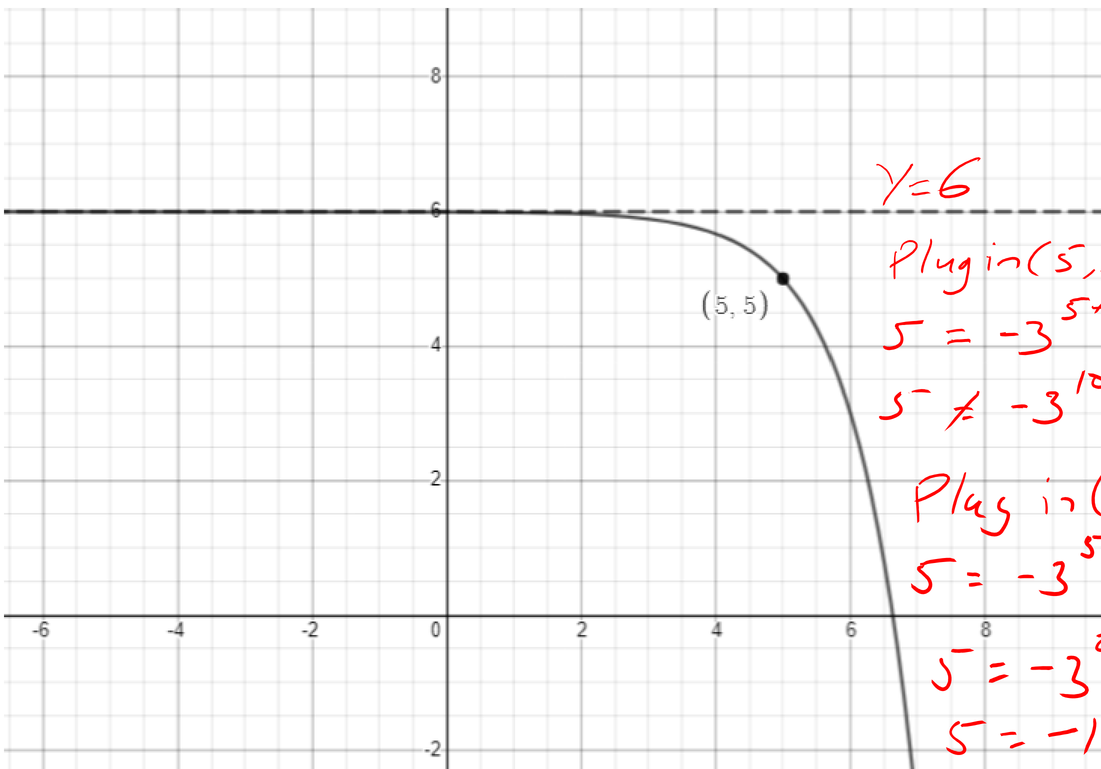
$y = 3^{x}$



our graph

$f(x) = 3^{x+a} + 6$

$f(x) = -3^{x+a} + 6$



$y = 6$

Plug in (5,5) to (b)

$5 = -3^{5+5} + 6$

$5 \neq -3^{10} + 6$

Plug in (5,5) to (f)

$5 = -3^{5-5} + 6$

$5 = -3^{0} + 6 \rightarrow -1 \cdot 3^{0}$

$5 = -1 + 6$

$5 = 5$

~~a)~~ $f(x) = 3^{x+5} - 6$ (HA)

~~b)~~ $f(x) = -3^{x+5} + 6$ (5,5)

~~c)~~ $f(x) = 3^{x-5} - 6$ (HA)

~~d)~~ $f(x) = (-3)^{x-5} - 6$ $a > 0$, $a \neq 1$

~~e)~~ $f(x) = 3^{x+5} + 6$ x-axis refl.

f) $f(x) = -3^{x-5} + 6$

~~g)~~ $f(x) = -3^{x+5} - 6$ (HA)

Give the equation of the asymptote of the following: $f(x) = log_{0.75}(2x - 7) + 6$

inside = 0

VA: $2x - 7 = 0$

$2x = 7$

$\boxed{x = 7/2}$

If the polynomial, $f(x) = x^4 + 3x^3 - x^2 + 27x - 90$, has one zero located at x = 2, find zeros of the function.

$$\begin{array}{r|rrrrr} & 1 & 3 & -1 & 27 & -90 \\ 2 & \downarrow & 2 & 10 & 18 & 90 \\ \hline & 1 & 5 & 9 & 45 & 0 \end{array}$$

$Q(x) = (x^3 + 5x^2) + (9x + 45)$

$x^2(x+5) + 9(x+5)$

$(x+5)(x^2+9)$

$x+5=0$

$x=-5$

$x^2+9=0$

$x^2=-9$

$x = \pm\sqrt{-9} = \pm 3i$

$\{-5, 2, \pm 3i\}$

Determine the translation of the key point, (0,1), of the function: $f(x) = -e^{x-7} + 4$

Horizontal shift: Right 7 $\quad e^{x-7}$

x-axis reflection: $\quad -e^{x-7}$

vertical shift: up 4 $\quad -e^{x-7}+4$

$(0,1) \rightarrow (7,1) \rightarrow (7,-1) \rightarrow (7,3)$

Determine the equation of the horizontal asymptote of: $f(x) = \frac{7-8x^4}{2x^4+5x^3-3x^2+8x-5}$

compare degrees:

$$\frac{\text{Deg } 4}{\text{Deg } 4} = \frac{-8x^4}{2x^4} = \frac{-8}{2} = -4$$

HA: $y = -4$

$\frac{\text{Large Deg}}{\text{small Deg}}$: No HA

$\frac{\text{Small Deg}}{\text{Large Deg}}$: $y=0$

Identify the illustrated function from the options listed. *(The vertical asymptote is provided.)*

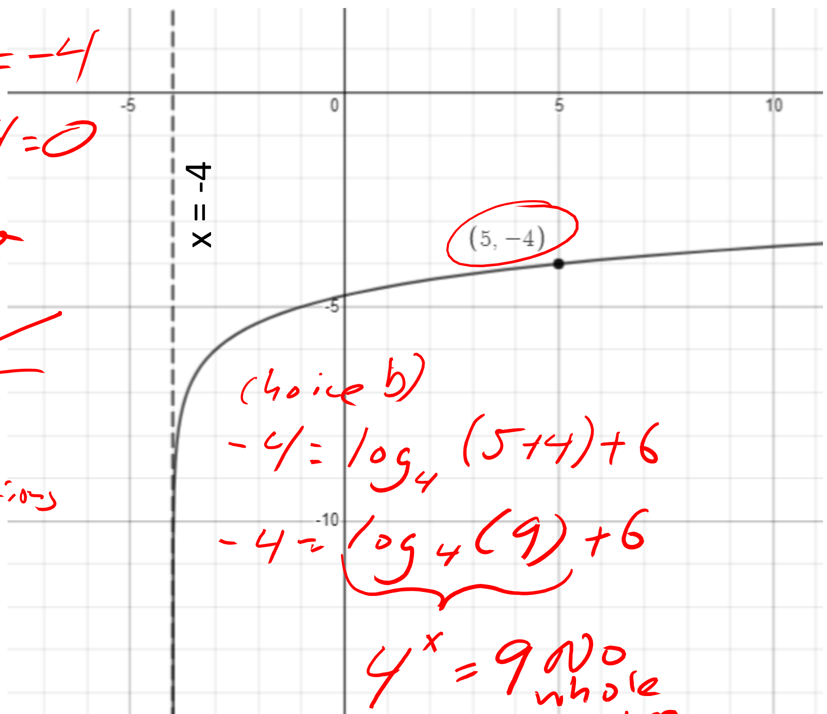
VA: $x = -4$

$x + 4 = 0$

inside

$y = \log_a x$

No Reflections



(choice b)

$-4 = \log_4 (5+4) + 6$

$-4 = \underbrace{\log_4 (9)} + 6$

$4^x = 9$ No whole number

Base $\neq 4$

~~a)~~ $f(x) = \log_3(x-4) - 6$ VA

~~b)~~ $f(x) = \log_4(x+4) + 6$ Base $\neq 4$

~~c)~~ $f(x) = -\log_3(x-4) - 6$ VA

d) $f(x) = \log_3(x+4) - 6$ $\log_3(5+4) - 6 =$

$\log_3 9 - 6$

$2 - 6$

-4

~~e)~~ $f(x) = \log_1(x+4) - 6$ Base $= 1$

f) $f(x) = \log_3(x+4) + 6$

~~g)~~ $f(x) = \log_{(-2)}(x-4) - 6$ Base $= -2$

~~h) $f(x) = \log_3(x+4) - 6$~~ Duplicate

~~i)~~ $f(x) = \log_4(x+4) - 6$ Base $\neq 4$

Determine the equation of the asymptote of for the following: $f(x) = -2.4 \cdot 5^{x+2} \underbrace{- 8}_{HA}$

HA: $y = -8$

Evaluate the following logarithms. *

- $\log_3(81)$ = X
  $3^X = 81 \rightarrow X = 4$
- $\log(0.001)$
  $\log_{10} \frac{1}{1000}$ = X
  $10^x = \frac{1}{1000}$
  Fract → Neg X = -3
  $10^{\square} = 1000 \rightarrow 3$
- $\cancel{\ln}(e^{17})$ = 17
  Base e
- $\cancel{8^{\log_8}}\sqrt{2}$ = $\sqrt{2}$
- $\cancel{0.3^{\log_{(0.3)}}}\ 9$ = 9
- $e^{\ln(-4)}$ No Solution
  cannot be Negative
- $\log_{(-3)}(^1/_{81})$ No Solution
  cannot be Negative
- $\log_5(625)$ = X
  $5^X = 625$
  X = 4