MATH 1314

Test 4 Review

18 Multiple Choice Questions

1. Sketch the graph of $P(x) = -2x(x+1)(3-x)^3$

$-2x(x+1)(-x+3)^3$

Leading Term: $(-2x)(x)(-x)^3 =$

Degree: 5 $= -2x \cdot x \cdot -x^3 = 2x^5$

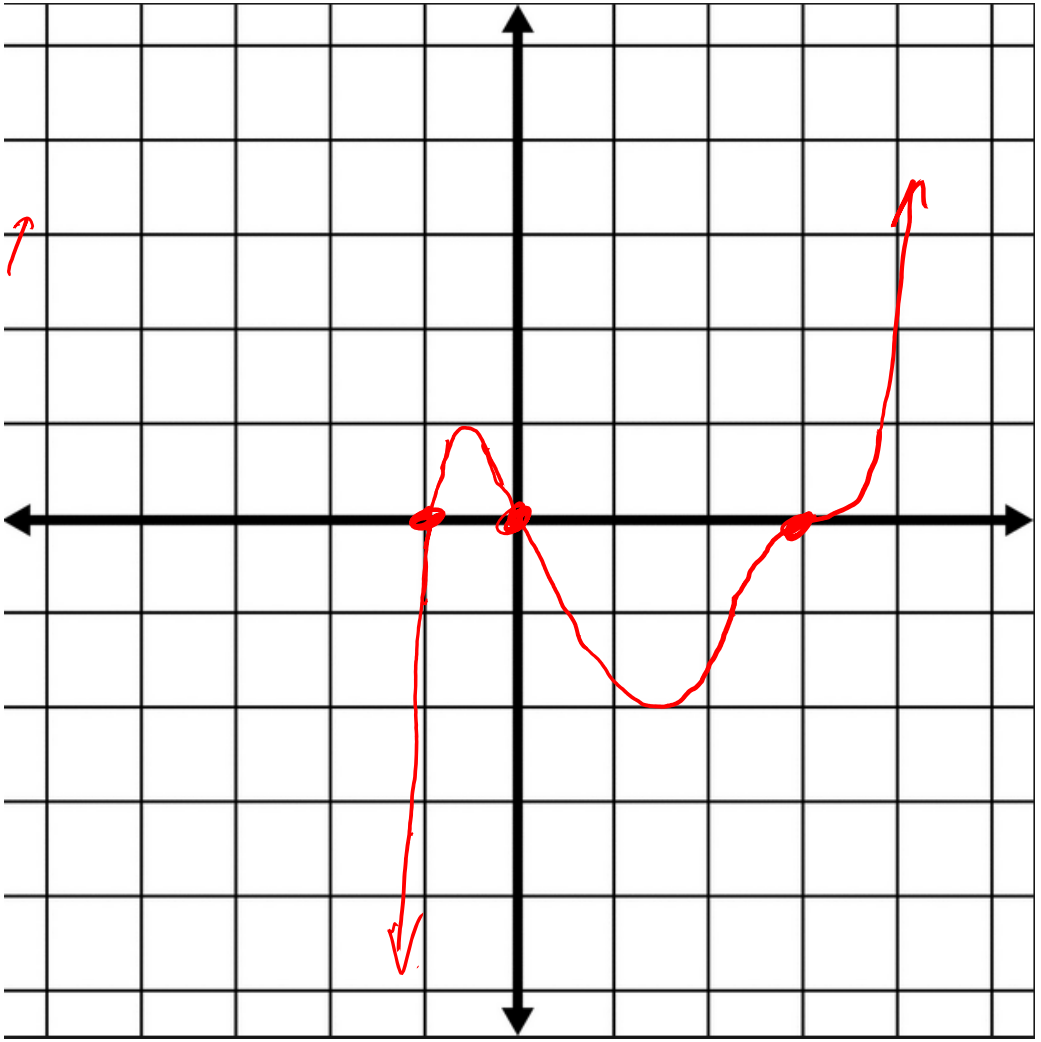
End Behavior (pos, odd): Left: ↓ Right: ↑

x-intercepts:

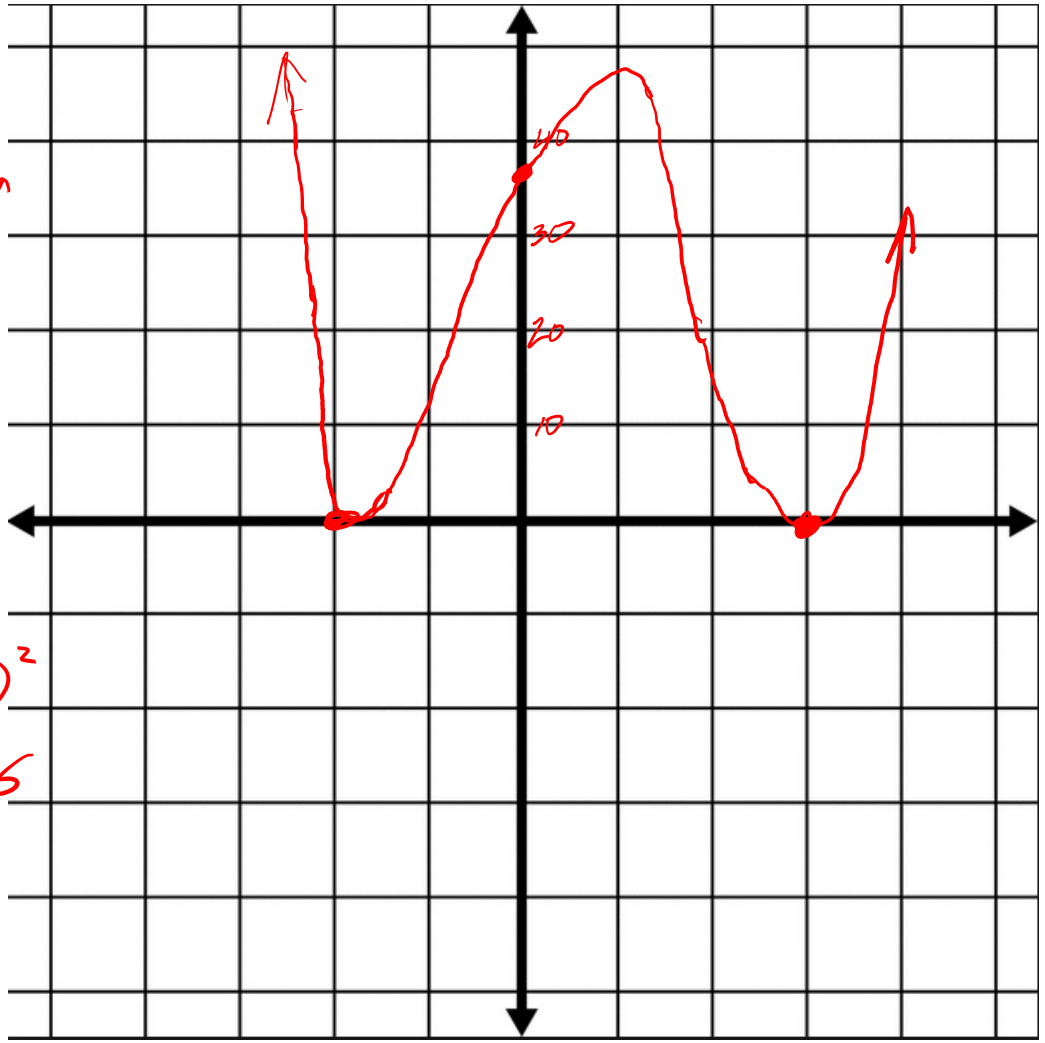
$-2x = 0$	$x+1 = 0$	$3-x = 0$
$x = 0$	$x = -1$	$-x = -3$
		$x = 3$
M: 1	M: 1	M: 3
(Linear)	(Linear)	(cubic)

y-intercept:

$P(0) = -2(0)(0+1)(3-0)^3 = 0$



2. **Sketch the graph of $P(x) = (x-3)^2(x+2)^2$**



Leading Term: $(x)^2(x)^2 = x^2 \cdot x^2 = x^4$

Degree: 4 End Behavior (Even, Pos): Left: ↑, Right: ↑

x-intercepts:

$x-3=0$ $\quad$ $x+2=0$

$x=3$ $\quad$ $x=-2$

M: 2 (Quadratic) $\quad$ M: 2 (Quadratic)

y-intercepts:

$P(0) = (0-3)^2(0+2)^2 = (-3)^2(2)^2$

$= 9 \cdot 4 = 36$

3. Find the quotient and remainder for $\dfrac{2x^3 - 13x^2 - 10x + 19}{2x + 3}$

Long Division

$x^2 - 8x + 7 \rightarrow Q(x)$

$2x+3 \;\overline{)\; 2x^3 - 13x^2 - 10x + 19}$

$\ominus 2x^3 \oplus 3x^2$ ↓

$-16x^2 - 10x + 19$

$\oplus 16x^2 \ominus 24x$ ↓

$14x + 19$

$\ominus 14x \oplus 21$

$-2 \rightarrow R(x)$

$Q(x) = x^2 - 8x + 7$

$R(x) = -2$

$x^2 - 8x + 7 - \dfrac{2}{2x+3}$

$Q(x)$; $\dfrac{R(x)}{D(x)}$

① $2x \cdot \square = 2x^3 \rightarrow x^2$

$x^2 \cdot (2x+3) \rightarrow 2x^3 + 3x^2$

subtract

② $2x \cdot \square = -16x^2 \rightarrow -8x$

$-8x \cdot (2x+3) \rightarrow -16x - 24x$

subtract

③ $2x \cdot \square = 14x \rightarrow 7$

$7 \cdot (2x+3) = 14x + 21$

subtract

④ Since degree of -2 is less than degree of $2x$, we are done.

4. Find the quotient and remainder for $\frac{x^3 - 2x + 12}{x - 4}$ $= \frac{x^3 + 0x^2 - 2x + 12}{x - 4}$ Synthetic Division

	1	0	−2	12
4	↓	4	16	56
	1	4	14	68

$Q(x)$ $R(x)$

$1x^2 + 4x + 14$

4 × 14
4 × (10 + 4)
40 + 16
56

$Q(x) = x^2 + 4x + 14$

$R(x) = 68$

$x^2 + 4x + 14 + \frac{68}{x-4}$

5. Find the zeros:

a. $P(x) = (x-2)^3(x^2-2x-8)$

$(x-2)^3(x-4)(x+2)$

$x-2=0$ $x=2$ M:3

$x-4=0$ $x=4$ M:1

$x+2=0$ $x=-2$ M:1

b. $P(x) = 4x^3+4x^2-x-1$

$4x^2(x+1)-1(x+1)$

$(x+1)(4x^2-1)$

$(x+1)(2x+1)(2x-1)$

$x+1=0$ $x=-1$ M:1

$2x+1=0$ $2x=-1$ $x=-1/2$ M:1

$2x-1=0$ $2x=1$ $x=1/2$ M:1

c. $P(x) = x^3+x^2+9x+9$

$x^2(x+1)+9(x+1)$

$(x+1)(x^2+9)$

$x+1=0$ $x=-1$ M:1

$x^2+9=0$ $x^2=-9$ $x=\pm\sqrt{-9}$ $x=\pm 3i$ M:1

6. **3rd degree polynomial with integer coefficient given 1, 6i and −6i with a constant coefficient of 72.**

zeros and

$x = 1$ $\quad x = 6i$ $\quad x = -6i$

$x - 1 = 0$ $\quad x - 6i = 0$ $\quad x + 6i = 0$

$P(x) = a(x-1)(x-6i)(x+6i)$

$P(x) = a(x-1)(x^2 + 6ix - 6ix - 36i^2)$

$+36$

$P(x) = a(x-1)(x^2+36)$

$P(x) = a(x^3 + 36x - x^2 - 36)$

$P(x) = a(x^3 - x^2 + 36x - 36)$

$P(x) = -2x^3 + 2x^2 - 72x + 72$

$-36 \times \boxed{-2} = 72$

$a = -2$

7. Use for questions a and b: $f(x) = \dfrac{x-4}{x+2}$

a. Find the x-intercept. (Numerator only)

$x - 4 = 0$

$x = 4$

$(4, 0)$

b. Find the y-intercept. Evaluate $f(0)$

$$f(0) = \frac{0-4}{0+2} = \frac{-4}{2} = -2$$

$(0, -2)$

8. Find the x and y intercepts, and horizontal asymptotes in the function:

vertical asymptotes, holes, sketch

$$f(x) = \frac{x^2 + x - 6}{2x^2 - 2x - 4} = \frac{(x+3)(x-2)}{2(x+1)(x-2)}$$

$2(x^2 - x - 2)$

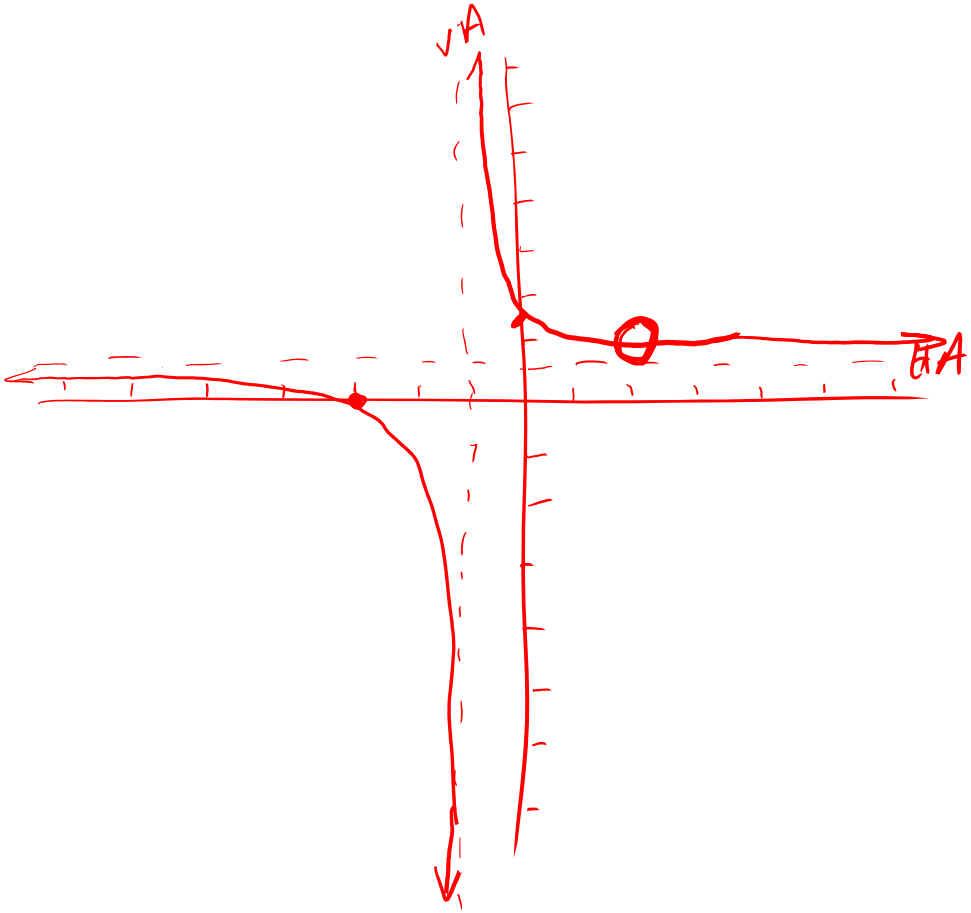
x-int: (Num, 0, by): $x+3=0 \rightarrow x=-3$

y-int: $f(0) = \frac{-6}{-4} = \frac{3}{2}$

Holes: (Num, Den): $x-2=0 \rightarrow x=2$

VA: (Den, 0, by) $x+1=0 \rightarrow x=-1$

HA: (compare Degrees) $\frac{\text{Deg}}{\text{Deg}} = \frac{1x^2}{2x^2}$ $y=\frac{1}{2}$



9. Find the vertical asymptote(s) and hole(s) for

$$f(x) = \frac{x^2 + 8x + 12}{x^2 + x - 30} = \frac{(x+2)(x+6)}{(x-5)(x+6)}$$

VA: $x - 5 = 0 \rightarrow x = 5$

Hole: $x + 6 = 0 \rightarrow x = -6$

10. State the following and clearly label the graph.

a. x-intercepts
b. hole(s)
c. y-intercepts
d. vertical asymptotes
e. horizontal asymptotes

$$f(x) = \frac{x-4}{x+2}.$$

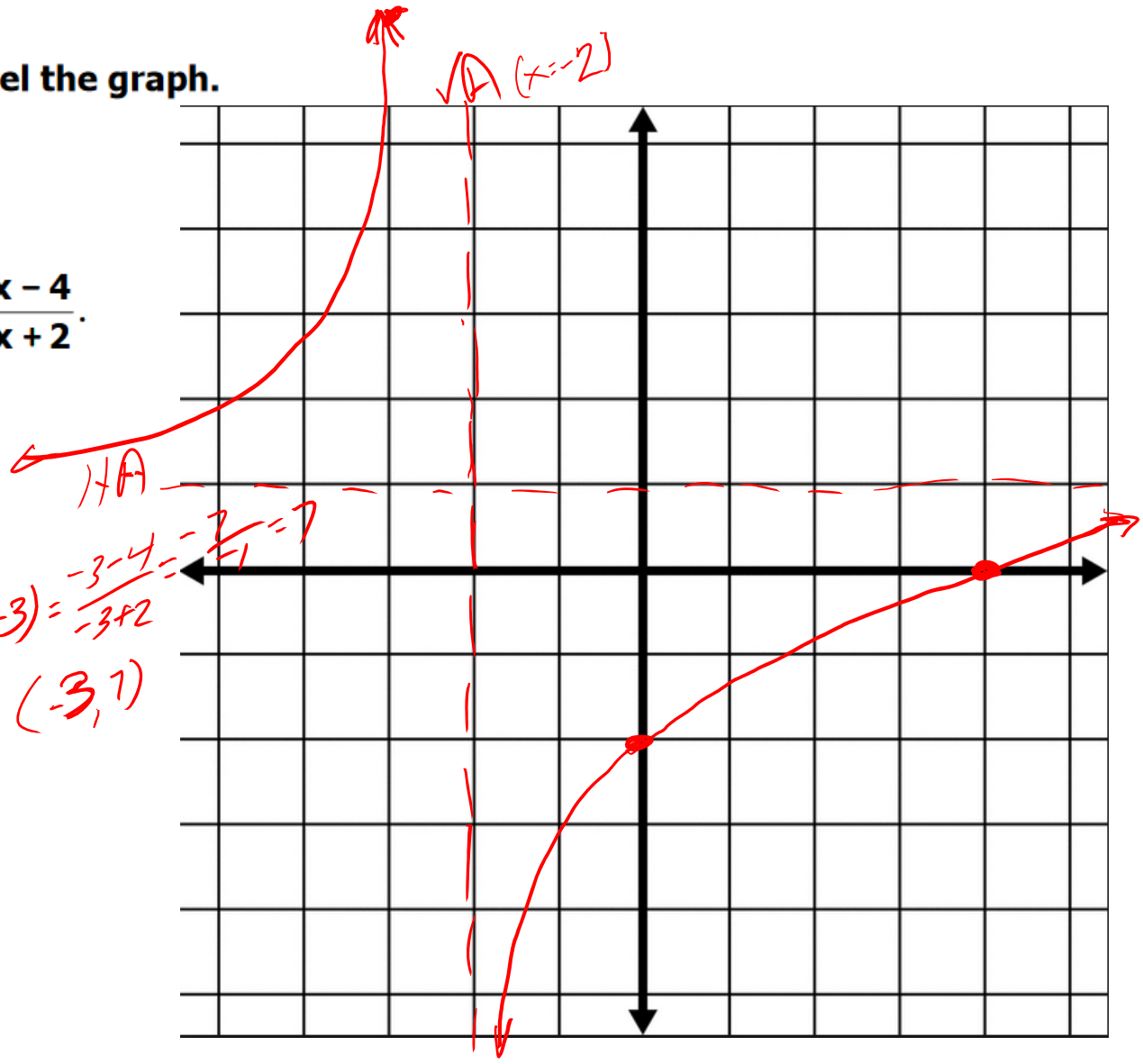
a) $x-4=0 \rightarrow x=4$

b) No Hole

c) $f(0) = \frac{-4}{2} = -2$

d) $x+2=0 \rightarrow x=-2$

e) $\frac{\text{Deg}1}{\text{Deg}1} \rightarrow \frac{1x}{1x}$ $y=1$



11. Find the exponential function of the form $f(x) = a^x$ which passes through the point (0, 1) and (2, 25).

$f(x) = a^x$ (Looking for a-value)

Plug in (0,1)

$1 = a^0$

$1 = 1$

(We learned nothing new from (0,1))

Plug in (2,25)

$\sqrt{25} = \sqrt{a^2}$

$\pm 5 = a$

But wait!

Base of an exponential function cannot be 0, 1, Neg

$a = 5$

$f(x) = 5^x$

12. Given $f(x) = 3^{x-2} + 2$

HA

Horizontal: Right 2

Vertical: up 2

a. Use transformations to determine the coordinates of key point (0, 1).

$(0,1) \rightarrow (2,1) \rightarrow (2,3)$

b. Asymptote? (HA)

$Y = 2$

c. Range Exponential Term is Positive → Above HA

$(2, \infty)$

d. Domain: $(-\infty, \infty)$

13. Given $f(x) = -e^{x+1}$ $+0$

Horizontal: Left 1
X-axis Refl: (neg)

a. Use the transformations to determine the coordinate of (0, 1).

$(0,1) \rightarrow (-1,1) \rightarrow (-1,-1)$

b. Asymptote? HA: $Y=0$

c. Range? Exp Term: Neg

$(-\infty, 0)$

d. Domain: $(-\infty, \infty)$

14. Find the y intercept for the following functions:

a. $f(x) = 4^{x+2} - 6$ $\quad f(0) = 4^{0+2} - 6 = 4^2 - 6 = 16 - 6 = 10 \rightarrow (0, 10)$

b. $f(x) = -e^x - 2$ $\quad f(0) = -e^0 - 2 = (-1)e^0 - 2 = -1(1) - 2 = -1 - 2 = -3$

$(0, -3)$

$-e^0$ is Not $(-e)^0$

15. Write as an exponential function:

a. $\log_3 x = y$ $3^y = x$

b. $\ln 4 = y \rightarrow \log_e 4 = y$ $e^y = 4$

c. $\log 100 = 2 \rightarrow \log_{10} 100 = 2$ $10^2 = 100$

16. Write in the logarithmic form:

a. $e^2 = x$ $\log_e x = 2 \rightarrow \ln x = 2$

b. $3^3 = 27$ $\log_3 27 = 3$

c. $5^{-2} = \frac{1}{25}$ $\log_5 \left(\frac{1}{25}\right) = -2$

17. Evaluate

a. $\log_2 4 = x$

$2^x = 4$

$x = 2$

b. $\log_2 \sqrt{2} = x$

$2^x = \sqrt{2}$

$x = 1/2$

c. $\log_{\frac{1}{4}} 16 = x$

$4^x = \frac{1}{16}$

Fract → Neg

$4^2 = 16 \rightarrow 2$

$x = -2$

d. $\ln(-3)$

No Solution

e. $2^{\log_2 6} = 6$

f. $\log_4 1 = x$

$4^x = 1$

$x = 0$

g. $\log_4 4 = x$

$4^x = 4$

$x = 1$

h. $e^{\ln 4} = 4$

i. $\log 0.01 = \log_{10}\left(\frac{1}{100}\right) = x$

$10^x = \frac{1}{100}$

Fract → Neg

$10^2 = 100 \rightarrow 2$

$x = -2$

j. $\log_{1/2}\left(\frac{1}{2}\right)^8 = 8$

k. $\log_6 6^{-3} = -3$

m. $9^{\log_9(-2)}$ ← Can never be Negative

No Solution

Horizontal: Left 2

Vertical: Down

18. Given $f(x) = \log_2(x+2) - 1$

a. Use the transformations to determine the coordinate of (1,0).

$(1,0) \rightarrow (-1,0) \rightarrow (-1,-1)$

b. Asymptote? VA: $x = -2$

c. Range? $(-\infty, \infty)$

d. Domain? $x+2 > 0$

$x > -2$

$(-2, \infty)$

19. Find the domain:

a. $f(x) = \ln(2 - x) - 2$

$2 - x > 0$

$-x > -2$

$x < 2$

$(-\infty, 2)$

b. $f(x) = \log_3(2x + 4) - 2$

$2x + 4 > 0$

$2x > -4$

$x > \frac{-4}{2}$

$x > -2$

$(-2, \infty)$

20. The polynomial $p(x) = x^3 - 7x^2 + 7x + 15$ has one root at $x = 5$. Determine the value of all roots.

(a factor of $x-5$)

	1	−7	7	15
5	↓	5	−10	−15
	1	−2	−3	0 → Expected to be zero.

$Q(x) = x^2 - 2x - 3$

$(x-3)(x+1)$

$x-3=0 \quad x+1=0$

$x=3 \quad x=-1$

$\{-1, 3, 5\}$

21. Determine the exponential equation of the following graph [in base 3]:

$f(x) = 3^{x+a} + b$ (b: HA)

$f(x) = 3^{x+a} + 1$ (Based on HA)

$y = 3^x$ → Reflect Left & Right → Y-axis reflection → 3^{-x+a}

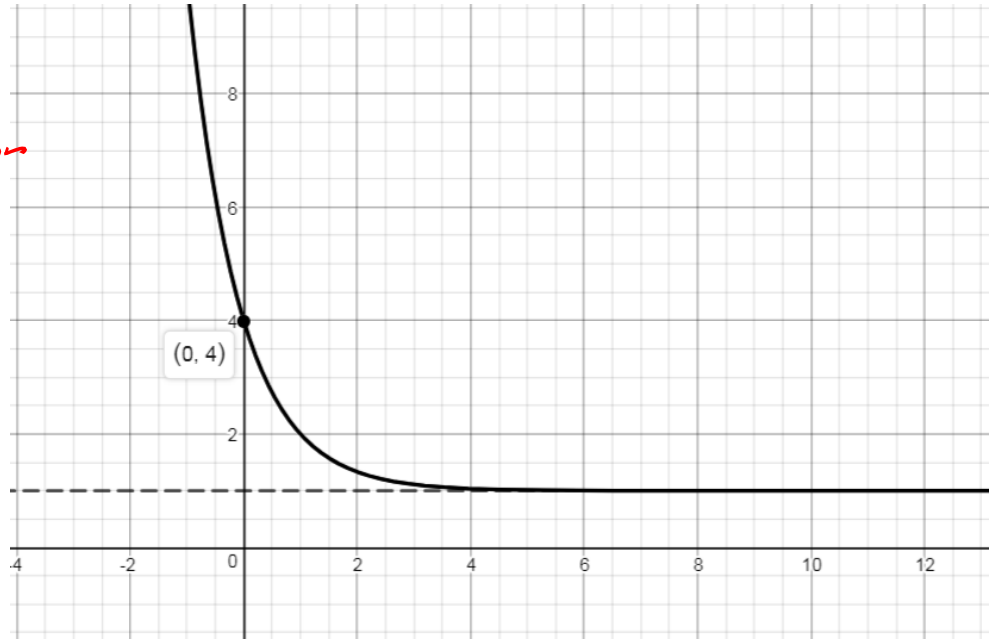
$f(x) = 3^{-x+a} + 1$

Plug in (0, 4) (Solve for a)

$4 = 3^{-0+a} + 1$

$4 = 3^a + 1 \rightarrow 3 = 3^a \rightarrow a = 1$

HA $y = 1$



$f(x) = 3^{-x+1} + 1$

22. Determine the equation of the logarithmic equation of the following graph [in base 8]:

$\log_8 8 = x$

$8^x = 8$

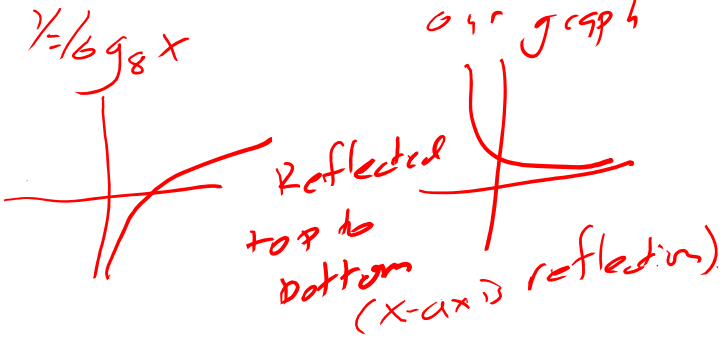
$x = 1$

$f(x) = \log_8(x+a) + b$

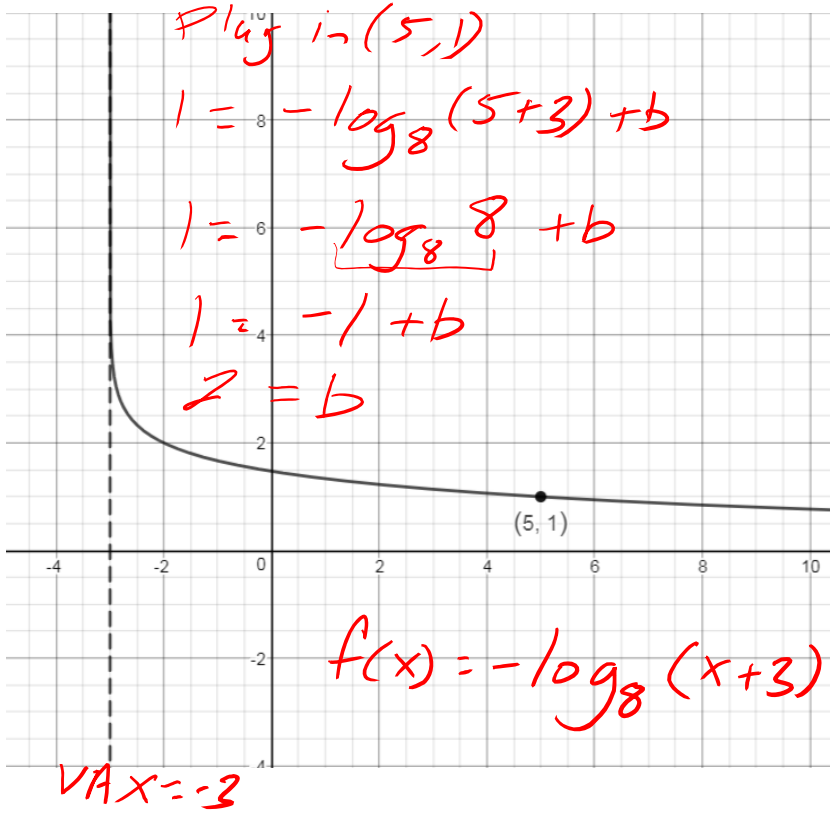
VA: $x = -3 \rightarrow x + 3 = 0$ (inside)

$f(x) = \log_8(x+3) + b$

$y = \log_8 x$ our graph



$f(x) = -\log_8(x+3) + b$



$f(x) = -\log_8(x+3) + 2$

23. Which of the following is a correct function form for the indicated parent function (there may be multiple answers).

Rules for Bases:
Cannot be: 0
1
neg

Exponential Function:

- $f(x) = 2 \cdot 3^x$ Base 3
- $g(x) = 2 \cdot (-3)^x$ Base (-3)
- $h(x) = 2 \cdot 1^x$ Base (1)
- $j(x) = 2 \cdot 0^x$ Base (0)

Logarithmic Function:

- $f(x) = 2\log(3x)$ Base 10
- $g(x) = 2\ln(3x)$ Base e
- $h(x) = 2\log_{-3}x$ Base -3
- $j(x) = 2\log_3 x$ Base 3

Popper 24:

Fill out choice D for questions 1 – 5