

# MATH 1314

Test 4 Review

18 Multiple Choice Questions

1. Sketch the graph of  $P(x) = -2x(x+1)(3-x)^3$

Leading Term:  $(-2x)(x)(-x)^3 = -2x(x+1)(-x+3)^3$

Degree: 5  $= -2x^1 \cdot x^1 \cdot -x^3 = 2x^5$

End Behavior (pos, odd): Left:  $\downarrow$  Right:  $\uparrow$

x-intercepts:

$$-2x = 0$$

$$x = 0$$

M:1  
(Linear)

$$x+1 = 0$$

$$x = -1$$

M:1  
(Linear)

$$3-x = 0$$

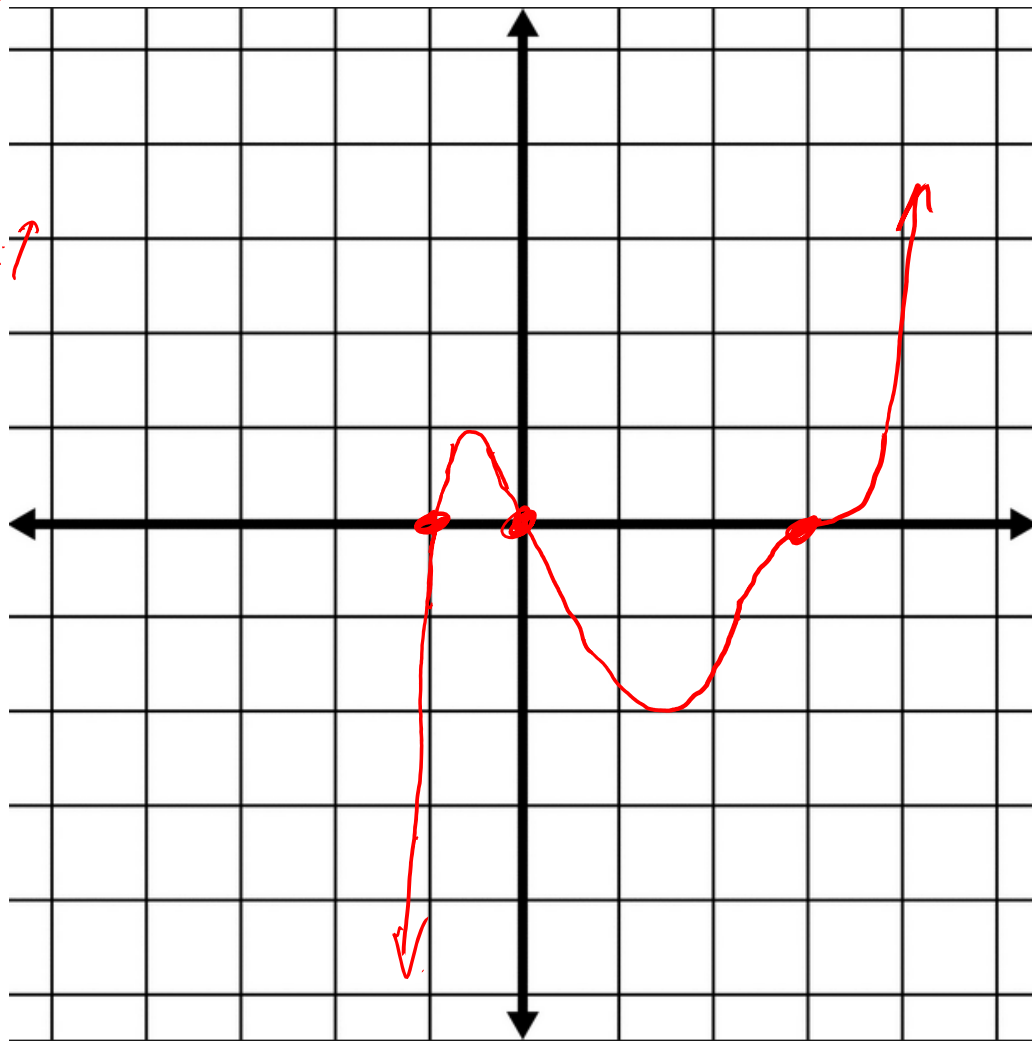
$$-x = -3$$

$$x = 3$$

M:3  
(cubic)

y-intercept:

$$P(0) = -2(0)(0+1)(3-0)^3 = 0$$



## 2. Sketch the graph of $P(x) = (x-3)^2(x+2)^2$

Leading Term:  $(x)^2(x)^2 = x^2 \cdot x^2 = x^4$

Degree: 4 End Behavior (Even, Pos).  
Left:  $\uparrow$ , Right:  $\uparrow$

X-intercepts:

$$x-3=0$$

$$x=3$$

$$M: 2$$

(Quadratic)

$$x+2=0$$

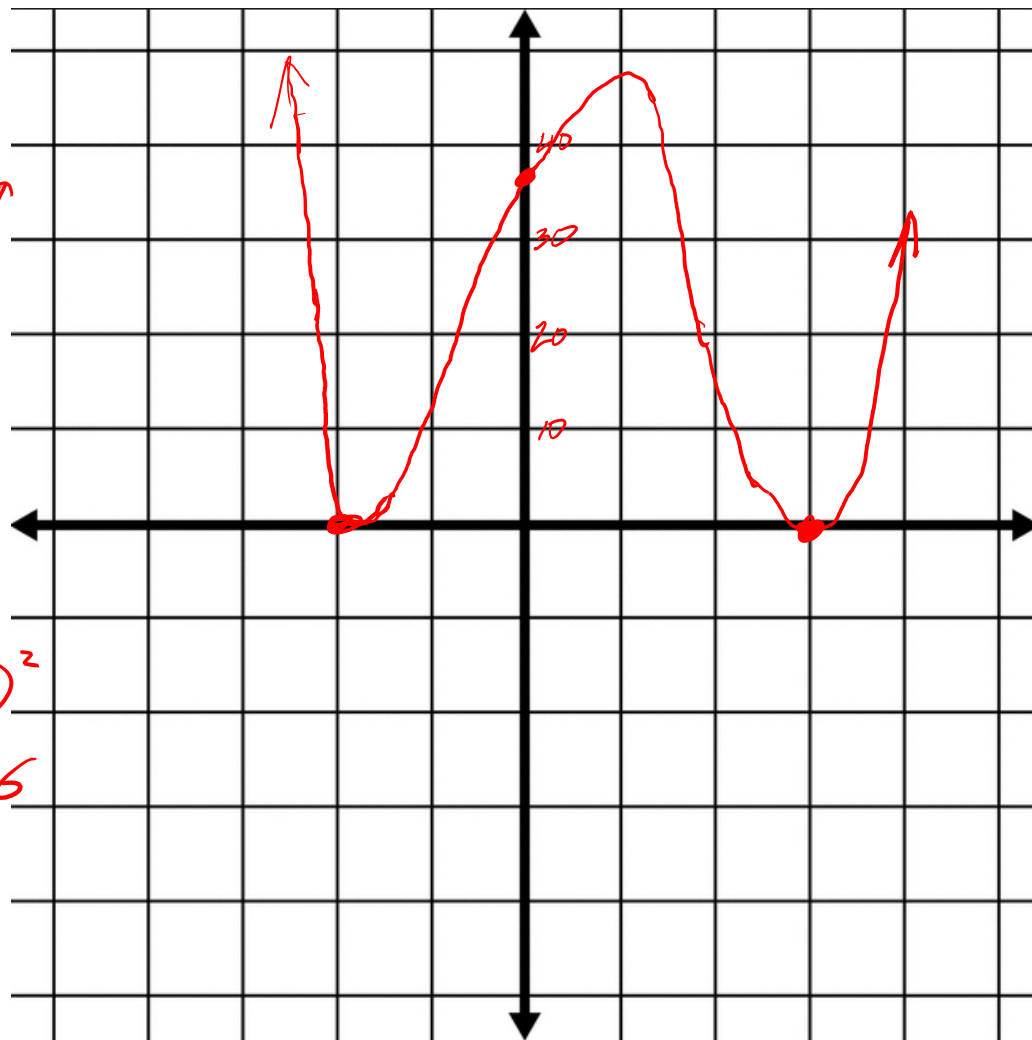
$$x=-2$$

$$M: 2$$

(Quadratic)

Y-intercept:

$$\begin{aligned} P(0) &= (0-3)^2(0+2)^2 = (-3)^2(2)^2 \\ &= 9 \cdot 4 = 36 \end{aligned}$$



3. Find the quotient and remainder for  $\frac{2x^3 - 13x^2 - 10x + 19}{2x + 3}$

$$\begin{array}{r}
 x^2 - 8x + 7 \rightarrow Q(x) \\
 2x+3 \overline{) 2x^3 - 13x^2 - 10x + 19} \\
 \underline{\ominus 2x^3 + 3x^2} \quad \downarrow \\
 -16x^2 - 10x + 19 \\
 \underline{\oplus 16x^2 + 24x} \quad \downarrow \\
 14x + 19 \\
 \underline{\ominus 14x + 21} \\
 -2 \rightarrow R(x)
 \end{array}$$

$$\left. \begin{array}{l} Q(x) = x^2 - 8x + 7 \\ R(x) = -2 \end{array} \right\} \underbrace{x^2 - 8x + 7}_{Q(x)} - \underbrace{\frac{2}{2x+3}}_{\frac{R(x)}{D(x)}}$$

Long Division

$$\textcircled{1} 2x \times \square = 2x^3 \rightarrow x^2$$

$$x^2 \times (2x+3) \rightarrow 2x^3 + 3x^2$$

subtract

$$\textcircled{2} 2x \times \square = -16x^2 \rightarrow -8x$$

$$-8x \times (2x+3) \rightarrow -16x^2 - 24x$$

subtract

$$\textcircled{3} 2x \times \square = 14x \rightarrow 7$$

$$7 \times (2x+3) = 14x + 21$$

subtract

\textcircled{4} Since degree of  $-2$  is less than degree of  $2x$ , we are done.

4. Find the quotient and remainder for  $\frac{x^3 - 2x + 12}{x - 4} = \frac{x^3 + 0x^2 - 2x + 12}{x - 4}$  Synthetic Division

$$4 \left| \begin{array}{cccc} 1 & 0 & -2 & 12 \\ & \downarrow & & \\ & 4 & 16 & 68 \end{array} \right.$$

$$\begin{array}{cccc} 1 & 4 & 14 & 68 \\ \hline & & & \downarrow \\ & & & R(x) \end{array}$$

$$1x^2 + 4x + 14$$

$$\left. \begin{array}{l} Q(x) = x^2 + 4x + 14 \\ R(x) = 68 \end{array} \right\} x^2 + 4x + 14 + \frac{68}{x-4}$$

$$\begin{array}{l} 4 \times 14 \\ 4 \times (10 + 4) \\ 40 + 16 \\ 56 \end{array}$$

## 5. Find the zeros:

a.  $P(x) = (x-2)^3(x^2 - 2x - 8)$

$$(x-2)^3(x-4)(x+2)$$

$$x-2=0$$

$$x=2$$

$$M:3$$

$$x-4=0$$

$$x=4$$

$$M:1$$

$$x+2=0$$

$$x=-2$$

$$M:1$$

b.  $P(x) = (4x^3 + 4x^2 - x - 1)$

$$4x^2(x+1) - 1(x+1)$$

$$(x+1)(4x^2 - 1)$$

$$(x+1)(2x+1)(2x-1)$$

$$x+1=0$$

$$x=-1$$

$$M:1$$

$$2x+1=0$$

$$2x=-1$$

$$x=-\frac{1}{2}$$

$$M:1$$

$$2x-1=0$$

$$2x=1$$

$$x=\frac{1}{2}$$

$$M:1$$

c.  $P(x) = (x^3 + x^2 + 9x + 9)$

$$x^2(x+1) + 9(x+1)$$

$$(x+1)(x^2 + 9)$$

$$x+1=0$$

$$x=-1$$

$$M:1$$

$$x^2 + 9 = 0$$

$$x^2 = -9$$

$$x = \pm\sqrt{-9}$$

$$x = \pm 3i$$

$$M:1$$

6. 3<sup>rd</sup> degree polynomial with integer coefficient given 1,  $6i$  and  $-6i$  with a constant coefficient of 72. zeros are

$$\begin{array}{lll} x=1 & x=6i & x=-6i \\ x-1=0 & x-6i=0 & x+6i=0 \end{array}$$

$$P(x) = a(x-1)(x-6i)(x+6i)$$

$$P(x) = a(x-1)(x^2 + 6ix - 6ix - 36i^2)$$

$\underbrace{\hspace{10em}}_{+36}$

$$P(x) = a(x-1)(x^2 + 36)$$

$$P(x) = a(x^3 + 36x - x^2 - 36)$$

$$P(x) = a(x^3 - x^2 + 36x - 36)$$

$$P(x) = -2x^3 + 2x^2 - 72x + 72$$

$$-36 \times \boxed{-2} = 72$$

$$a = -2$$

Popper 29:

Fill out choice A for questions 1 – 5



7. Use for questions a and b:  $f(x) = \frac{x-4}{x+2}$

a. Find the x-intercept. (Numerator only)

$$\begin{aligned}x-4 &= 0 & (4, 0) \\x &= 4\end{aligned}$$

b. Find the y-intercept. Evaluate  $f(0)$

$$f(0) = \frac{0-4}{0+2} = \frac{-4}{2} = -2$$

$$(0, -2)$$

8. Find the x and y intercepts, and horizontal asymptotes in the function:

\*

vertical asymptotes  
holes, sketch

$$f(x) = \frac{x^2 + x - 6}{2x^2 - 2x - 4} = \frac{(x+3)(x-2)}{2(x+1)(x-2)}$$

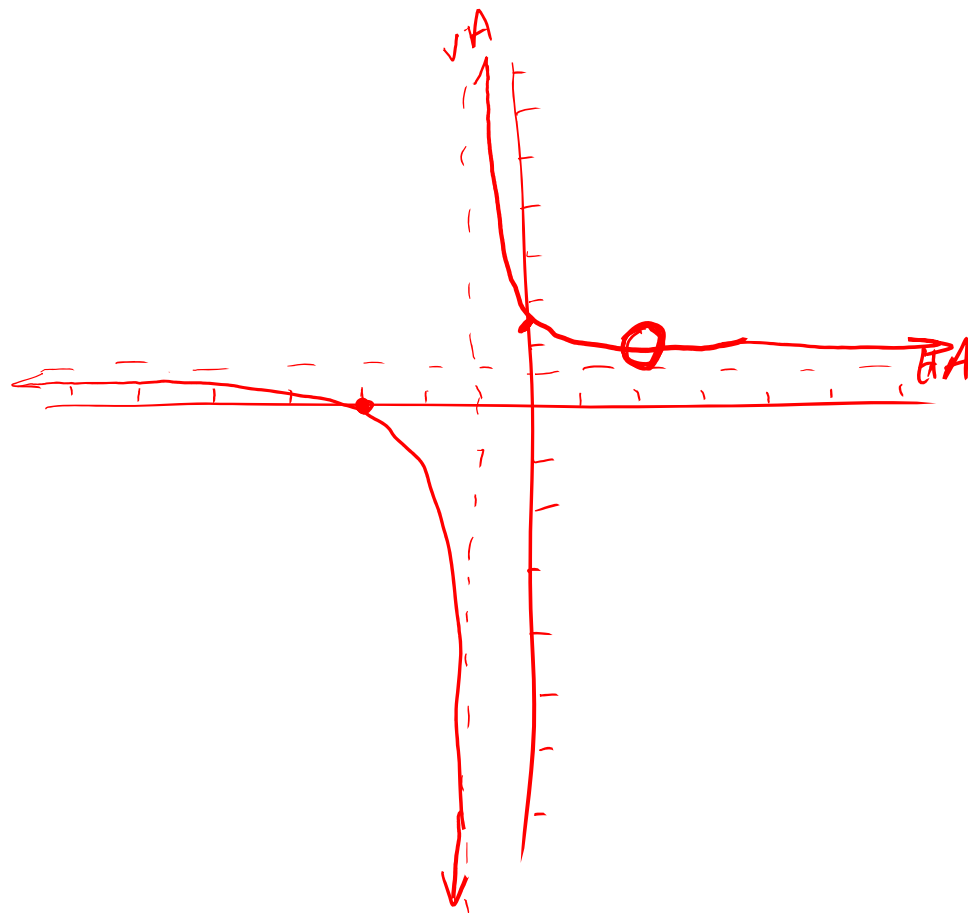
x-int: (Num Only):  $x+3=0 \rightarrow x=-3$

y-int:  $f(0) = \frac{-6}{-4} = \frac{3}{2}$

Holes: (Num & Den):  $x-2=0 \rightarrow x=2$

VA: (Den Only):  $x+1=0 \rightarrow x=-1$

HA: (compare Degs):  $\frac{\text{Deg } 2}{\text{Deg } 2} = \frac{1x^2}{2x^2} \quad y = \frac{1}{2}$



**9. Find the vertical asymptote(s) and hole(s) for**

$$f(x) = \frac{x^2 + 8x + 12}{x^2 + x - 30} = \frac{(x+2)(x+6)}{(x-5)(x+6)}$$

$$\text{VA: } x - 5 = 0 \rightarrow x = 5$$

$$\text{Hole: } x + 6 = 0 \rightarrow x = -6$$

10. State the following and clearly label the graph.

a. x-intercepts

b. hole(s)

c. y-intercepts

d. vertical asymptotes

e. horizontal asymptotes

$$f(x) = \frac{x-4}{x+2}$$

a)  $x-4=0 \rightarrow x=4$

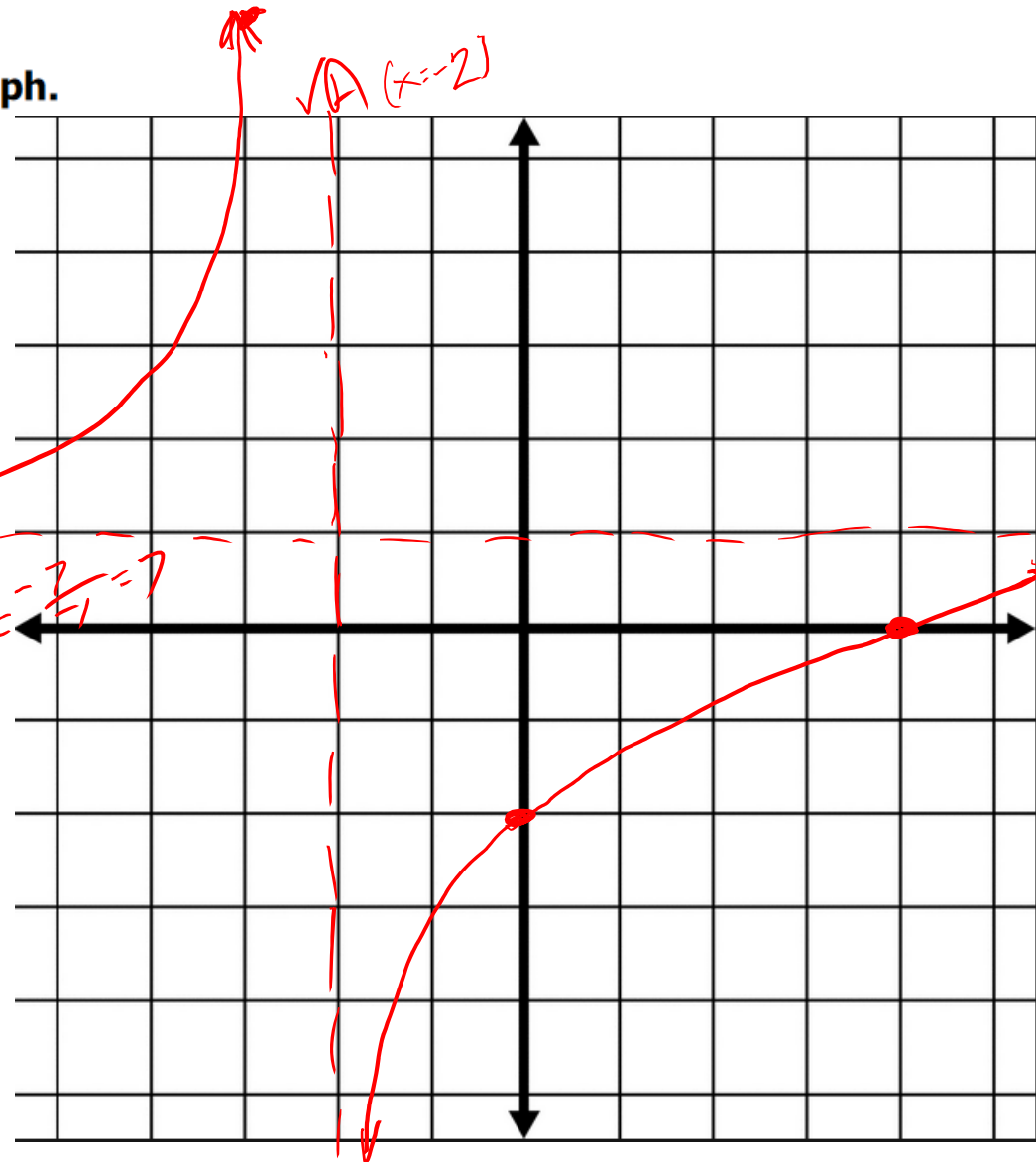
b) No Holes

c)  $f(0) = \frac{-4}{2} = -2$

d)  $x+2=0 \rightarrow x=-2$

e)  $\frac{\text{Deg } 1}{\text{Deg } 1} \rightarrow \frac{1x}{1x} \quad y=1$

$f(-3) = \frac{-3-4}{-3+2} = \frac{-7}{-1} = 7$   
 $(-3, 7)$



11. Find the exponential function of the form  $f(x) = a^x$  which passes through the point  $(0, 1)$  and  $(2, 25)$ .

$$f(x) = a^x \quad (\text{Looking for } a\text{-value})$$

Plug in  $(0, 1)$

$$1 = a^0$$

$$1 = 1$$

(We learned nothing new from  $(0, 1)$ )

Plug in  $(2, 25)$

$$\sqrt{25} = \sqrt{a^2}$$

$$\pm 5 = a$$

But wait!

Base of an exponential function cannot be 0, 1, Neg

$$a = 5$$

$$f(x) = 5^x$$

12. Given  $f(x) = 3^{x-2} + 2$

HA

- a. Use transformations to determine the coordinates of key point (0, 1).

$$(0, 1) \rightarrow (2, 1) \rightarrow (2, 3)$$

- b. Asymptote? (HA)

$$y = 2$$

- c. Range Exponential Term is Positive  $\rightarrow$  Above HA

$$(2, \infty)$$

d. Domain:  $(-\infty, \infty)$

Horizontal: Right 2

Vertical: up 2

13. Given  $f(x) = -e^{x+1}$  +0

a. Use the transformations to determine the coordinate of (0, 1).

$$(0, 1) \rightarrow (-1, 1) \rightarrow (-1, -1)$$

b. Asymptote? H A:  $y = 0$

c. Range? Exp Term: Neg

$$(-\infty, 0)$$

d. Domain:  $(-\infty, \infty)$

Horizontal: Left 1  
X-axis Ref.: (neg)

14. Find the y intercept for the following functions:

a.  $f(x) = 4^{x+2} - 6$

$$f(0) = 4^{0+2} - 6 = 4^2 - 6 = 16 - 6 = 10 \rightarrow (0, 10)$$

b.  $f(x) = -e^x - 2$

$$f(0) = -e^0 - 2 = (-1)e^0 - 2 = -1(1) - 2 = -1 - 2 = -3$$

$(0, -3)$

$-e^0$  is Not  $(-e)^0$



**15. Write as an exponential function:**

a.  $\log_3 x = y$

$$3^y = x$$

b.  $\ln 4 = y$

$$\rightarrow \log_e 4 = y \quad e^y = 4$$

c.  $\log_{10} 100 = 2$

$$\rightarrow \log_{10} 100 = 2 \quad 10^2 = 100$$

16. Write in the logarithmic form:

a.  $e^2 = x$        $\log_e x = 2 \rightarrow \ln x = 2$

b.  $3^3 = 27$        $\log_3 27 = 3$

c.  $5^{-2} = \frac{1}{25}$        $\log_5 \left(\frac{1}{25}\right) = -2$

## 17. Evaluate

a.  $\log_2 4 = x$

$$2^x = 4$$
$$x = 2$$

b.  $\log_2 \sqrt{2} = x$

$$2^x = \sqrt{2}$$
$$x = 1/2$$

c.  $\log_4 \frac{1}{16} = x$

$$4^x = \frac{1}{16}$$

Fract  $\rightarrow$  Neg  
 $4^2 = 16 \rightarrow 2$

$$x = -2$$

d.  $\ln(-3)$

No Solution

e.  ~~$2^{\log_2 6} = 6$~~

f.  $\log_4 1 = x$

$$4^x = 1$$
$$x = 0$$

g.  $\log_4 4 = x$

$$4^x = 4$$
$$x = 1$$

h.  ~~$e^{\ln 4} = 4$~~

i.  $\log 0.01 = \log_{10} \left(\frac{1}{100}\right) = x$

$10^x = \frac{1}{100}$  Fract  $\rightarrow$  Neg  
 $10^2 = 100 \rightarrow 2$   
 $x = -2$

j.  ~~$\log_{1/2} \left(\frac{1}{2}\right)^8 = 8$~~

k.  ~~$\log_6 6^{-3} = -3$~~

m.  $9^{\log_9(-2)}$   $\rightarrow$  Can never be Negative  
No Solution

18. Given  $f(x) = \log_2(x + 2) - 1$

a. Use the transformations to determine the coordinate of (1,0).

$$(1, 0) \rightarrow (-1, 0) \rightarrow (-1, -1)$$

b. Asymptote?  $\checkmark$  VA:  $x = -2$

c. Range?  $(-\infty, \infty)$

d. Domain?  $x + 2 > 0$

$$x > -2$$

$$(-2, \infty)$$

Horizontal: Left 2  
Vertical: Down 1

19. Find the domain:

a.  $f(x) = \ln(2 - x) - 2$

$$2 - x > 0$$

$$-x > -2$$

$$x < 2$$

$$(-\infty, 2)$$

b.  $f(x) = \log_3(2x + 4) - 2$

$$2x + 4 > 0$$

$$2x > -4$$

$$x > \frac{-4}{2}$$

$$x > -2$$

$$(-2, \infty)$$

20. The polynomial  $p(x) = x^3 - 7x^2 + 7x + 15$  has one root at  $x = 5$ . Determine the value of all roots.

(a factor of  $x - 5$ )

$$\begin{array}{r|rrrr} & 1 & -7 & 7 & 15 \\ 5 & \downarrow & 5 & -10 & -15 \\ \hline & 1 & -2 & -3 & 0 \end{array} \rightarrow \text{Expected to be zero.}$$

$$Q(x) = x^2 - 2x - 3$$

$$(x - 3)(x + 1)$$

$$x - 3 = 0 \quad x + 1 = 0$$

$$x = 3$$

$$x = -1$$

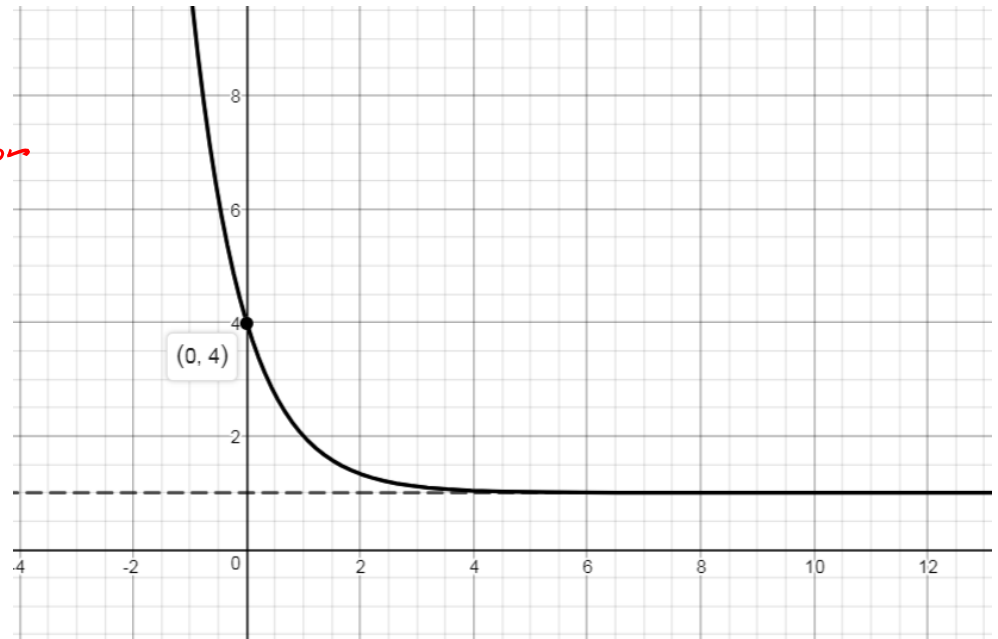
$$\{-1, 3, 5\}$$

21. Determine the exponential equation of the following graph [in base 3]:

$$f(x) = 3^{x+a} + b$$

$\underbrace{+b}_{HA}$

$$f(x) = 3^{x+a} + 1 \quad (\text{Based on HA})$$



$$f(x) = 3^{-x+a} + 1$$

Plug in  $(0, 4)$  (solve for  $a$ )    HA  $y = 1$   
 $4 = 3^{-0+a} + 1$

$$4 = 3^a + 1 \rightarrow 3 = 3^a \rightarrow a = 1$$

$$f(x) = 3^{-x+1} + 1$$

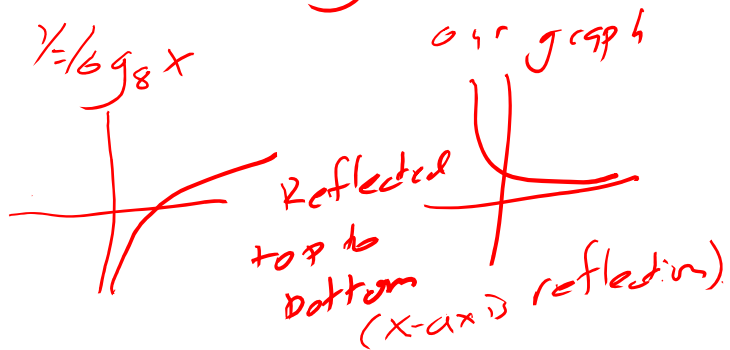
22. Determine the equation of the logarithmic equation of the following graph [in base 8]:

$\log_8 8 = x$   
 $8^x = 8$   
 $x = 1$

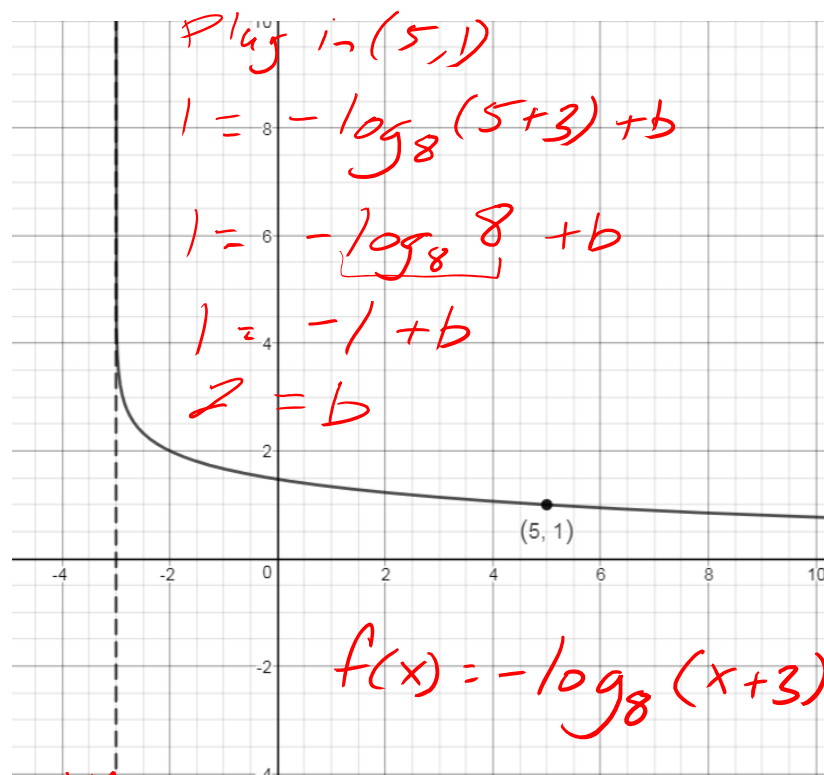
$$f(x) = \log_8(x+a) + b$$

$$VA: x = -3 \rightarrow x+3 = 0 \text{ (inside)}$$

$$f(x) = \log_8(x+3) + b$$



$$f(x) = -\log_8(x+3) + b$$



$$VA x = -3$$



23. Which of the following is a correct function form for the indicated parent function (there may be multiple answers).

Rules for Bases:

Cannot be: 0

)  
neg

Exponential Function:

Logarithmic Function:

- $f(x) = 2 \cdot 3^x$  Base 3
- $g(x) = 2 \cdot (-3)^x$  Base (-3)
- $h(x) = 2 \cdot 1^x$  Base (1)
- $j(x) = 2 \cdot 0^x$  Base (0)

- $f(x) = 2 \log(3x)$  Base 10
- $g(x) = 2 \ln(3x)$  Base e
- $h(x) = 2 \log_{-3} x$  Base -3
- $j(x) = 2 \log_3 x$  Base 3

Popper 30:

Fill out choice D for questions 1 – 5