

MATH 1314

Test 4 Review

18 Multiple Choice Questions

1. Sketch the graph of $P(x) = -2x(x+1)(3-x)^3$

Leading Term: $(-2x)(x)(-x)^3 = -2x(x+1)(-x+3)^3$

Degree: 5 $= -2x^1 \cdot x^1 \cdot -x^3 = 2x^5$

End Behavior (pos, odd): Left: \downarrow Right: \uparrow

x-intercepts:

$$-2x = 0$$

$$x = 0$$

M:1
(Linear)

$$x+1 = 0$$

$$x = -1$$

M:1
(Linear)

$$3-x = 0$$

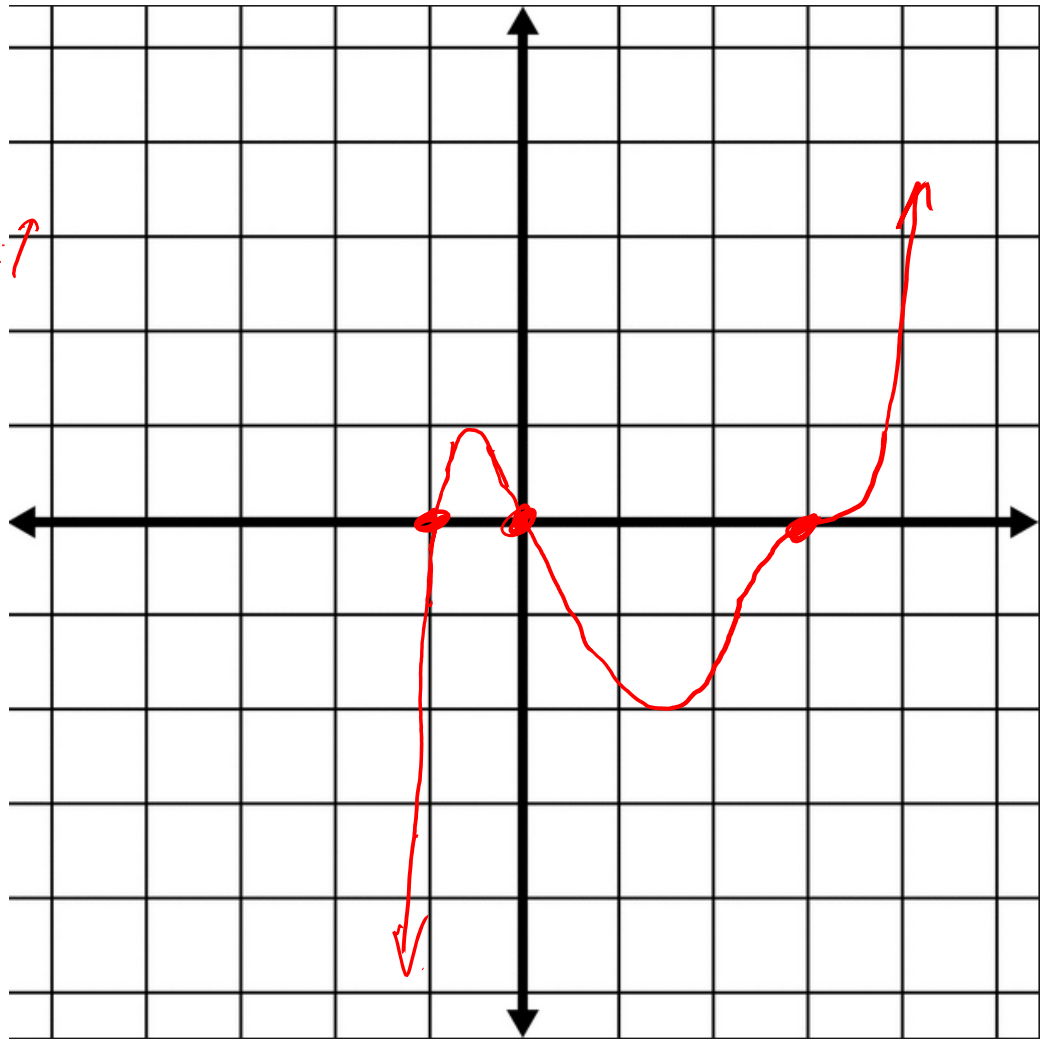
$$-x = -3$$

$$x = 3$$

M:3
(cubic)

y-intercept:

$$P(0) = -2(0)(0+1)(3-0)^3 = 0$$



2. Sketch the graph of $P(x) = (x-3)^2(x+2)^2$

Leading Term: $(x)^2(x)^2 = x^2 \cdot x^2 = x^4$

Degree: 4 End Behavior (Even, Pos).
Left: \uparrow , Right: \uparrow

X-intercepts:

$$x-3=0$$

$$x=3$$

$$M: 2$$

(Quadratic)

$$x+2=0$$

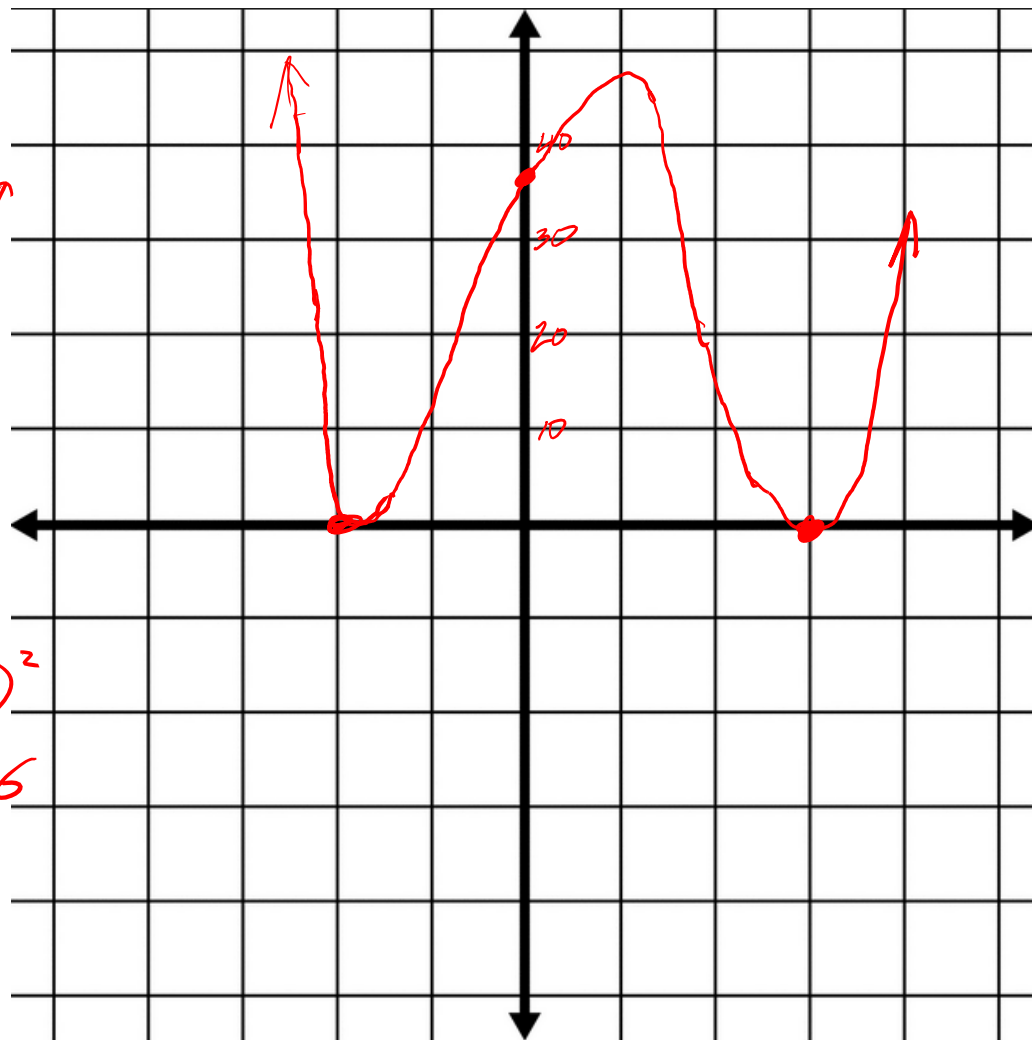
$$x=-2$$

$$M: 2$$

(Quadratic)

Y-intercept:

$$\begin{aligned} P(0) &= (0-3)^2(0+2)^2 = (-3)^2(2)^2 \\ &= 9 \cdot 4 = 36 \end{aligned}$$



3. Find the quotient and remainder for $\frac{2x^3 - 13x^2 - 10x + 19}{2x + 3}$

$$\begin{array}{r}
 x^2 - 8x + 7 \rightarrow Q(x) \\
 2x+3 \overline{) 2x^3 - 13x^2 - 10x + 19} \\
 \underline{\ominus 2x^3 + 3x^2} \quad \downarrow \\
 -16x^2 - 10x + 19 \\
 \underline{\oplus 16x^2 + 24x} \quad \downarrow \\
 14x + 19 \\
 \underline{\ominus 14x + 21} \\
 -2 \rightarrow R(x)
 \end{array}$$

$$\left. \begin{array}{l} Q(x) = x^2 - 8x + 7 \\ R(x) = -2 \end{array} \right\} \underbrace{x^2 - 8x + 7}_{Q(x)} - \underbrace{\frac{2}{2x+3}}_{\frac{R(x)}{D(x)}}$$

Long Division

$$\textcircled{1} 2x \times \square = 2x^3 \rightarrow x^2$$

$$x^2 \times (2x+3) \rightarrow 2x^3 + 3x^2$$

subtract

$$\textcircled{2} 2x \times \square = -16x^2 \rightarrow -8x$$

$$-8x \times (2x+3) \rightarrow -16x^2 - 24x$$

subtract

$$\textcircled{3} 2x \times \square = 14x \rightarrow 7$$

$$7 \times (2x+3) = 14x + 21$$

subtract

\textcircled{4} Since degree of -2 is less than degree of $2x$, we are done.

4. Find the quotient and remainder for $\frac{x^3 - 2x + 12}{x - 4} = \frac{x^3 + 0x^2 - 2x + 12}{x - 4}$ Synthetic Division

$$\begin{array}{r|rrrr} 4 & 1 & 0 & -2 & 12 \\ & \downarrow & 4 & 16 & 68 \\ \hline & 1 & 4 & 14 & 68 \end{array}$$

$$\begin{array}{r} 4 \times 14 \\ 4 \times (10+4) \\ 40+16 \\ 56 \end{array}$$

$$Q(x) = x^2 + 4x + 14$$

$$R(x) = 68$$

$$\left. \begin{array}{l} Q(x) = x^2 + 4x + 14 \\ R(x) = 68 \end{array} \right\} x^2 + 4x + 14 + \frac{68}{x-4}$$

5. Find the zeros:

a. $P(x) = (x-2)^3(x^2 - 2x - 8)$

$$(x-2)^3(x-4)(x+2)$$

$$x-2=0$$

$$x=2$$

$$M:3$$

$$x-4=0$$

$$x=4$$

$$M:1$$

$$x+2=0$$

$$x=-2$$

$$M:1$$

b. $P(x) = (4x^3 + 4x^2 - x - 1)$

$$4x^2(x+1) - 1(x+1)$$

$$(x+1)(4x^2 - 1)$$

$$(x+1)(2x+1)(2x-1)$$

$$x+1=0$$

$$x=-1$$

$$M:1$$

$$2x+1=0$$

$$2x=-1$$

$$x=-\frac{1}{2}$$

$$M:1$$

$$2x-1=0$$

$$2x=1$$

$$x=\frac{1}{2}$$

$$M:1$$

c. $P(x) = (x^3 + x^2 + 9x + 9)$

$$x^2(x+1) + 9(x+1)$$

$$(x+1)(x^2 + 9)$$

$$x+1=0$$

$$x=-1$$

$$M:1$$

$$x^2 + 9 = 0$$

$$x^2 = -9$$

$$x = \pm\sqrt{-9}$$

$$x = \pm 3i$$

$$M:1$$

6. 3rd degree polynomial with integer coefficient given 1, $6i$ and $-6i$ with a constant coefficient of 72. zeros are

$$\begin{array}{lll} x=1 & x=6i & x=-6i \\ x-1=0 & x-6i=0 & x+6i=0 \end{array}$$

$$P(x) = a(x-1)(x-6i)(x+6i)$$

$$P(x) = a(x-1)(x^2 + 6ix - 6ix - 36i^2)$$

$\underbrace{\hspace{10em}}_{+36}$

$$P(x) = a(x-1)(x^2 + 36)$$

$$P(x) = a(x^3 + 36x - x^2 - 36)$$

$$P(x) = a(x^3 - x^2 + 36x - 36)$$

$$P(x) = -2x^3 + 2x^2 - 72x + 72$$

$$-36 \times \boxed{-2} = 72$$

$$a = -2$$

Popper 29:

Fill out choice A for questions 1 – 5