MATH 1310

Test 4 Review

18 Multiple Choice

1. Sketch the graph of $P(x) = -2x(x+1)(3-x)^3$

$-2x(x+1)(-x+3)^3$

Leading Term: $(-2x)(x)(-x)^3 =$

Degree: 5 $= -2x \cdot x \cdot -x^3 = 2x^5$

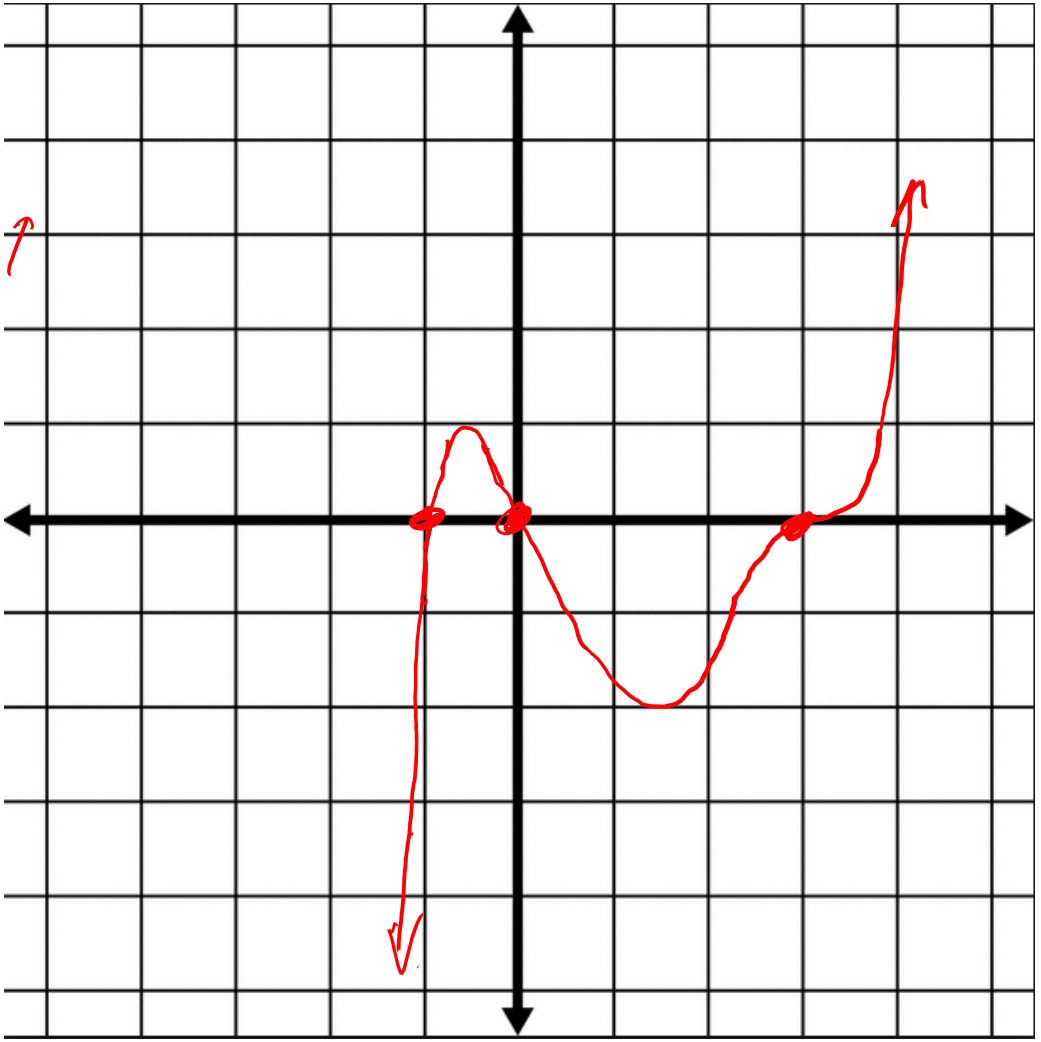
End Behavior (pos, odd): Left: ↓ Right: ↑

x-intercepts:

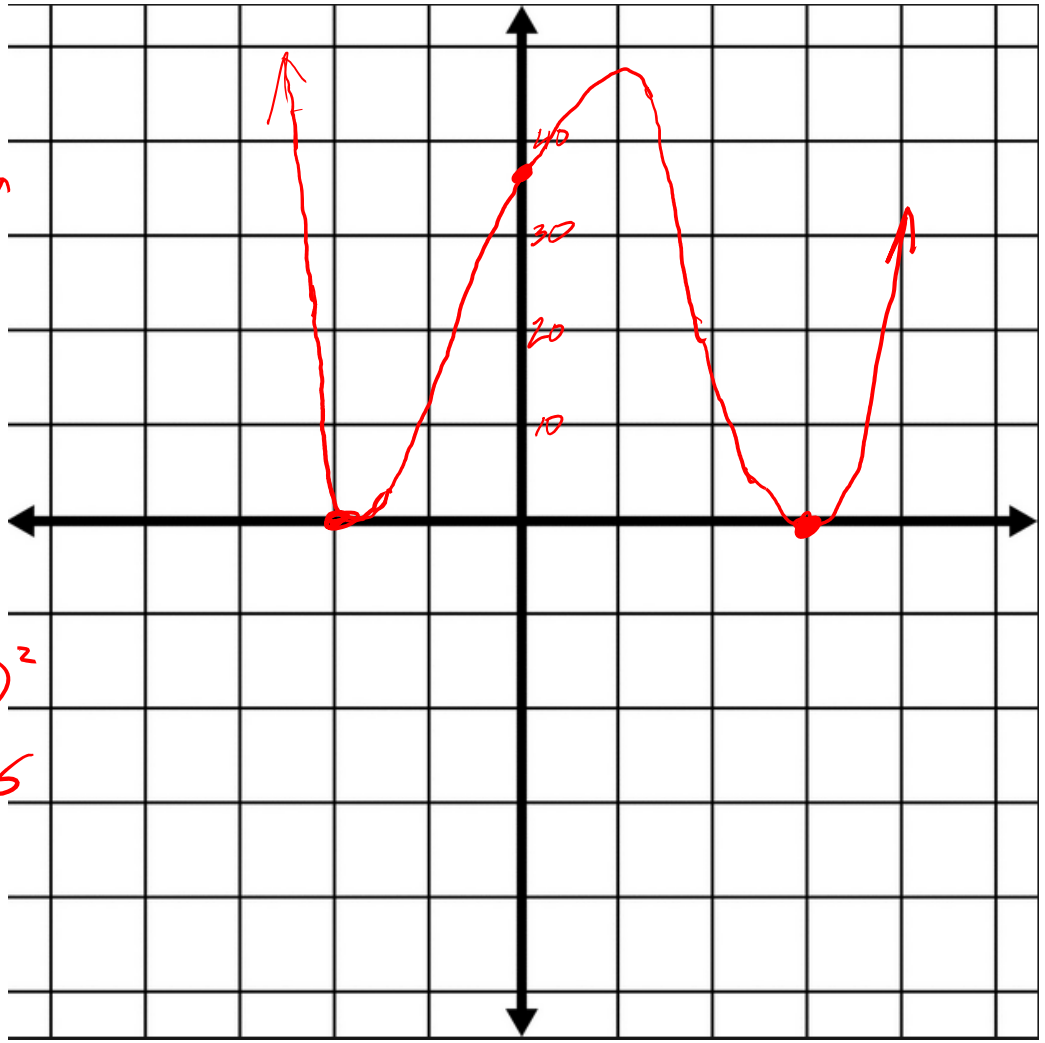
$-2x = 0$	$x+1 = 0$	$3-x = 0$
$x = 0$	$x = -1$	$-x = -3$
		$x = 3$
M: 1	M: 1	M: 3
(Linear)	(Linear)	(cubic)

y-intercept:

$P(0) = -2(0)(0+1)(3-0)^3 = 0$



2. **Sketch the graph of $P(x) = (x-3)^2(x+2)^2$**



Leading Term: $(x)^2(x)^2 = x^2 \cdot x^2 = x^4$

Degree: 4 End Behavior (Even, Pos): Left: ↑, Right: ↑

x-intercepts:

$x-3=0$ $\quad$ $x+2=0$

$x=3$ $\quad$ $x=-2$

M: 2 (Quadratic) $\quad$ M: 2 (Quadratic)

y-intercepts:

$P(0) = (0-3)^2(0+2)^2 = (-3)^2(2)^2$

$= 9 \cdot 4 = 36$

3. Find the quotient and remainder for $\dfrac{2x^3 - 13x^2 - 10x + 19}{2x + 3}$

Long Division

$x^2 - 8x + 7 \rightarrow Q(x)$

$2x+3 \;\overline{)\; 2x^3 - 13x^2 - 10x + 19}$

$\ominus 2x^3 \oplus 3x^2$ ↓

$-16x^2 - 10x + 19$

$\oplus 16x^2 \ominus 24x$ ↓

$14x + 19$

$\ominus 14x \oplus 21$

$-2 \rightarrow R(x)$

$Q(x) = x^2 - 8x + 7$

$R(x) = -2$

$x^2 - 8x + 7 - \dfrac{2}{2x+3}$

$Q(x)$; $\dfrac{R(x)}{D(x)}$

① $2x \cdot \square = 2x^3 \rightarrow x^2$

$x^2 \cdot (2x+3) \rightarrow 2x^3 + 3x^2$

subtract

② $2x \cdot \square = -16x^2 \rightarrow -8x$

$-8x \cdot (2x+3) \rightarrow -16x - 24x$

subtract

③ $2x \cdot \square = 14x \rightarrow 7$

$7 \cdot (2x+3) = 14x + 21$

subtract

④ Since degree of -2 is less than degree of $2x$, we are done.

4. Find the quotient and remainder for $\frac{x^3 - 2x + 12}{x - 4}$ $= \frac{x^3 + 0x^2 - 2x + 12}{x - 4}$ Synthetic Division

	1	0	−2	12
4	↓	4	16	56
	1	4	14	68

$Q(x)$ $R(x)$

$1x^2 + 4x + 14$

4 × 14
4 × (10 + 4)
40 + 16
56

$Q(x) = x^2 + 4x + 14$

$R(x) = 68$

$x^2 + 4x + 14 + \frac{68}{x-4}$

5. Find the zeros:

a. $P(x) = (x-2)^3(x^2-2x-8)$

$(x-2)^3(x-4)(x+2)$

$x-2=0$ $x=2$ M:3

$x-4=0$ $x=4$ M:1

$x+2=0$ $x=-2$ M:1

b. $P(x) = 4x^3+4x^2-x-1$

$4x^2(x+1)-1(x+1)$

$(x+1)(4x^2-1)$

$(x+1)(2x+1)(2x-1)$

$x+1=0$ $x=-1$ M:1

$2x+1=0$ $2x=-1$ $x=-1/2$ M:1

$2x-1=0$ $2x=1$ $x=1/2$ M:1

c. $P(x) = x^3+x^2+9x+9$

$x^2(x+1)+9(x+1)$

$(x+1)(x^2+9)$

$x+1=0$ $x=-1$ M:1

$x^2+9=0$ $x^2=-9$ $x=\pm\sqrt{-9}$ $x=\pm 3i$ M:1

6. **3rd degree polynomial with integer coefficient given 1, 6i and −6i with a constant coefficient of 72.**

zeros and

$x = 1$ $\quad x = 6i$ $\quad x = -6i$

$x - 1 = 0$ $\quad x - 6i = 0$ $\quad x + 6i = 0$

$P(x) = a(x-1)(x-6i)(x+6i)$

$P(x) = a(x-1)(x^2 + 6ix - 6ix - 36i^2)$

$+36$

$P(x) = a(x-1)(x^2+36)$

$P(x) = a(x^3 + 36x - x^2 - 36)$

$P(x) = a(x^3 - x^2 + 36x - 36)$

$P(x) = -2x^3 + 2x^2 - 72x + 72$

$-36 \times \boxed{-2} = 72$

$a = -2$

7. Use for questions a and b: $f(x) = \dfrac{x-4}{x+2}$

a. Find the x-intercept. (Numerator only)

$x - 4 = 0$

$x = 4$

$(4, 0)$

b. Find the y-intercept. Evaluate $f(0)$

$$f(0) = \frac{0-4}{0+2} = \frac{-4}{2} = -2$$

$(0, -2)$

8. Find the x and y intercepts, and horizontal asymptotes in the function:

vertical asymptotes, holes, sketch

$$f(x) = \frac{x^2 + x - 6}{2x^2 - 2x - 4} = \frac{(x+3)(x-2)}{2(x+1)(x-2)}$$

$2(x^2 - x - 2)$

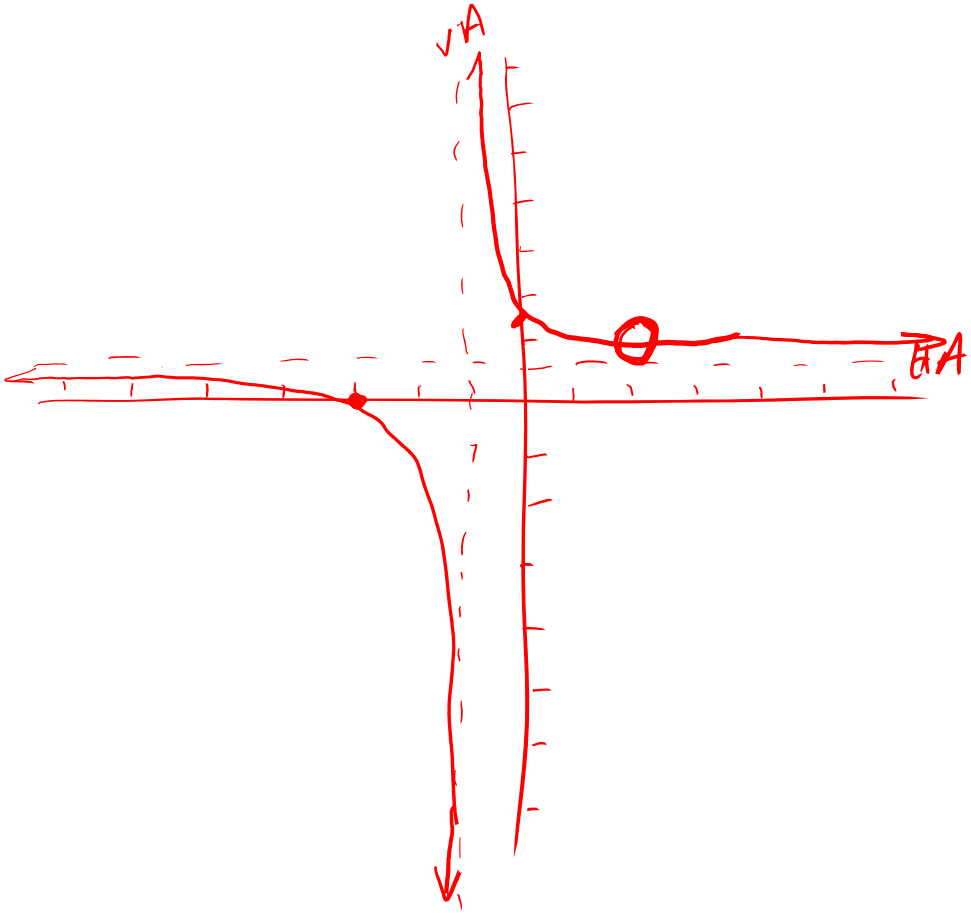
x-int: (Num, 0, by): $x+3=0 \rightarrow x=-3$

y-int: $f(0) = \frac{-6}{-4} = \frac{3}{2}$

Holes: (Num, Den): $x-2=0 \rightarrow x=2$

VA: (Den, 0, by) $x+1=0 \rightarrow x=-1$

HA: (compare Degrees) $\frac{\text{Deg}}{\text{Deg}} = \frac{1x^2}{2x^2}$ $y=\frac{1}{2}$



9. Find the vertical asymptote(s) and hole(s) for

$$f(x) = \frac{x^2 + 8x + 12}{x^2 + x - 30} = \frac{(x+2)(x+6)}{(x-5)(x+6)}$$

VA: $x - 5 = 0 \rightarrow x = 5$

Hole: $x + 6 = 0 \rightarrow x = -6$

10. State the following and clearly label the graph.

a. x-intercepts
b. hole(s)
c. y-intercepts
d. vertical asymptotes
e. horizontal asymptotes

$$f(x) = \frac{x-4}{x+2}.$$

a) $x-4=0 \rightarrow x=4$

b) No Hole

c) $f(0) = \frac{-4}{2} = -2$

d) $x+2=0 \rightarrow x=-2$

e) $\frac{\text{Deg}1}{\text{Deg}1} \rightarrow \frac{1x}{1x}$ $y=1$