MATH 1311

Section 4.1

Exponential Growth and Decay

As we saw in the previous chapter, functions are linear if adding or subtracting the same value will get you to different coordinate points.

Exponential functions involve multiplying the same value repeated times to obtain different coordinate points.

Exponential Growth

Exponential Growth occurs when a value is increasing by a predictable multiplier.

For example, the number of bacteria grown in a petri dish doubles every hour from an initial count of 3000 cells.

Hour	Number of bacteria
0	3000
1	$2 \times 3000 = 6000$
2	$2 \times 6000 = 12{,}000$
3	$2 \times 12{,}000 = 24{,}000$

Exponential Growth

Try to create a formula for this situation. Keep in mind, you are using repeated multiplications. *We will use N(t) as the number of bacteria cells and t to represent the time in hours.*

Hour	Number of bacteria	
0	3000	
1	$2 \times 3000 = 6000$	$\times 2$
2	$2 \times 6000 = 12{,}000$	$\times 2$
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$N(t)=3000 \times 2^t$

Identify the initial value and the multiplier (known as the base).

Exponential Decay:

Exponential Decay occurs when the base is a number between 0 and 1.

For example, in chemistry, radioactive decay is the when a radioactive element becomes stable. This happens at a predicable rate, known as a half-life.

If you have a 1000 grams of a certain radioactive isotope with a half-life of 1 year, create a table of the remaining amount of this substance for the first 5 years. Then create a formula.

Exponential Decay:

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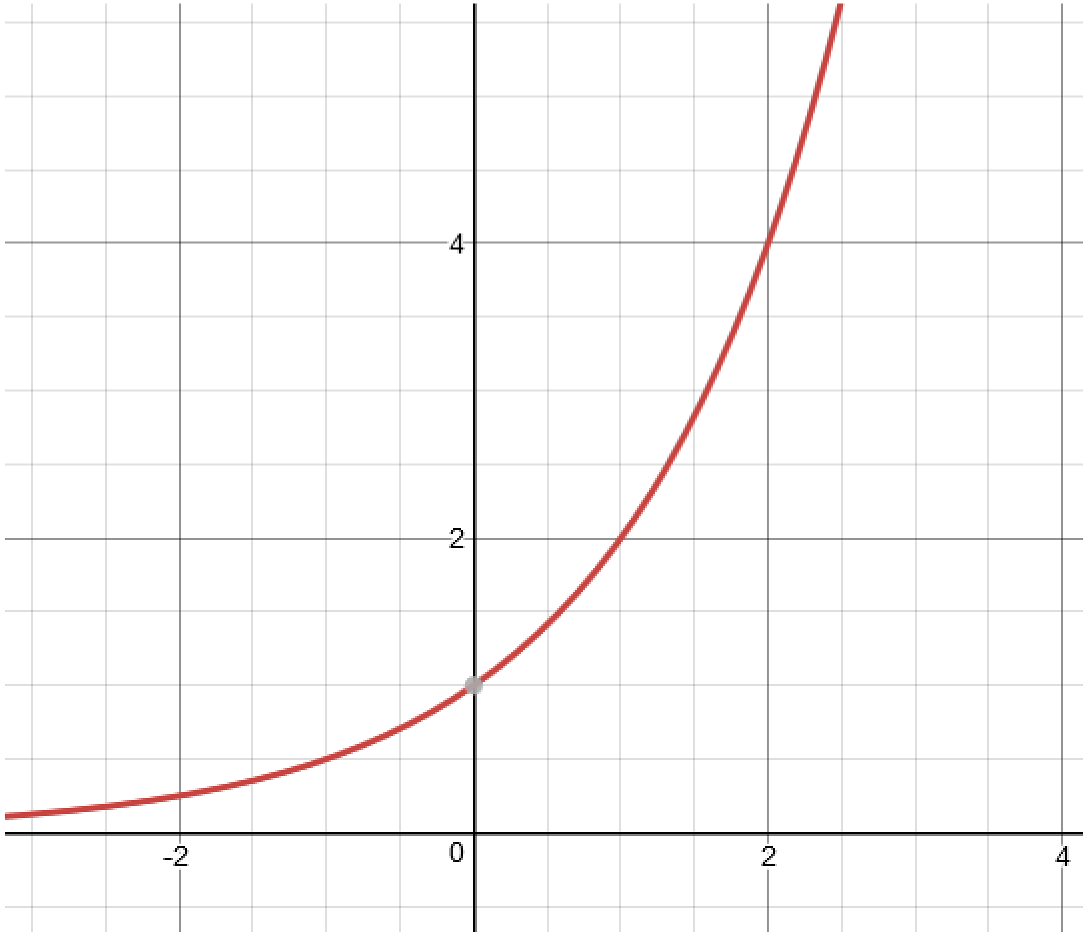
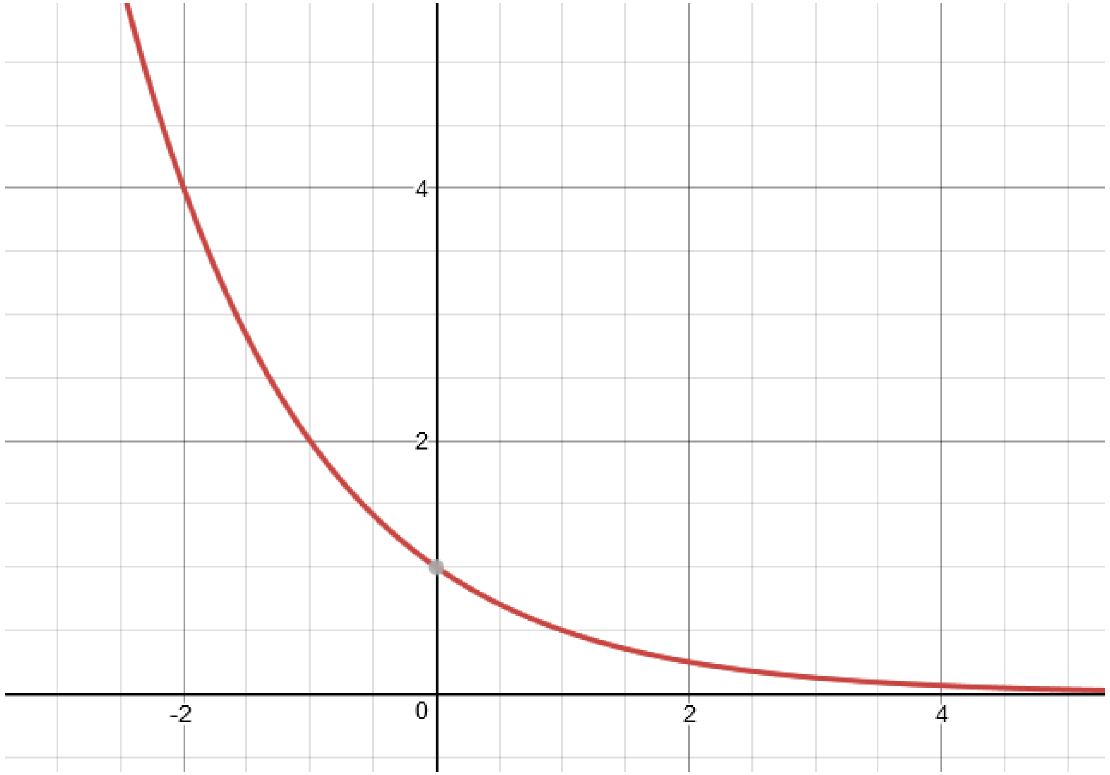
Time (years)	Amount of Substance (Grams)
0	
1	
2	
3	
4	
5	

Graphs of Exponential Functions





Properties of Exponential Functions

KEY IDEA 4.1 **EXPONENTIAL FUNCTIONS**

A function $N = N(t)$ is exponential with base a if N changes in constant multiples of a. That is, if t is increased by 1, the new value of N is found by multiplying by a.

1. The formula for an exponential function with base a and initial value P is
$$N = Pa^t.$$
2. If $a > 1$, then N shows exponential growth with *growth factor* a. The graph of N will be similar in shape to that in Figure 4.1.
3. If $a < 1$, then N shows exponential decay with *decay factor* a. The graph of N will be similar in shape to that in Figure 4.2. The limiting value of such a function is 0.

The population growth of a city is exponential. Currently the city has 30,000 people. It is estimated that it will increase in population by 5% annually. Determine a growth equation and find the population in 5 years.

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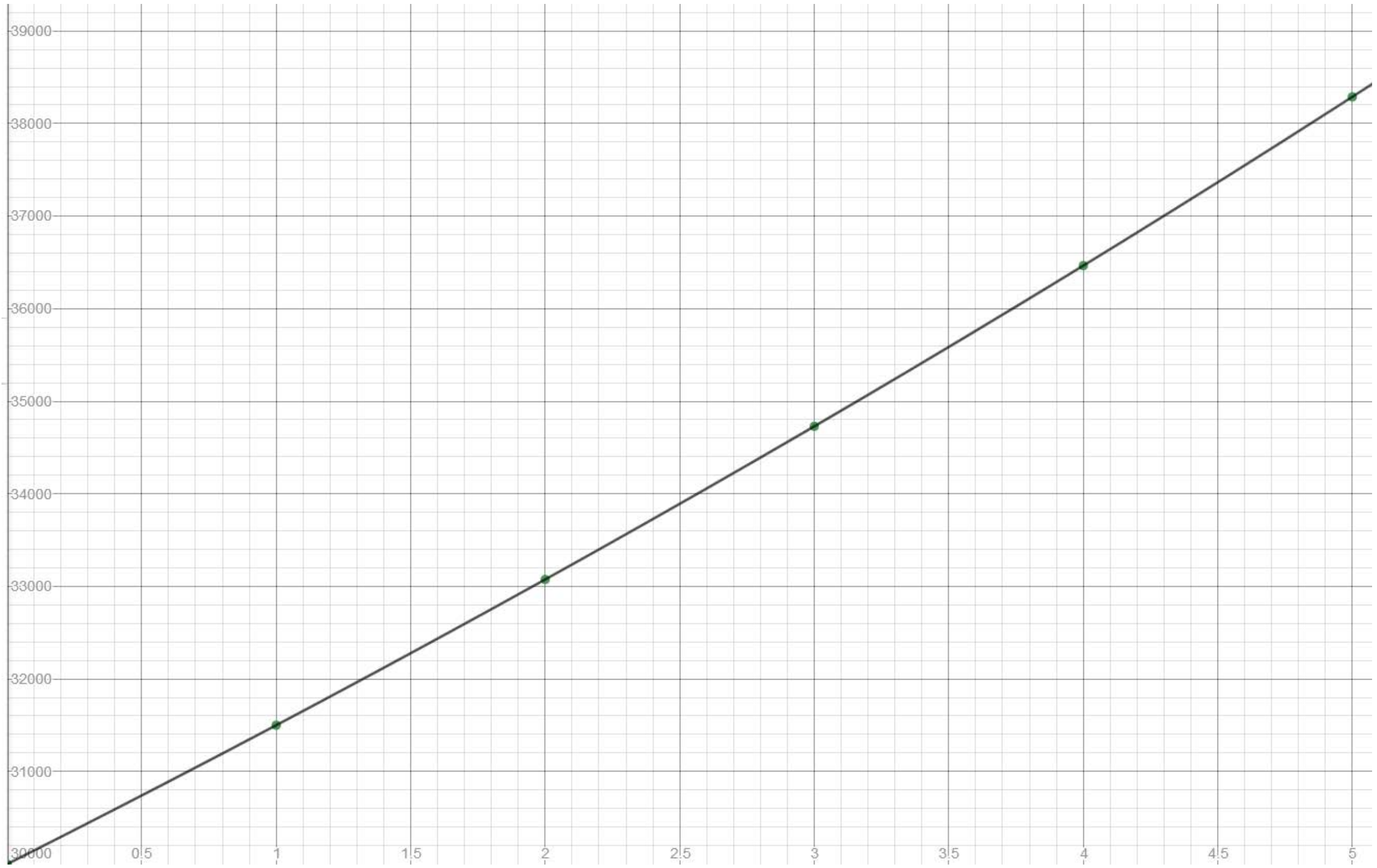
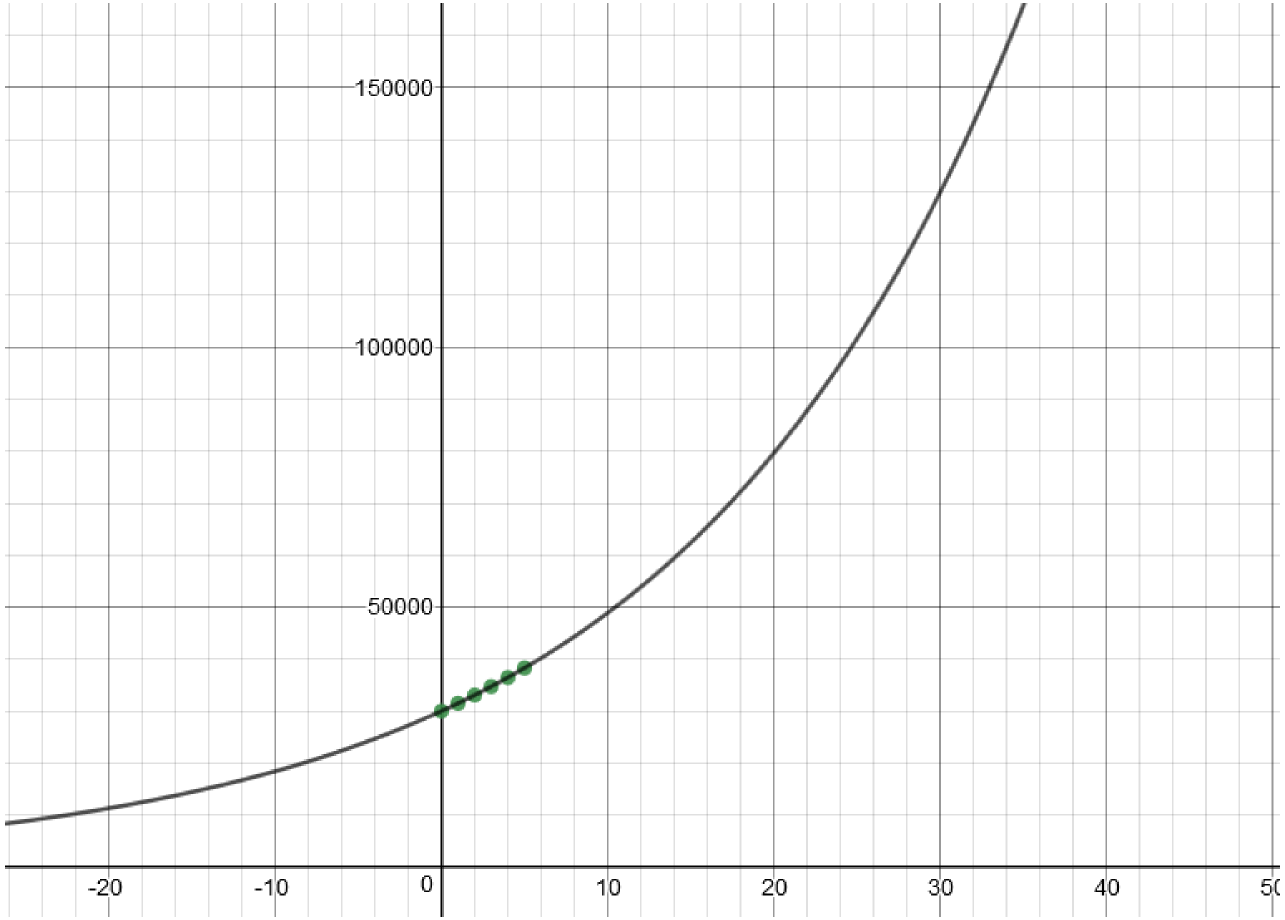
Create a table and a graph of this function.

Table and Graph

x_1	$30000 \cdot 1.05^{x_1}$
0	30000
1	31500
2	33075
3	34728.75
4	36465.188
5	38288.447



Graph (zoomed out)



The number of trees in a certain forest is decreasing at an exponential rate. The forest currently has 25,000 trees. By next year, it is expected to have 20,000 trees.

1. Is this exponential growth or decay?

2. What is the initial value?

3. What is the base value (the multiplier)?

4. What is the exponential function?

Complete the Table:

Year (t)	Number of Trees (N)
0	25000
1	20000
2	
3	
4	

Unit Conversion for Growth Factors:

Sometimes it is convenient to convert the growth or decay factor to a standard unit (such as 2 or ½) and have the non-integer value as the exponent.

For example: The half-life of plutonium is 81 million years. This means that if we have an initial value of 10000 grams, the table would look like:

Number of Half Lives	Amount (grams)
0	10000
1	5000
2	2500
3	1250
4	625

Time (Millions of Years)	Amount (grams)
0	10000
81	5000
162	2500
243	1250
324	625

How to convert:

Since it takes 81 million years to cut the initial value in half, rather than counting by number of half-lives, we can convert the exponent:

$n(h) = 10000 \times (½)^h$

$m(t) = 10000 \times (½)^t$

Where t is measured in millions of years, rather than number of half-lives. *When doing this, t = 1/(time frame).*

How to Convert Units:

KEY IDEA 4.2 UNIT CONVERSION FOR GROWTH FACTORS

If the growth or decay factor for 1 period of time is a, then the growth or decay factor A for k periods of time is given by

$$A = a^k.$$

This relationship may be illustrated with the following diagram.

$$\text{GROWTH FACTOR FOR ONE PERIOD OF TIME} \;\underset{\text{Raise to } 1/k\text{th Power}}{\overset{\text{Raise to } k\text{th Power}}{\rightleftarrows}}\; \text{GROWTH FACTOR FOR } k \text{ PERIODS OF TIME}$$

Try it out:

It is estimated that the alligators in a certain swamp will double every 15 years. Currently, the swamp has 250 alligators in it.

What is the initial value?

What is the base value?

What is the exponent?

Create the exponential formula.

Try it out:

It is estimated that the alligators in a certain swamp will double every 15 years. Currently, the swamp has 250 alligators in it.

Determine how many alligators are present after 10 years. *Round to the nearest whole number.*

Try this:

It is estimated that the number of bacteria in a sample (after administration of antibiotic) will decrease by 1/100 every minute. You want to create a function to show how it will decrease by the hour. The initial sample has 15000 bacteria cells.

What is the initial value?

What is the base value?

What is the exponent?

What is the exponential function?

Verify with tables:

x_2	$15000 \cdot \left(\frac{99}{100}\right)^{x_2}$
0	15000
1	14850
2	14701.5
3	14554.485
4	14408.94
5	14264.851

This table shows the decay in terms of minutes.

x_1	$15000 \cdot \left(\frac{99}{100}\right)^{60x_1}$
0	15000
$\frac{1}{60}$	14850
$\frac{2}{60}$	14701.5
$\frac{3}{60}$	14554.485
$\frac{4}{60}$	14408.94

This table shows the decay in terms of hours.

A sapling will grow new leaves at an exponential rate. It begins with 2 leaves, and that number is expected to triple every 6 weeks. You want to create a function that will explain the growth rate of the leaves in terms of single weeks.

1. What is the initial value?
2. What is the base value?
3. What is the exponent?
4. What is the exponential formula?

A sapling will grow new leaves at an exponential rate. It begins with 2 leaves, and that number is expected to triple every 6 weeks. You want to create a function that will explain the growth rate of the leaves in terms of single weeks.

5. What is the number of leaves after 6 weeks (using formula)?

6. What is the number of leaves in 12 weeks (using formula)?

7. Do your answers for #5 and #6 confirm the accuracy of your formula?