MATH 1311

Section 4.5

Logarithmic Functions

A logarithm is a function that is the inverse function of a exponential function. This means that it is the opposite function.

Anytime you are given a logarithm, you can translate it into an exponential:

$$\log_n a = b$$

can be rewritten as:

$$n^b = a$$

For example:

To evaluate:

$$\log_{10} 100$$

you can turn it into a exponential function:

$$\log_{10} 100 = x$$

$$10^x = 100$$

and you can see that the answer would be 2.

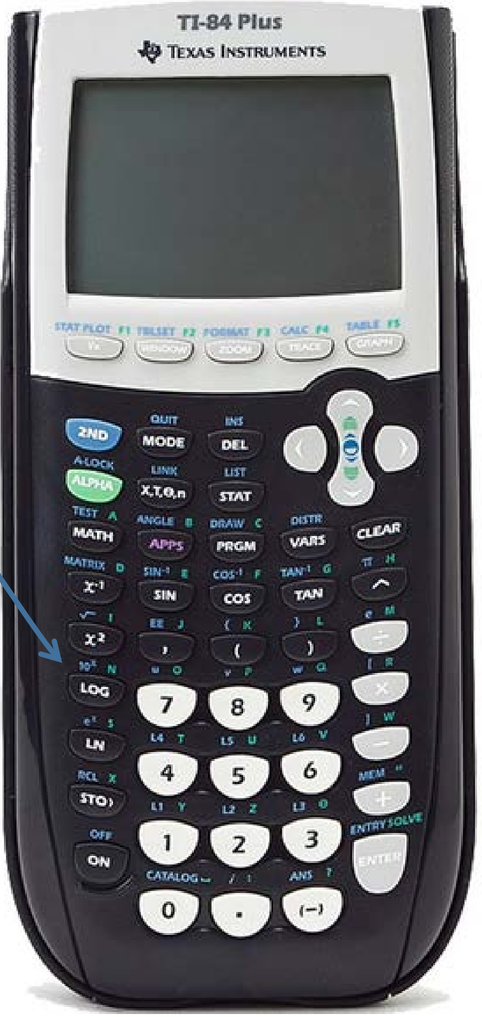
The common logarithmic function:

KEY IDEA 4.6 THE COMMON LOGARITHM

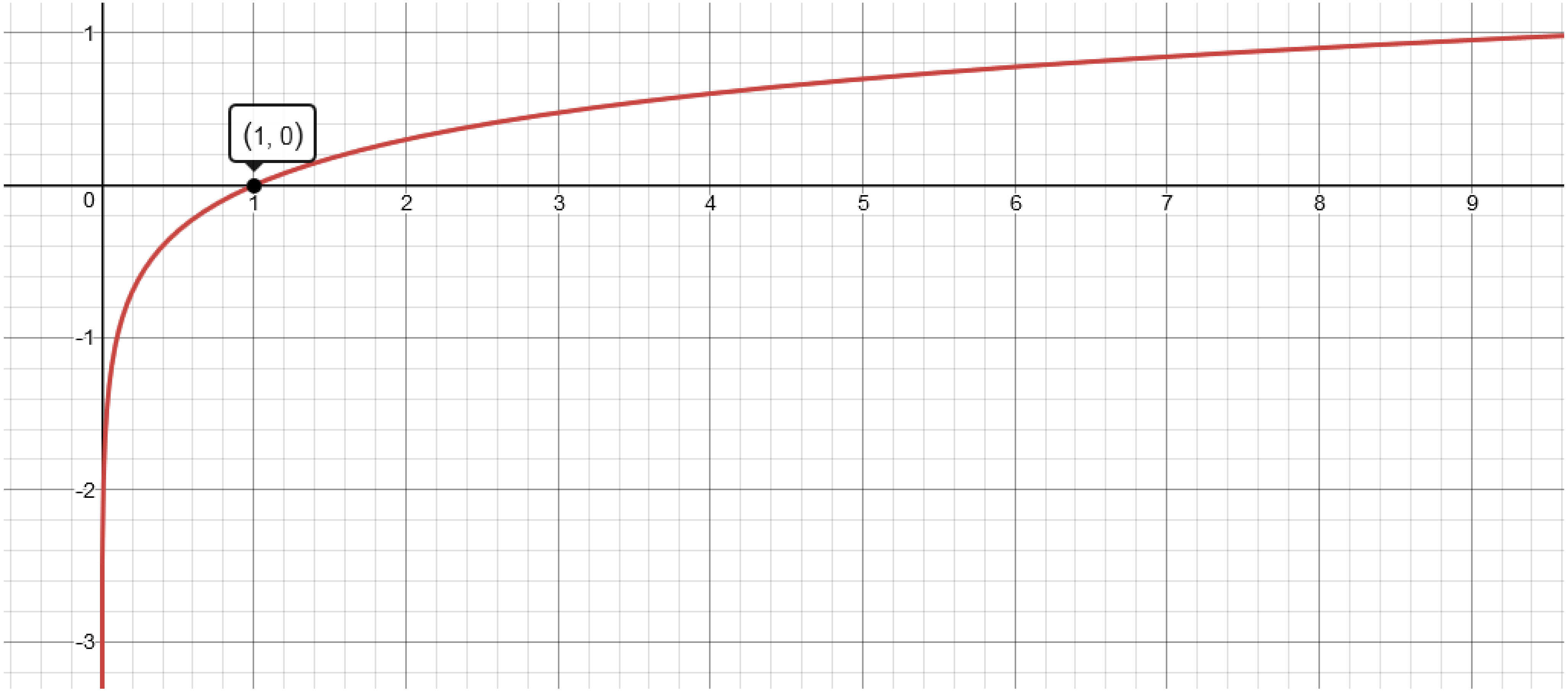
- The common logarithm, $\log x$, is the power of 10 that gives x. Alternatively, $\log x$ is the solution for t of the equation $10^t = x$.
- If we multiply a number by 10^t, the logarithm is increased by t units.
- The function $\log x$ increases very slowly, and its graph is concave down.

Evaluate logarithms (with base 10).

LOG



Graph of y = log x



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$$7500 = 5000 \cdot 10^{t/12}$$

$$1.5 = 10^{t/12}$$

$$\log_{10} 1.5 = \frac{t}{12}$$

$$12 \cdot \log_{10} 1.5 = t$$

$$t \approx 2.113$$

Solving for a logarithm:

To solve a logarithm, you can *exponentiate* the function. This means rewrite the function as exponents to the same base.

$$\log(x - 1) = 0.23$$

unless otherwise written, log is base 10

$$10^{\log(x-1)} = 10^{0.23}$$

$$x - 1 = 10^{0.23}$$

$$x = 1 + 10^{0.23} \approx 2.698$$

Example:

According to research the number of brain cells in a newborn increases at a logarithmic rate, given by the function:

$$b(t) = \log(t + 0.01) + 4$$

Where t is measured in years and b(t) is measured in billions of brain cells.

Determine the age at which the child will reach 4 billion brain cells.

Determine the age at which the child will reach 5 billion brain cells.

Natural Logarithms:

While common logarithms are always considered to be Base 10, Natural Logarithms are always considered to be Base e, for the irrational number e ≈ 2.71828…

$$\log_{10} x = \log x$$

$$\log_e x = \ln x$$

Note: All other base values, must be written out!

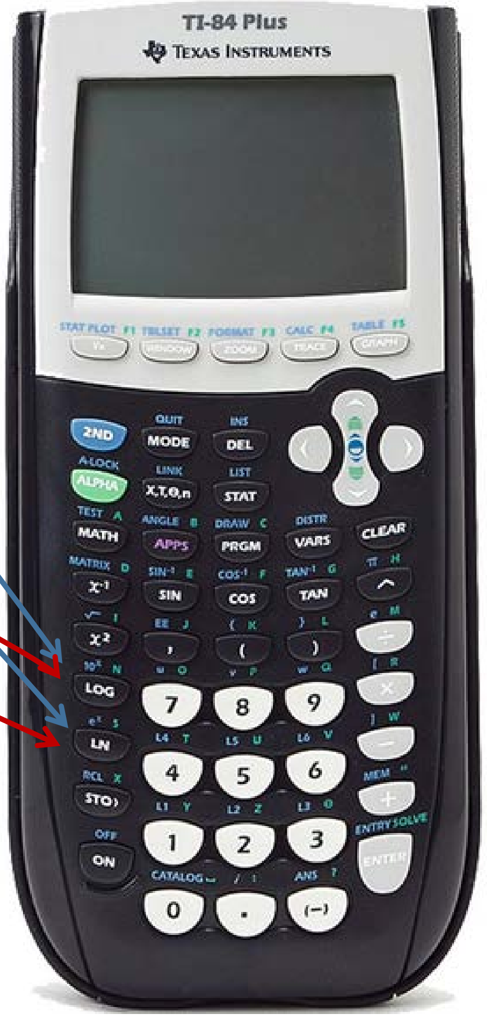
On the Calculator:

log (common base): LOG

ln(base e): LN

10^x: 2^{nd} LOG

e^x: 2^{nd} LN



The spread of a rumor in a school is modelled by the following equation:

$$r(t) = 3\ln(t+1) + 1$$

where t is measured in minutes and r(t) is number of people having heard the rumor.

1. What is the initial value?

2. How many people would have hear the rumor after 10 minutes?

3. How long will it take the rumor to be heard by 5 people?

The number of jellyfish in a portion of ocean water is increasing at a rate modelled by the formula:

$$j(t) = 2 \cdot 10^t$$

where t is measured in minutes and j(t) is measured in number of jellyfish.

4. What is the initial value of the function?

5. How long will it take for the initial value to double?