MATH 1311

Section 5.1

Logistic Functions

Logistical Functions are functions which level out to two different values. Often these are used to represent long-term population growth.

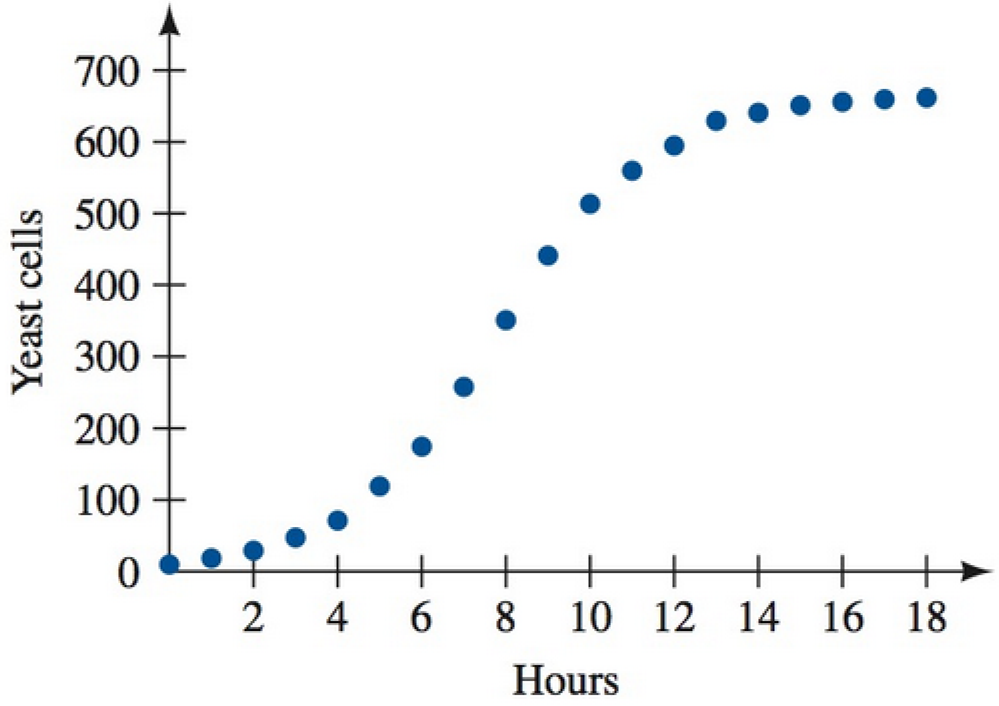


FIGURE 5.1 Population growth data for yeast

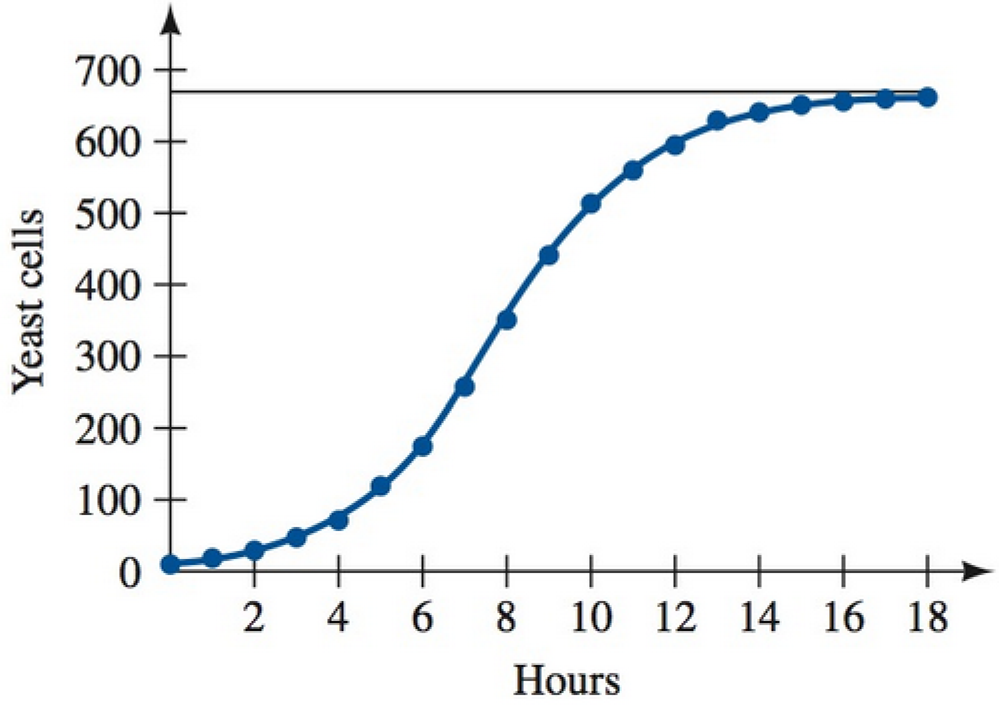


FIGURE 5.2 Yeast data fitted with a logistic curve

Key Features:

There is initial rapid growth:

The graph takes on the appearance of an exponential function for small x-values.

Point of Inflection:

The graph will change from concave up to concave down.

Leveling out:

The graph will level out to a different value than what it began at.

KEY IDEA 5.1 LOGISTIC CURVE

The logistic growth curve has the following properties:

- Initially the growth is rapid, nearly exponential.
- The inflection point represents the time of most rapid growth.
- After the inflection point, the growth rate declines. The function has a limiting value, known as the *carrying capacity*.
- The point of inflection occurs at half of the carrying capacity. This is the level of maximum growth. In the case of harvested populations, this level is known as the *optimum yield level*.

The Logistic Formula:

KEY IDEA 5.2 LOGISTIC MODEL

The formula for a logistic model is

$$N = \frac{K}{1 + be^{-rt}}.$$

- The constant K is the carrying capacity. It is the limiting value of N. The point of inflection occurs at $N = K/2$.
- The constant b is determined by $b = \frac{K}{N(0)} - 1$.
- The r value is the intrinsic exponential growth rate. In the absence of limiting factors, growth would be exponential according to the formula $N = N(0)e^{rt}$. The corresponding growth factor for this exponential function is $a = e^r$, so we can find r from a using $r = \ln a$.

The number of plant buds in a flower pot is modelled by the logistic function, where t is measured in months:

$$b(t) = \frac{250}{1 + 4e^{-.2t}}$$

How many buds are initially in the pot?

What will the carrying capacity be of this pot?

What would be the growth rate per month?

At what time will the monthly growth be the greatest?

Fitting Logistic Data to a model:

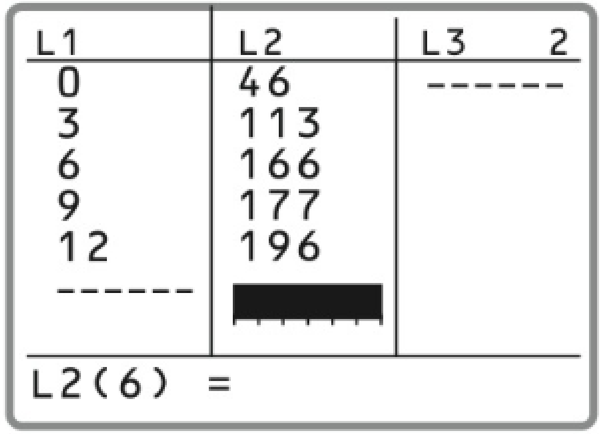
Just like before, after entering data into L1 and L2 (STAT → Edit), go to the STAT → Calc → LogisticReg and press ENTER.

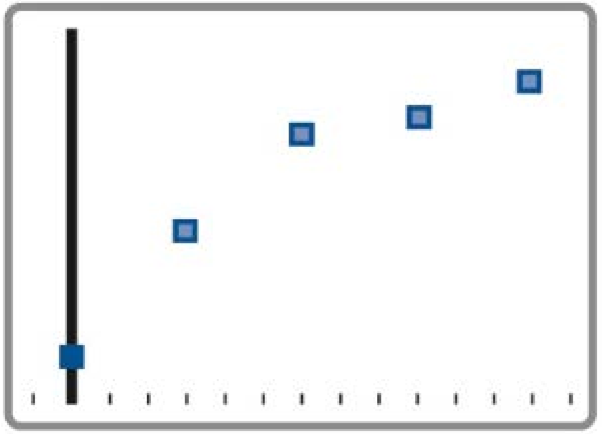
The accompanying table shows the number, in millions, of adults in the United States who had Internet access (home, work, or other) in the given year. The data are taken from the *Statistical Abstract of the United States*.

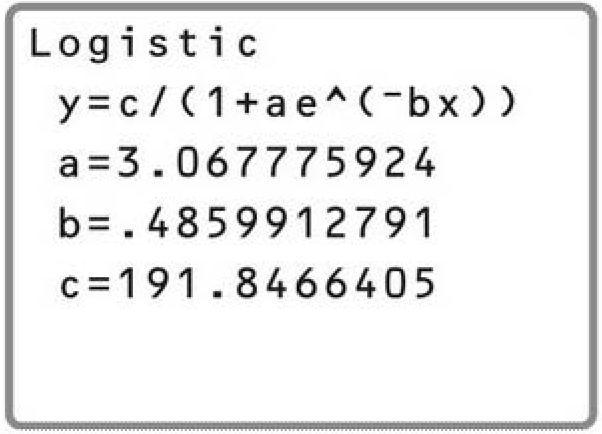
Year	1997	2000	2003	2006	2009
Number, in millions	46	113	166	177	196

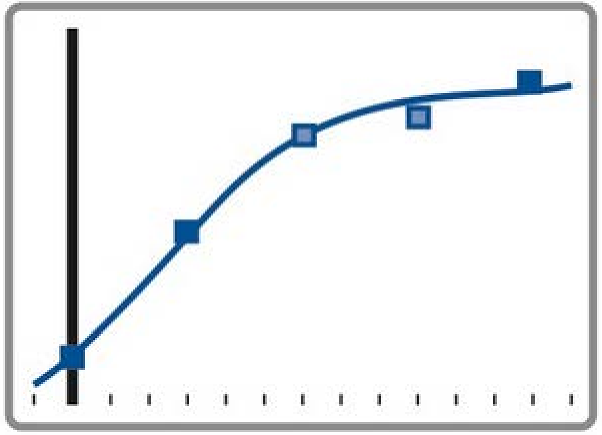
Convert the Independent Variable to years from 1997.

Data, Scatter Plot, Regression and Graph









The following table shows the population of a certain town from 1950.

time(years)	0	10	20	30	40	50
Population (thousands)	7	32.61085	34.9531	34.99914	34.99998	35

1. What is the population of the town in 1950?

2. Use Regression to find the equation of the Logistic Function:

3. What is the carrying capacity of the town?

4. When does the town obtain its largest growth rate?

5. What is the annual growth rate?