

Math 1314

Lesson 11: Exponential Functions as Mathematical Models

Functions whose equations contain a variable in the exponent are called **exponential functions**.

Uninhibited Exponential Growth

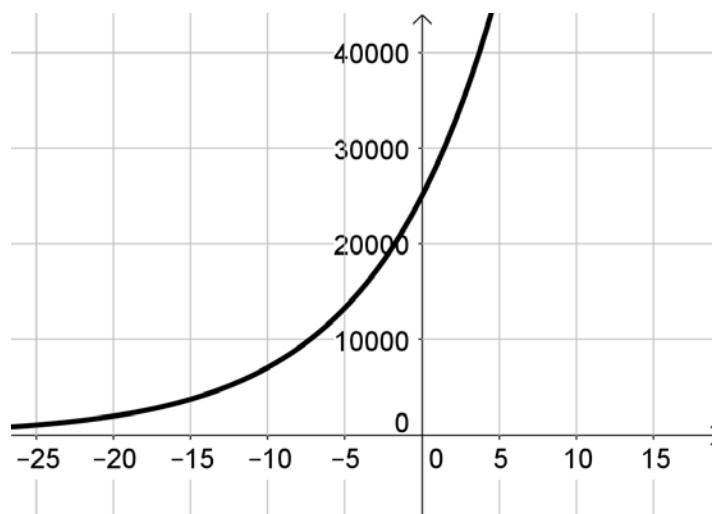
Some common exponential applications model **uninhibited exponential growth**. This means that there is no “upper limit” on the value of the function. It can simply keep growing and growing. Problems of this type include population growth problems and growth of investment assets.

- The function $f(x) = a \cdot e^{bx}$ is an **exponential growth function**.
- For a function of the form $g(x) = a \cdot b^x$, the function is an **exponential growth function** if $b > 1$.
- These type of exponential functions are always increasing.

In both equations the variable a is called the **initial amount** and b is called the **growth constant**.

Example 1: The function $f(x) = 25000e^{0.1263x}$ describes the bacteria population in a culture x hours after an experiment begins.

Enter the function into GGB.



a. How many bacteria were initially present?

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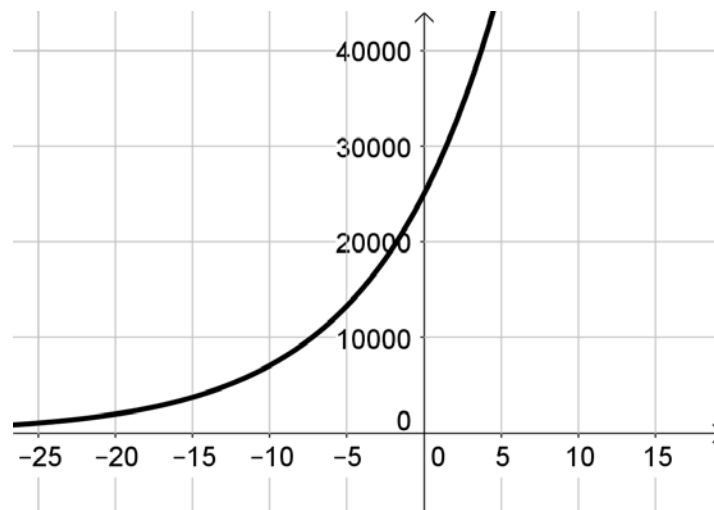
Answer:

b. How many bacteria would be present in the culture 10 hours after beginning the experiment?
Command: Answer:

c. How quickly is the bacteria growing 20 hours after the start of the experiment?
Command: Answer:

d. How long after the start of the experiment did the bacteria population reach 35,000?

Command: Answer:



Example 2: At the beginning of a population study, the population of an East Texas county was 167,000. Two years later, the population was 175,000. Assume the population grows exponentially.

a. How fast is the population of the county increasing 6 years after the start of the study?

Command:

Answer:

b. How long after the start of the study will the population be twice as much as when the study began?

Command:

Answer:

Exponential Decay

- The function $f(x) = a \cdot e^{-bx}$ is an **exponential decay function**.
- For a function of the form $g(x) = a \cdot b^x$, the function is an **exponential decay function** if $0 < b < 1$.
- These type of exponential functions are always decreasing.

In both equations the variable a is called the **initial amount** and b is called the **decay constant**.

Exponential decay problems frequently involve the half-life of a substance. The **half-life** of a substance is the time it takes to reduce the amount of the substance by one-half.

Example 3: A certain medicine has a half-life of 6 hours. Suppose you take a dose of 600 milligrams of the medicine at noon.

a. How quickly is the medication decreasing in your bloodstream after 8 hours?

Command:

Answer:

d. At what time will you have 100 mg remaining in your bloodstream?

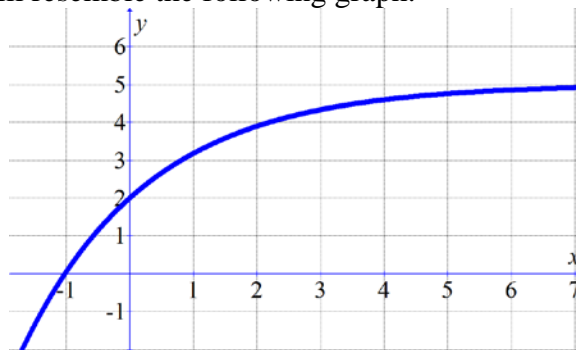
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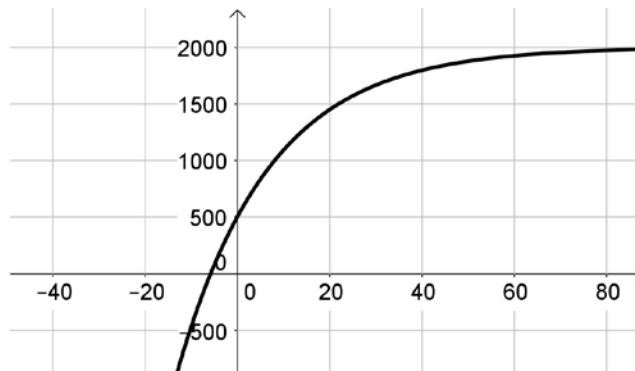
Limited Growth Models

A worker on an assembly line performs the same task repeatedly throughout the workday. With experience, the worker will perform at or near an optimal level. However, when first learning to do the task, the worker's productivity will be much lower. During these early experiences, the worker's productivity will increase dramatically. Then, once the worker is thoroughly familiar with the task, there will be little change to his/her productivity.

The function that models this situation will have the form $Q(t) = C - Ae^{-kt}$ and their graphs called **learning curves** will resemble the following graph.



Example 4: The monthly demand for a certain computer t months after placing the line on the market was given by $D(t) = 2000 - 1500e^{-0.05t}$



a. How many units should an experienced worker be able to assemble?

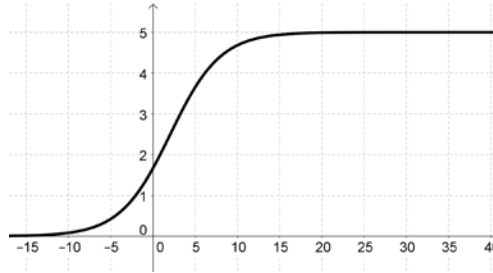
b. At what rate is an employee's productivity changing 4 months after starting to work?

Command:

Answer:

Logistic Functions

The last growth model that we will consider involves the logistic function. The general form of the equation is $Q(t) = \frac{A}{1 + Be^{-kt}}$ and the graph looks something like this:



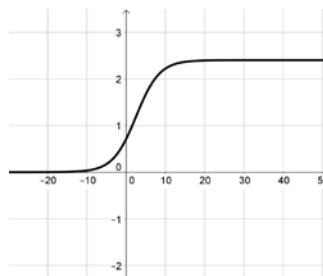
Logistic functions have some of the features of both exponential models and limited growth models.

Note that the graph above has a limiting value at $y = 5$. In the context of a logistic function, this asymptote is called the **carrying capacity** (maximum number). In general the carrying capacity is A from the formula above: $Q(t) = \frac{A}{1 + Be^{-kt}}$.

Logistic curves are used to model various types of phenomena and other physical situations such as population management. Suppose a number of animals are introduced into a protected game reserve, with the expectation that the population will grow. Various factors will work together to keep the population from growing exponentially (in an uninhibited manner). The natural resources (food, water, protection) may not exist to support a population that gets larger without bound. Often such populations grow according to a logistic model.

Example 5: A population study was commissioned to determine the growth rate of the fish population in a certain area of the Pacific Northwest. The function given below models the population where t is measured in years and N is measured in millions of tons. *Enter the function in GGB.*

$$N(t) = \frac{2.4}{1 + 2.39e^{-0.338t}}$$



a. What is the maximum number of fish present, using this model?

Command:

Answer:

c. What is the fish population after 3 years?

Command:

Answer:

d. How fast is the fish population changing after 2 years?

Command:

Answer: