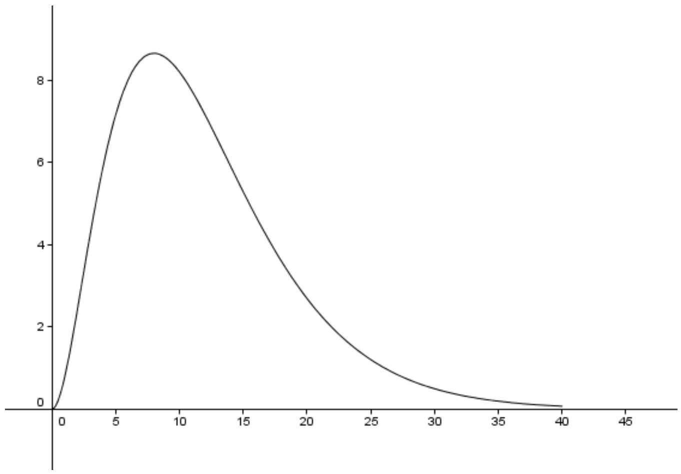
Math 1314
Lesson 12
Curve Analysis (Polynomials)

This lesson will cover analyzing polynomial functions using GeoGebra.

Suppose your company embarked on a new marketing campaign and was able to track sales based on it. The graph below gives the number of sales in thousands shown *t* days after the campaign began.



Now suppose you are assigned to analyze this information. We can use calculus to answer the following questions:

When are sales increasing or decreasing?
What is the maximum number of sales in the given time period?
Where does the growth rate change?
Etc.

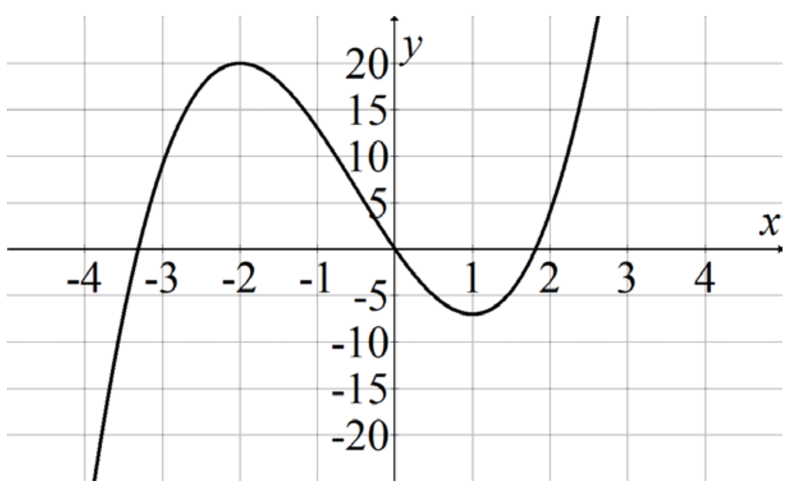
Calculus can't answer the "why" questions, but it can give you some information you need to start that inquiry.

Intervals on Which a Function is Increasing/Decreasing

A function is **increasing** on an interval (*a*, *b*) if, for any two numbers x_1 and x_2 in (*a*, *b*), $f(x_1) < f(x_2)$, whenever $x_1 < x_2$. A function is **decreasing** on an interval (*a*, *b*) if, for any two numbers x_1 and x_2 in (*a*, *b*), $f(x_1) > f(x_2)$, whenever $x_1 < x_2$.

In other words, if the *y* values are getting bigger as we move from left to right across the graph of the function, the function is increasing. If they are getting smaller, then the function is decreasing. We will state intervals of increase/decrease using interval notation. The interval notation will consists of corresponding *x*-values wherever *y*-values are getting bigger/smaller.

Example 1: Given the following graph of a function, state the intervals on which the function is:
a. increasing. b. decreasing.

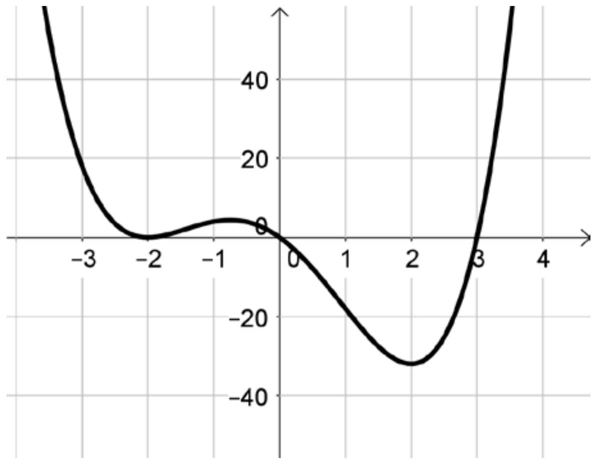


Do you notice WHERE increasing/decreasing CHANGES?

A relative maximum or a relative minimum can only occur at a **critical number**.

The **critical numbers** of a polynomial function are all values of *x* that are in the domain of *f* where $f'(x) = 0$ (the tangent line to the curve is horizontal).

Example 2: Find any critical numbers of $f(x) = x^4 + x^3 - 8x^2 - 12x$.
Enter the function into GGB.



Command: Answer:

We can use calculus to determine where a function changes from increasing to decreasing or from decreasing to increasing, this occurs at its **critical numbers**.

We know relative extrema can only occur at critical numbers so:

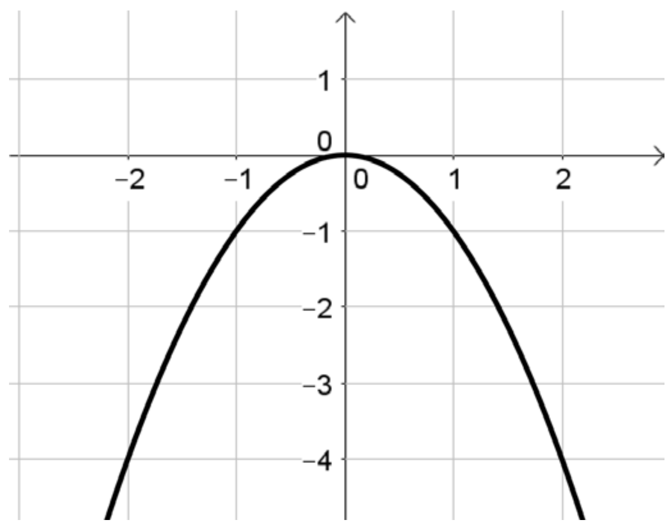
- A function has a relative maximum if it changes from increasing to decreasing.
- A function has a relative minimum if it changes from decreasing to increasing.

Example 3: The graph below is the graph of a polynomial function *f*. Examine the graph and state:

a. where the slope of the tangent line would be zero.

b. over which intervals the slopes of the tangent lines would be positive.

c. over which intervals the slopes of the tangent lines would be negative.



After answering parts b and c, do you notice where the orignal function is increasing/decreasing?

We can then make the following conclusions:

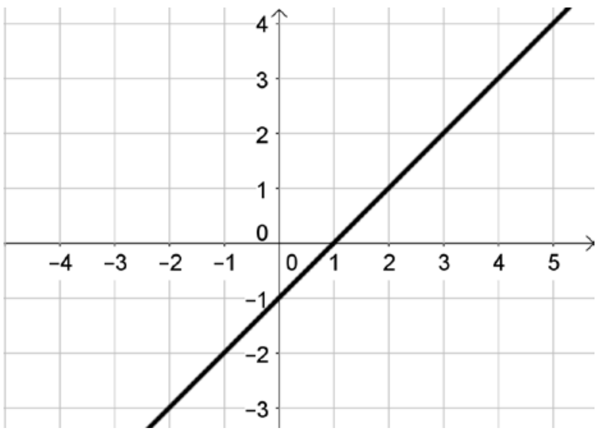
If	**i.e.**	**Then**
slopes of the tangent lines are positive	$f' > 0$	f is increasing
slopes of the tangent lines are negative	$f' < 0$	f **is decreasing**

Finding Intervals of Increase/Decrease Given the Graph of the First Derivative of a Function

Given the ***graph of the first derivative*** of a function:

1. Find any critical numbers.
2. If the graph of the first derivative is above the x-axis, that is where the orignial funciton is increasing.
3. If the graph of the first derivative is below the x-axis, that is where the orignial funciton is decreasing.

Example 4: The graph given below is the *first derivative* of a function, *f*. Find any critical numbers, intervals of increase/decrease and any x-values of relative extremum of the function *f*.



Critical Numbers:

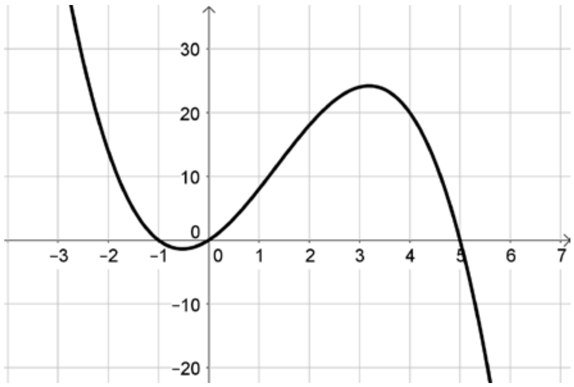
Increasing:

Decreasing:

Relative Maximum(s) occur at:

Relative Minimum(s) occur at:

Example 5: The graph given below is the *first derivative* of a function, f. Find any critical numbers, intervals of increase/decrease and any x-values of relative extremum of the function f.



Critical Numbers:

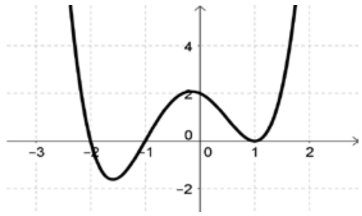
Increasing:

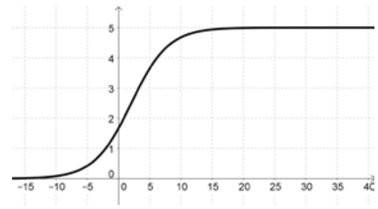
Decreasing:

Relative Maximum(s) occur at:

Relative Minimum(s) occur at:

In business, for example, the first derivative might tell us that our sales are increasing, but the second derivative will tell us if the pace of the increase or decrease is increasing or decreasing.





From these graphs, you can see that the shape of the curve change differs depending on whether the slopes of tangent lines are increasing or decreasing. This is the idea of **concavity**.

A point in the domain of the function where concavity changes is called a ***point of inflection***.

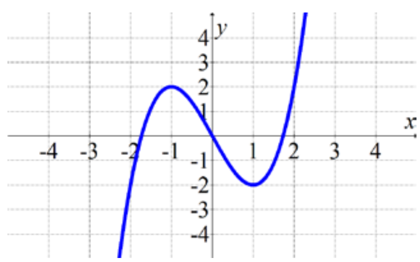
GGB command is: inflectionpoint[<polynomial>]

We will state intervals of concavity using interval notation. The interval notation will consists of corresponding x-values wherever the function is concave up/down.

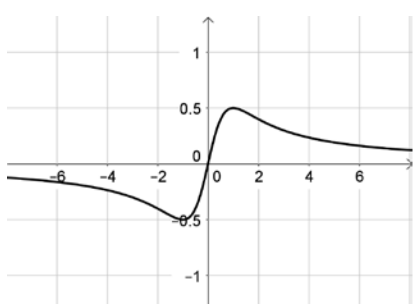
Example 6: The graph given below is the graph of a function *f*. Determine the interval(s) on which the function is:

a. concave upward.

b. concave downward.



Example 7: The graph given below is the graph of a function, state whether each of the statements below is true or false.

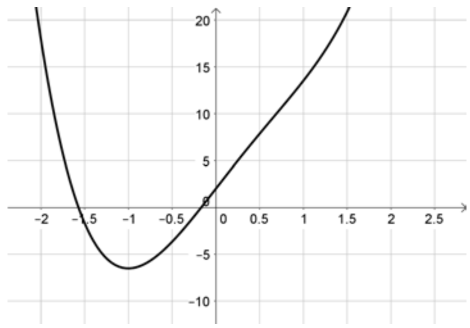


I. The function is concave up over two intervals and concave down over two intervals.

II. The function has three points of inflection.

Example 8: Find any intervals of concavity and inflection points of $f(x)=\frac{3}{2}x^4-2x^3+12x+2$.

Enter the function into GGB.



Command: Answer:

Concave up: Concave down:

Points of Inflection:

Since we know that concavity is about if the pace of the increase/decrease is increasing or decreasing (rate of change of a rate of change), then the second derivative gives us information about the concavity of the original function.

If	**Then**
$f'' > 0$	f is concave up
$f'' < 0$	f is concave down

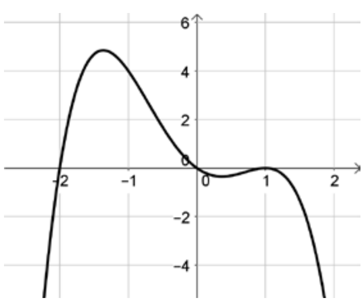
We can find concavity intervals by analyzing the *second derivative* of the function. The analysis is very similar to the method we used to find increasing/decreasing intervals.

Finding Intervals of Concavity Given the Graph of the Second Derivative of a Function

Given the ***graph of the second derivative*** of a function:

1. Find when $f''(x) = 0$.
2. If the graph of the second derivative is above the x-axis, that is where the orignial fucniton is concave up.
3. If the graph of the second derivative is below the x-axis, that is where the orignial fucniton is concave down.

Example 9: The graph below is the graph of the second derivative of a function *f*. Determine where *f* is concave up/down and x-values of any points of inflection.



Concave up:

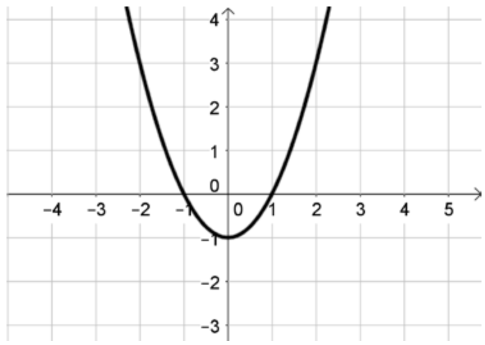
Concave down:

x-values of any points of inflection:

We may also find intervals of concavity by simply using the first derivative.

Fact: The function f is concave upward on the interval(s) where f' is increasing. The function f is concave downward on the interval(s) where f' is decreasing.

Example 10: The graph below is the graph of the first derivative of a polynomial function f.



a. Find the interval(s) on which the function f is concave upward.

b. Find the interval(s) on which the function f is concave downward.

c. Find the x coordinate of any inflection points of the function f.

d. Use the sign chart above and the sign chart for increasing/decreasing to sketch a possible original graph of function f.